REL Appalachia Ask-A-REL Response

Educator Effectiveness, Math
June 2017

Question:

What type of mathematical skills and knowledge predict success in algebra I? What does the research say about strategies or interventions to improve algebra readiness (particularly in middle school)?

Response:

Thank you for your request to our REL Reference Desk regarding evidence-based information about algebra readiness. Ask-A-REL is a collaborative reference desk service provided by the 10 Regional Educational Laboratories (RELs) that, by design, functions much in the same way as a technical reference library. Ask-A-REL provides references, referrals, and brief responses in the form of citations in response to questions about available education research.

Following an established REL Appalachia research protocol, we searched for research reports and descriptive study articles on algebra readiness. We focused on identifying resources that specifically addressed factors that predict success in algebra I and interventions that improve algebra readiness. The sources included ERIC and other federally funded databases and organizations, research institutions, academic research databases, and general Internet search engines. For more details, please see the methods section at the end of this document.

The research team did not evaluate the quality of the resources provided in this response; we offer them only for your reference. Also, the search included the most commonly used research databases and search engines to produce the references presented here, but the references are not necessarily comprehensive, and other relevant references and resources may exist.

Research References

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology*, *37*(4), 247–253. Retrieved from https://eric.ed.gov/?id=EJ977998; full text available at https://www.researchgate.net/publication/257178514_Fractions_Could_they_really_be_the_gatekeeper%27s_doorman

From the abstract: "The National Mathematics Advisory Panel (NMAP, 2008) asserts that a foundational knowledge of fractions is crucial for students' success in algebra; however, empirical evidence for this claim is relatively nonexistent. In the present study, we examine

the impact of middle school students' fraction and whole number magnitude knowledge on various components of their algebra readiness. Results suggest that fraction knowledge is related to algebra readiness, more so than number magnitude knowledge in general; students' magnitude knowledge of unit fractions (i.e., those with a numerator of 1) appears particularly important. Findings confirm the intuition of the NMAP (2008) and support the recommendation of the Common Core Standards (National Governors Association Center for Best Practices [NGA Center], 2010) that students' fraction knowledge should be cultivated using number lines."

Booth, J. L., Newton, K. J., & Twiss-Garrity, L. K. (2014). The impact of fraction magnitude knowledge on algebra performance and learning. *Journal of Experimental Child Psychology*, *118*, 110–118. Retrieved from https://www.researchgate.net/publication/257812608_The_impact_of_fraction_magnitude_knowledge_on_algebra_performance_and_learning

From the abstract: "Knowledge of fractions is thought to be crucial for success with algebra, but empirical evidence supporting this conjecture is just beginning to emerge. In the current study, Algebra 1 students completed magnitude estimation tasks on three scales (0–1 [fractions], 0–1,000,000, and 0–62,571) just before beginning their unit on equation solving. Results indicated that fraction magnitude knowledge, and not whole number knowledge, was especially related to students' pretest knowledge of equation solving and encoding of equation features. Pretest fraction knowledge was also predictive of students' improvement in equation solving and equation encoding skills. Students' placement of unit fractions (e.g., those with a numerator of 1) was not especially useful for predicting algebra performance and learning in this population. Placement of nonunit fractions was more predictive, suggesting that proportional reasoning skills might be an important link between fraction knowledge and learning algebra."

Clark, T. F., Arens, S. A., & Stewart, J. (2015). *Efficacy study of a pre-algebra supplemental program in rural Mississippi: Preliminary findings*. Society for Research on Educational Effectiveness Spring 2015 Conference, Washington, D.C. Retrieved from https://eric.ed.gov/?id=ED562175

