IES Institute of Education Sciences

Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades

Practice Guide Summary

What Works Clearinghouse™

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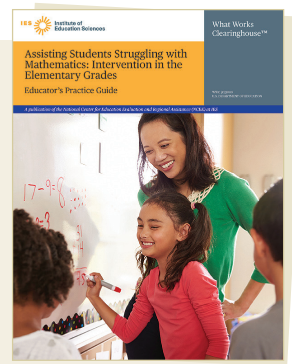
According to the Nation's Report Card, test scores in mathematics have remained stagnant over time for elementary students who scored in the lowest 25th percentile. These students now trail further behind their peers than they have in over a decade as the gap between the highest and lowest performing students on the National Assessment of Educational Progress (NAEP) math assessment has widened.

Recent research has identified interventions that improved achievement for students with low test scores in mathematics. The Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades practice guide, developed by the What Works Clearinghouse™ (WWC) in conjunction with an expert panel, distills this recent research into six easily comprehensible and practical recommendations for educators to use when teaching elementary students with low test scores in mathematics. This practice guide focuses on practices and principles underlying effective small-group interventions for students in grades K-6.

This summary introduces the six recommendations described in the full practice guide and the supporting evidence for each recommendation. These recommendations are designed to help educators, including mathematics general and special education educators, mathematics specialists and coaches; school, district, and state personnel; and parents.

For a full description of the recommendations and more implementation tips, including the panel's guidance for addressing potential obstacles or roadblocks to implementation, download your free copy of the guide.



Recommendations in this practice guide:

1. Provide systematic instruction during intervention to develop student understanding of mathematical ideas.
2. Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts.
3. Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.
4. Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics.
5. Provide deliberate instruction on word problems to deepen students' mathematical understanding and support their capacity to apply mathematical ideas.
6. Regularly include timed activities as one way to build fluency in mathematics.

Recommendation 1: Systematic Instruction

Provide systematic instruction during intervention to develop student understanding of mathematical ideas.

Effective interventions for improving mathematics achievement for students struggling with mathematics share one key feature: the design of the curricular materials and the instruction provided are systematic. The term systematic indicates that instructional elements intentionally build students' knowledge over time toward an identified learning outcome(s). Systematic intervention materials are designed to cover topics in an incremental and intentional way. Systematic interventions most often include a "bundle" of practices used to build and support student learning strategically.

How to carry out the recommendation

1. **Review and integrate previously learned content throughout intervention to ensure that students maintain understanding of concepts and procedures.** The panel recommends that interventions include systematic review of content by including a mix of previously and newly learned material within and across lessons. Review previously taught material before introducing new, related content. Help students understand the link between the previous content and the new content. To avoid having students overgeneralize new concepts or procedures to previously learned material, regularly present a variety of problems that require students to discriminate among problem types.

   Mathematical ideas are complex, and virtually all learners need to use, discuss, and explain the ideas multiple times over a long period of time to understand them. Provide students with opportunities to use and explain previous or newly learned mathematics concepts or procedures.

2. **When introducing new concepts and procedures, use accessible numbers to support learning.** When teaching a new concept or procedure, use single-digit or easy-to-process numbers so that students can focus on the new concept or procedure rather than on difficult calculations. For example, when teaching students to find equivalent fractions, first work on equivalencies to unit fractions. Start with fractions equivalent to one-half, one-third, and one-fourth that are familiar and accessible to students. When students have a grasp of the concept, systematically add other fraction types to prevent students from overgeneralizing that equivalencies are only applicable to unit fractions.

3. **Sequence instruction so that the mathematics students are learning builds incrementally.** Present mathematics concepts in a cohesive and logical way. Introduce concepts strategically so that the new learning relates to concepts previously taught. An intentional sequence of instruction capitalizes on prior learning and ensures that students have the knowledge necessary to learn new content effectively.

   Focus lessons on smaller tasks needed to solve complex problems before pulling it all together. This may apply to highly procedural multi-digit computation problems, or when teaching students to solve word problems. In the view of the panel, the key to building knowledge in this incremental way is to help students become comfortable with simpler subtasks of problem solving so they can eventually connect them to solve complex problems. For example, focus students on simpler tasks by using worked-out examples. Then exclude steps in a worked-out example and ask students to provide those steps until they become more comfortable with the procedures in solving problems.