From the abstract: "Mastering mathematics is important for all students, not only because such success increases college and career options and prospects for future income, but also because mathematics literacy helps citizens and policy leaders to make sound judgments (NMAP, 2008). Research suggests that the rural achievement gap can be addressed with modifiable school-based strategies that increase students' opportunity to learn. Therefore, even though rural students lag behind in mathematics, it is likely that their achievement can be increased by providing additional resources. Given this assumption, the authors sought to understand whether a low-cost supplemental program could result in increases in student learning. 'Every Day Counts Algebra Readiness' ('EDC Algebra Readiness') is a resource developed to improve the mathematics proficiency of middle school students and help prepare them for success in high school algebra. The program is a supplemental curriculum for use in the first 10–15 minutes of class, focusing on the most important pre-

algebra concepts. This experimental efficacy study examines the impact of 'EDC Algebra Readiness' on the algebra readiness of 7th grade students in rural schools in Mississippi. The study also examines the program's impact on the Algebra I achievement of these students, as measured by results on the state's end-of-course test. Table 1 is appended."

Huang, C.-W., Snipes, J., & Finkelstein, N. (2014). *Using assessment data to guide math course placement of California middle school students* (REL 2014–040). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory West. Retrieved from https://eric.ed.gov/?id=ED546776

*From the abstract: "*Questions about how best to place students into appropriate middle grade math courses have been central to ongoing education policy and practice discussions in California and across the United States. Recent studies have shown that enrolling in algebra I in grade 8 works well for some students but backfires for others. This REL West report provides findings from a study of placements that were based on different test scores available for middle school students in California. Findings indicate that placement into grade 8 algebra I based solely on students' grade 6 California Standards Test (CST) proficiency status results in some students taking the course who have less than a 40 percent chance of succeeding. Using a higher cut point on the grade 6 CST scale score—as opposed to simply using CST proficiency status—would avoid placing students into grade 8 algebra I who have a lower than 50 percent chance of success, and would increase the overall success rate (from 69 to 75 percent) for students placed into grade 8 algebra I. Prediction accuracy is even higher using grade 7 CST scale scores (78 percent); however, grade 7 scale scores are typically not available until after initial algebra I placements are made. The study also finds that a newly available assessment of algebra readiness developed as part of the Math Diagnostic Testing Project (MDTP) makes a valuable contribution to decisions about algebra I placement. Placements based on the MDTP result in a success rate (77 percent) that is comparable to that of placements based on the grade 7 math CST. Furthermore, the MDTP test can be administered online at any time during the school year, and MDTP test results are available immediately after students take the test, whereas CST results are not available until the next school year. The following are appended: (1) Detailed study design; and (2) Model comparisons."

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education. Retrieved from https://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

From pages 17–18 of the report: "Based on all these considerations, the Panel proposes three clusters of concepts and skills—called the Critical Foundation of Algebra—reflecting their judgment about the most essential mathematics for students to learn thoroughly prior to algebra course work.

1. Fluency with Whole Numbers. By the end of Grade 5 or 6, children should have a robust sense of number. This sense of number must include an understanding of place value and

the ability to compose and decompose whole numbers. It must clearly include a grasp of the meaning of the basic operations of addition, subtraction, multiplication, and division. It must also include use of the commutative, associative, and distributive properties; computational facility; and the knowledge of how to apply the operations to problem solving. Computational facility requires the automatic recall of addition and related subtraction facts, and of multiplication and related division facts. It also requires fluency with the standard algorithms for addition, subtraction, multiplication, and division. Fluent use of the algorithms not only depends on the automatic recall of number facts but also reinforces it. A strong sense of number also includes the ability to estimate the results of computations and thereby to estimate orders of magnitude, e.g., how many people fit into a stadium or how many gallons of water are needed to fill a pool.

2. Fluency with Fractions. Before they begin algebra course work, middle school students should have a thorough understanding of positive as well as negative fractions. They should be able to locate positive and negative fractions on a number line; represent and compare fractions, decimals, and related percent; and estimate their size. They need to know that sums, differences, products, and quotients (with nonzero denominators) of fractions are fractions, and they need to be able to carry out these operations confidently and efficiently. They should understand why and how (finite) decimal numbers are fractions and know the meaning of percent. They should encounter fractions in problems in the many contexts in which they arise naturally, for example, to describe rates, proportionality, and probability. Beyond computational facility with specific numbers, the subject of fractions, when properly taught, introduces students to the use of symbolic notation and the concept of generality, both being integral parts of algebra.