4. **Provide visual and verbal supports.** Verbal supports may include teacher prompting or questioning to help students remember the connections between prior learning and new concepts. These verbal supports may be accompanied by a visual which could include a gesture or a concrete or semi-concrete representation. For example, when teaching division, a teacher might gesture using a motion like a dealer dealing out cards to show the action of divvying concrete items (such as cards) into equal groups. A visual may also include a picture or diagram to be used as a "hint" for a next step or as a reminder to think about a certain concept.

5. **Provide immediate, supportive feedback to students to address any misunderstandings.** If students are not able to explain their understanding of key mathematical concepts or do not execute procedures correctly, provide them with immediate feedback. When students solve problems, encourage them to articulate their thinking so that you can identify their strengths. Ask probing questions to identify any misconceptions and build on their strengths to correct those misunderstandings.

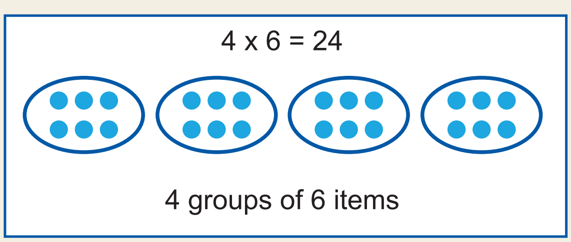
Representations illustrate the value of numbers and the relationship between quantities.

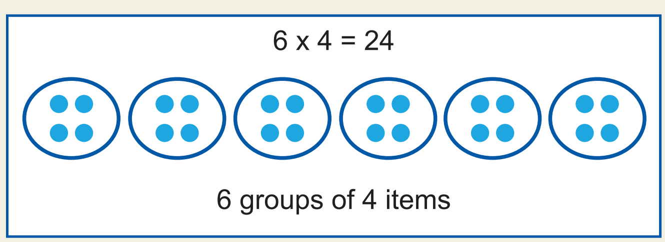
- **Concrete representations** are three-dimensional (3D) physical materials or actions that students can organize, act upon, or manipulate (e.g., regrouping with base 10 blocks, using fraction tiles to compare two fractions, and role playing a problem situation).
- **Semi-concrete representations** are two-dimensional (2D) visual depictions such as strip diagrams, simple drawings, tables, arrays, graphs, and number lines.
- **Abstract representations** are mathematical notations that can include numbers, equations, operations, relational symbols, and expressions (such as $4 \times 4 = 16$).

Structure questions in such a way as to help students self-identify where their thinking went wrong. It might be helpful for students to use representations to help them articulate what they are thinking. Correcting misunderstandings early can prevent the confusion from becoming an enduring problem.

Example 1. Reviewing previously learned content with verbal and visual supports*.

The teacher reviews what the students have previously learned and reminds students of key concepts, including fact families (sets of mathematically related number sentences, such as 3x5=15 and 15/5=3) and the inverse relationship between multiplication and division. After explicitly reviewing what students know, the teacher asks a student to explain how the equal-sized group model can be used to solve the problem 4×6. Because the student has recently practiced solving multiplication and division problems with a visual representation, the student draws 4 circles with 6 dots in each and explains how she created 4 groups of 6 and skip counted to solve the multiplication problem. If needed, the teacher is poised to prompt the student if she misses a key point and provide corrective feedback.





*This is an abbreviated version of the example presented in the practice guide.

Recommendation 2: Mathematical Language

Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts

Mathematical language is academic language that precisely conveys mathematical ideas, including the vocabulary, terminology, and language structures used when thinking about, talking about, and writing about mathematics.

Understanding mathematical language is critical to students' learning because it is used in textbooks, curricular and assessment materials, and teachers' instruction.

When teachers use conversational or informal language instead of mathematical language, students may get confused. For example, some teachers may refer to the commutative property ($a + b = b + a$) as the "flip-flop property." Although this creative property name may be viewed as a fun memory device, replacing accurate terms with informal language can confuse students later when they don't know the connection to the correct formal term *commutative property*. Using and practicing correct terminology from the start can eliminate later challenges.

How to carry out the recommendation

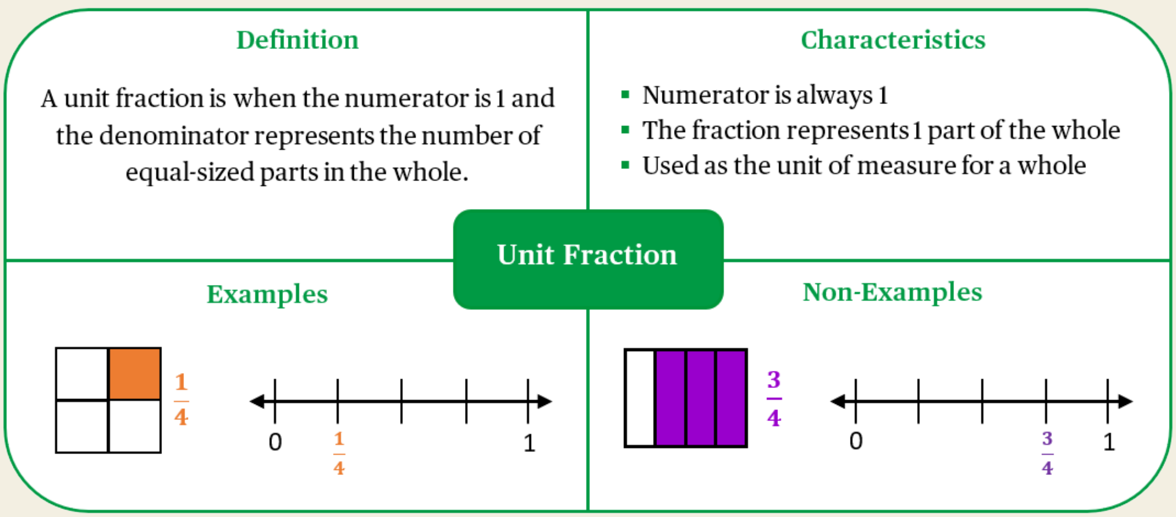
1. **Routinely teach mathematical vocabulary to build students' understanding of the mathematics they are learning.** Introduce new mathematical vocabulary during instruction and provide context and meaning to the words. Use student-friendly definitions with simple and familiar mathematical words. Simply providing a definition of a term is not sufficient for developing students' understanding of mathematical vocabulary and concepts. Link new vocabulary to a variety of examples when possible, including concrete or semi-concrete representations. A graphic organizer, such as in Example 2, can provide a student-friendly definition and visually and symbolically depict a vocabulary word's meaning, characteristics, examples, and non-examples. Hand gestures and role-playing can also provide context and meaning to mathematical vocabulary.

   To support learning across grade levels and settings, schools should consider creating a list of mathematical terminology that is shared among teachers and strategically becomes more sophisticated with each grade.

2. **Use clear, concise, and correct mathematical language throughout lessons to reinforce students' understanding of important mathematical vocabulary words.** Consistent use of mathematical language helps students learn how the terms should be used and develop a deeper understanding of the terms. Use and emphasize clear, concise, and correct mathematical language throughout instruction: when referring to a new or previously learned topic, when discussing homework, and when responding to questions. Model precise mathematical language when explaining your thought process and demonstrating how to solve a problem.

3. **Support students in using mathematically precise language during their verbal and written explanations of their problem solving.** Have students provide verbal and written explanations of mathematics concepts during intervention. Offer students a framework for providing explanations, such as sentence starters or a set of guiding questions. It is also helpful for teachers to restate the students' explanations using correct language when students do not. Remind students to include the mathematical language modeled and taught during instruction by displaying mathematical vocabulary on the classroom wall.

Example 2. Graphic organizer that depicts a student-friendly definition, characteristics, examples, and non-examples for the term unit fraction.



Recommendation 3: Representations

Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.

Students who struggle to learn mathematics need additional, focused instruction using representations to model mathematical ideas. Choose representations carefully and connect them explicitly to the abstract representations (mathematical notation). It is also important to provide students with many opportunities to use representations.

How to carry out the recommendation

1. **Provide students with the concrete and semi-concrete representations that effectively represent the concept or procedure being covered.** Not all representations work for every mathematical concept, so choosing representations must be intentional to be effective. Provide students with the representations that most accurately model the concept or procedure being addressed. When appropriate, use representations that are proportional. For example, when teaching place value, the representation for ones should be one-tenth the size of the representation for tens.

2. **When teaching concepts and procedures, connect concrete and semi-concrete representations to abstract representations.** When demonstrating concepts and procedures with concrete and semi-concrete representations, present the mathematical notation simultaneously. It is also important to connect concrete and semi-concrete representations to each other. It is helpful to make these connections when introducing new material, when reviewing previously learned content, and when using familiar representations in a new way. Example 3 demonstrates how the representations can be aligned vertically to demonstrate their connections.

3. **Provide ample and meaningful opportunities for students to use representations to help solidify the use of representations as "thinking tools."** Students need many opportunities to work with representations before they will successfully use them to model concepts and procedures and solve problems. Over time, students will begin to more deeply understand mathematics concepts and grasp how representations can be used as "thinking tools", which are tools to model and solve problems.

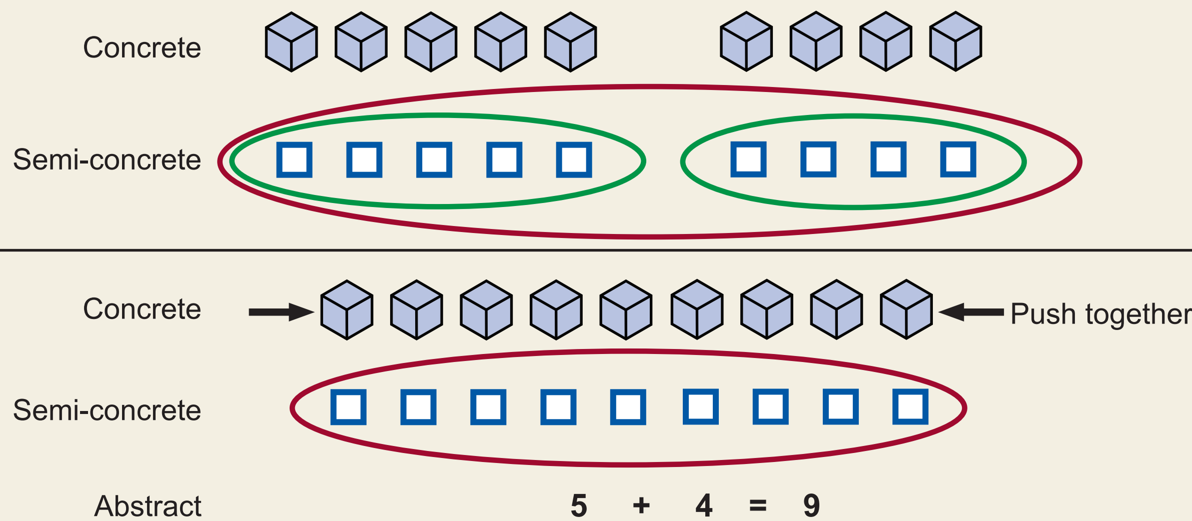
   Representations can be used when students explain their thinking. At first, students may need help articulating how they used the representations to depict the mathematical concepts. Pose prompting questions to help students explain how they represented the concepts and/or procedures. As students become more comfortable using representations, routinely ask them to use the representations to explain their solution approach. This helps reinforce the mathematics not only for the student explaining their thinking, but also for the students who are listening to the explanation the student is giving.

4. **Revisit concrete and semi-concrete representations periodically to reinforce and deepen understanding of mathematical ideas.** Systematically revisit concrete and semi-concrete representations to reinforce and deepen students' understanding of mathematical concepts. Also, if students are not able to correctly solve problems or are uncertain about how to approach a problem, encourage them to use a concrete or semi-concrete representation.

Example 3. Teacher shows how summing two numbers relates to concrete and semi-concrete representations and to an equation.

$$5 + 4 = \_\_\_\_\_$$

Teacher: When looking at this problem we see that we need to add or combine the 4 and the 5. I can use counters to count out a group of 4 and another group of 5. Then I can combine them and count how many I have. I can also draw 4 squares to represent the 4 and 5 squares for the 5, and then count how many squares I drew in all. I find that I drew 9 squares, so the answer to the problem 4 + 5 equals 9.



Recommendation 4: Number Lines

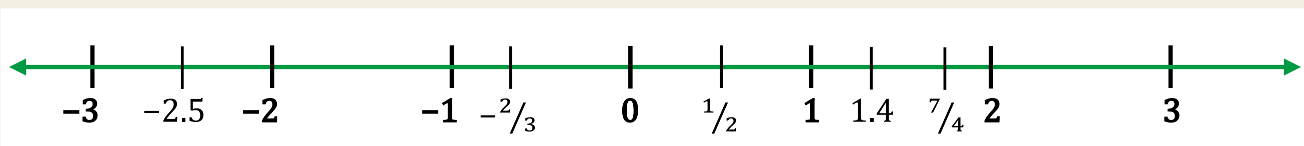
Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics.

The number line is a semi-concrete mathematical representation that can concurrently represent all real numbers, including whole numbers and rational numbers, positive and negative numbers, and other sets of numbers (Example 4). The ability to represent different sets of numbers makes the number line a powerful tool for helping students develop a unified understanding of numbers and for supporting their learning of advanced mathematics. Number lines are an important tool for teaching and understanding magnitude and operations for both whole numbers and fractions, graphing coordinates, and displaying and analyzing data.

Students who are proficient in mathematics often construct a mental number line as they solve problems. When a teacher consistently uses number lines during intervention, students gradually develop the ability to visualize a number line when considering the magnitude of a number, determining strategies for solving mathematics problems, or evaluating the reasonableness of their answers after solving problems. It also sets the stage for more advanced work in middle and high school mathematics.

Each step in this recommendation provides guidance for whole numbers in early elementary (grades K-2) and fractions and decimals in upper elementary (grades 3-6).

Example 4. Number line representing magnitudes of whole, positive, negative, and rational numbers.



How to carry out the recommendation

1. **Represent whole numbers, fractions, and decimals on a number line to build students' understanding of numerical magnitude. Early elementary (grades K-2).** Before using a number line, introduce students to a concrete version of a number line. For example, use a walkable number path, board games, or clotheslines. This may help students begin to form a visual image of a number line.

   After exposing students to the concrete number line with a series of individual units lined up on a path, connect that idea to a number line on paper or projected on a screen. Ask students to identify similarities and differences between the two representations. Draw their attention to the distance from zero to one and how that distance is the same length as one unit. This connection will help students understand that the 1 on a number line is not merely a tick mark, but also represents the full one-unit distance from zero.

   Upper elementary (grades 3-6). Once students understand the concept of a fraction with concrete representations, show students how to represent fractions on a number line.

Demonstrate the location of fractions on the number line, starting with familiar fractions that are less than one. Reinforce the idea that the denominator represents the number of partitions in one whole. Number lines can be used to:

- Demonstrate the pattern of unit fractions and their corresponding magnitude.
- Teach students that not all fractions are less than one; depict fractions equal to or greater than one.
- Show students that whole numbers can be represented as fractions and that some fractions are located between other whole numbers.
- Explain that some fractions are positioned at the same location on the number line and are therefore equivalent (for example, 1/2, 2/4, and 4/8).
- Expand the idea of equivalencies to include decimals and percentages so that students understand that these numbers are also equivalencies.

2. **Compare numbers and determine their relative magnitude using a number line to help students understand quantity. Early elementary (grades K–2).** Use number lines to teach the relative magnitude of whole numbers. Start by putting two numbers on a number line using equal units. Explain that each number's distance from zero represents the number's magnitude. Explain how to compare the two numbers and determine which is greater based on which is more equal units away from zero.

   Upper elementary (grades 3–6). Use number lines to compare the magnitude of fractions and decimals. Reinforce for students that fraction and decimal magnitude, like whole-number magnitude, is represented by how far to the right or left of zero a number is positioned. Help students compare fraction magnitude by locating "benchmark numbers," starting with 0, 1/2, and 1, when thinking of fractions between 0 and 1.

3. **Use the number line to build students' understanding of the concepts underlying operations.** Early elementary (grades K–2). Show students how to use number lines for addition and subtraction of whole numbers by looking at the distance between whole numbers. Teach students to focus on the unit length, or distance between two tick marks, rather than counting tick marks. When moving to the left students see that starting with 8 and moving 3 units to the left is equal to 5, showing the subtraction equation 8 – 3 = 5.

   Upper elementary (grades 3–6). A number line is a powerful visual for demonstrating addition and subtraction of fractions. Start by adding fractions with the same denominator using one number line. When adding and subtracting fractions with unlike denominators, use double number lines to make the equivalences more visible for students.

Recommendation 5: Word Problems

Provide deliberate instruction on word problems to deepen students' mathematical understanding and support their capacity to apply mathematical ideas.

Learning to solve word problems is an important part of the elementary mathematics curriculum because word problems help students apply the mathematics they are learning, develop critical thinking skills, and begin to connect mathematics to a variety of scenarios or contexts. Becoming successful at solving word problems can deepen students' understanding of grade-level content and set students up for success in advanced mathematics courses and the workforce.

Unfortunately, learning calculations alone does not necessarily help students successfully solve word problems. To set up and solve word problems successfully, students need to read and understand the problem's narrative, determine what the problem is asking them to find, and identify one or more mathematical operations that will solve the problem. Students who struggle with mathematics often have difficulty with one or more of these steps.

How to carry out the recommendation

1. **Teach students to identify word problem types that include the same type of action or event.** Introduce one problem type at a time. Begin by introducing a new problem type with a story that includes all quantities. This helps students think about what the quantities represent without needing to solve for an unknown. Next, present the same story with a missing quantity (that is, a word problem). For instance:

   - Story with all quantities: There were 18 children on the bus. 7 children got off the bus at the first stop. 11 children are still on the bus.
   - Word problem with a missing quantity: There were 18 children on the bus. 7 children got off the bus at the first stop. How many children are still on the bus?

   Connect the quantities between the story and the word problem so that the students see how they are the same. Use role-playing, gestures, or concrete and/or semi-concrete manipulatives to help students visualize the problem and identify relevant information. This helps students see how the quantities relate to each other.

2. **Teach students a solution method for solving each problem type.** Introduce a solution method using a worked-out example. Talk through the problem-solving process and connect the relevant problem information to the worked-out example. Say out loud the decisions that were made to solve the problem at each step. Then demonstrate how to apply the solution method by solving a similar problem with students using that method. Discuss each decision you make and ask students guiding questions to engage them as you solve the problem.

   Provide students with a visual guide detailing steps to reference as they solve word problems. Some parts of the guide may apply to understanding the problem before solving it, such as "read the problem," "name the problem type," "identify the question," and "find relevant information." Other parts may be geared toward choosing a solution method that is specific to the problem type. Over time, gradually fade the use of visual guides so students do not become overly reliant on them.

3. **Expand students' ability to identify relevant information in word problems by presenting problem information differently.** Once students can recognize and solve the most accessible problems within a type, present word problems of that same type that are less familiar so that students broaden their understanding of that problem type. For example, teachers can vary the unknown quantity to help students understand the mathematical structure in each problem type. Other problems that look different may require additional steps to solve or include irrelevant numerical information or information presented differently in a chart, graph, or diagram.

   Once students have learned several variations of each problem type, teachers can provide ongoing support to students in identifying which quantities are relevant for solving problems by:

   - Helping students visualize the problem by using concrete or semi-concrete representations.
   - Encouraging students to reread the problem more than once and restate the problem in their own words. This may help students determine which information is relevant versus irrelevant.
   - Asking students to write down, circle, or underline information that will be used solve the problem and to cross out information that is not useful.

4. **Teach vocabulary or language often used in word problems to help students understand the problem.** When first introducing word problems, choose problems where all the words in the story are familiar to students. Before you start teaching, anticipate which words are critical for understanding the problem. Pay particular attention to words that relate to one another that may help students identify which information in the problem is relevant and which is irrelevant. Teach the meaning of words and continue to discuss them during problem solving to solidify their meaning. Over time, include word problems with more difficult language.

5. **Include a mix of previously and newly learned problem types throughout intervention.** After a problem type has been taught, distribute previously learned problem types throughout lessons. By revisiting previously learned problems, students practice discriminating among problem types as they learn new ones. Include an activity where students identify and name problem types without solving them. This reinforces the importance of reading and thinking about each problem before solving it. Students may need support to remember the salient features of different problem types, like a prompt card listing the features of a problem or a gesture that evokes the action in the problem.

Recommendation 6: Timed Activities

Regularly include timed activities as one way to build students' fluency in mathematics.

Quickly retrieving basic arithmetic facts is not easy for students who struggle with mathematics. Automatic retrieval gives students more mental energy to understand relatively complex mathematical tasks and execute multi-step mathematical procedures. Thus, building automatic fact retrieval in students is one (of many) important goals of intervention.

In addition to basic facts, timed activities may address other mathematical subtasks important for solving complex problems. This could include, for example, recalling equivalencies for fraction benchmarks of 1/2 and 1, or quickly evaluating and estimating place value. The goal of these activities is to move students toward accurate and efficient performance of these smaller mathematical tasks so that this knowledge can be easily accessed when necessary for solving problems.

How to carry out the recommendation

1. **Identify already-learned topics for activities to support fluency and create a timeline.** When planning activities to support fluency, think through what students need to know how to do in order to understand and more easily apply the mathematics they are learning. Consider the mathematics topic that is the focus of intervention and whether reteaching basic facts and/or other subtasks might help students understand and perform that task more fluently. Think about which complex strategies or procedures the students will learn and break those into a series of smaller subtasks. Plan activities to support fluency in one of those areas.

   Pick one topic to build over time. For each topic, plan a schedule for introducing and conducting the activity to support fluency. At the start, choose easier items for the activity. To help students remain engaged in the topic, increase the difficulty of the items as students become more fluent with the easier items. For example, if working on addition facts, you might start with n + 1 or doubles. Then, increase the difficulty of the items to include other more difficult combinations.

2. **Choose the activity and accompanying materials to use in the timed activity and set clear expectations.** Timed activities are brief (usually 1–5 minutes) and require students to generate many correct responses in that short amount of time. Activities that support fluency can be done using flash cards, computer programs, or worksheets. Using these materials, activities can be structured for students to work together as a group or individually. In a small-group intervention setting, set up the activity with clear expectations of who responds and when. Periodically incorporate game-like features, such as keeping score or having students cooperate as a team to increase their score.

3. **Ensure that students have an efficient strategy to use as they complete the timed activity.** Plan timed activities that focus on previously learned content. Include the strategies you want students to use during timed activities during other portions of the intervention lessons. For example, when teaching addition facts, instruction may be organized around teaching number

combinations, doubles, doubles plus one, or various combinations of 10 or other numbers. Be sure that students are competent in using these strategies before students begin the timed activity.

4. **Encourage and motivate students to work hard by having them chart their progress.** Remind students that the goal of fluency activities isn't to simply generate answers quickly, but instead to generate accurate answers in a short amount of time.

   To keep students focused and motivated during these activities, have students record their scores over time on a chart or graph. As students see their scores improve over time, they may feel more excited and motivated to set goals and work hard. Goals to "meet or beat" a previously earned fluency score can be set for individuals or as a collective score for the intervention group. Working toward a goal as a group can reduce the pressure on individual students.

5. **Provide immediate feedback by asking students to correct errors using an efficient strategy.** When using flash cards or other activities that allow immediate feedback from a teacher, students may self-correct before feedback can be provided. This might indicate that they are moving toward fluency. If students do not self-correct, immediately ask them to fix their incorrect answer and explain why the new answer is correct. If the student struggles, remind them of the efficient strategy they have learned. The student is responsible for using the taught strategy and correcting their answer before moving on.

   Often, computer-based programs provide students with immediate feedback. However, immediate feedback is sometimes not possible with worksheets, even in small groups. In the opinion of the panel, teachers should score and return worksheets as soon as possible and then review with students the problems that need to be corrected and the effective strategies that could be used.

Summary of Evidence by Recommendation

	Rec. 1 *Systematic instruction*	Rec. 2 *Systematic instruction*	Rec. 3 *Representa-tions*	Rec. 4 *Number lines*	Rec. 5 *Word problems*	Rec. 6 *Time activities*
Number of Studies						
	43	16	28	14	18	27
Level of Evidence						
	Strong	Strong	Strong	Strong	Strong	Strong
Meta-Analysis Results by Outcome Domain						
Algebra and Algebraic Reasoning	+	0	+	X	N/A	+
Geometry	X	X	X	X	N/A	X
General Mathematics Achievement	+	+	+	+	N/A	+
Rational Numbers Computation	+	+	+	+	N/A	+
Rational Numbers Knowledge	+	+	+	+	N/A	+
Rational Numbers Magnitude Understanding/Relative Magnitude Understanding	+	+	+	+	N/A	+
Rational Numbers Word Problems/ Problem Solving	+	N/A	N/A	N/A	+	N/A
Statistics	X	X	X	X	N/A	X
Whole Numbers Computation	+	+	+	+	N/A	+
Whole Numbers Knowledge	+	X	+	X	N/A	+
Whole Numbers Magnitude Understanding/ Relative Magnitude Understanding	0	0	0	0	N/A	+
Whole Numbers Word Problems/ Problem Solving	+	N/A	N/A	N/A	+	N/A

+ = Statistically significant and positive; 0 = Indeterminate; X = No studies meeting WWC standards; N/A = Not applicable

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