3. Particular Aspects of Geometry and Measurement. Middle grade experience with similar triangles is most directly relevant for the study of Algebra: Sound treatments of the slope of a straight line and of linear functions depend logically on the properties of similar triangles. Furthermore, students should be able to analyze the properties of two- and three-dimensional shapes using formulas to determine perimeter, area, volume, and surface area. They should also be able to find unknown lengths, angles, and areas."

Siegler, R. S., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., Thompson, L., & Wray, J. (2010). *Developing effective fractions instruction for kindergarten through 8th grade: A practice guide* (NCEE 2010-4039). Washington DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, US Department of Education. Retrieved from https://eric.ed.gov/?id=ED512043

*From the abstract: "*This practice guide presents five recommendations intended to help educators improve students' understanding of, and problem-solving success with, fractions. Recommendations progress from proposals for how to build rudimentary understanding of fractions in young children; to ideas for helping older children understand the meaning of fractions and computations that involve fractions; to proposals intended to help students apply their understanding of fractions to solve problems involving ratios, rates, and proportions. Improving students' learning about fractions will require teachers' mastery of the subject and their ability to help students master it; therefore, a recommendation

regarding teacher education also is included. Appendices include: (1) Postscript from the Institute of Education Sciences; (2) About the Authors; (3) Disclosure of Potential Conflicts of Interest; (4) Rationale for Evidence Ratings; and (5) Evidence Heuristic. A glossary and index are also provided. (Contains 5 tables, 10 figures, and 250 endnotes.)"

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, *23*(7), 691–697. Retrieved from https://eric.ed.gov/?id=ED552898

*From the abstract: "*Identifying the types of mathematics content knowledge that are most predictive of students' long-term learning is essential for improving both theories of mathematical development and mathematics education. To identify these types of knowledge, we examined long-term predictors of high school students' knowledge of algebra and overall mathematics achievement. Analyses of large, nationally representative, longitudinal data sets from the United States and the United Kingdom revealed that elementary school students' knowledge of fractions and of division uniquely predicts those students' knowledge of algebra and overall mathematics achievement in high school, 5 or 6 years later, even after statistically controlling for other types of mathematical knowledge, general intellectual ability, working memory, and family income and education. Implications of these findings for understanding and improving mathematics learning are discussed."

Snipes, J., Huang, C.-W., Jaquet, K., & Finkelstein, N. (2015). *The effects of the Elevate Math summer program on math achievement and algebra readiness* (REL 2015–096). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory West. Retrieved from https://eric.ed.gov/?id=ED558157

From the abstract: "The effects of the Elevate Math summer program on math achievement and algebra readiness: This randomized trial examined the effects of the Elevate Math summer program on math achievement and algebra readiness, as well as math interest and self-efficacy, among rising 8th grade students in California's Silicon Valley. The Elevate Math summer math program targets students who score in the range between 'high basic' and 'low proficient' on state math tests. It consists of 19 days of mathematics instruction, consisting of three hours per day in traditional classroom instruction and one hour per day using Khan Academy (a free online learning system). During summer 2014, students were randomly assigned to a treatment group that received access to the program at the beginning of the summer or to a control group that received access to the program later in the summer. End-of-program test scores and survey responses of students in the treatment group were compared with those of students in the control group prior to their exposure to the program. Treatment group students scored significantly higher than the control group (4 points or 0.7 standard deviation) on a test of algebra readiness. They were also significantly more likely (29 percent versus 12 percent) to reach achievement thresholds associated with success in algebra I. However, treatment and control groups did not show significant differences in terms of math interest or self-efficacy. The results show that the

Elevate Math summer program can significantly improve student math achievement and algebra readiness; however, 70 percent of program participants were still not ready for algebra I content. This suggests that summer math programs such as Elevate Math's may be important tools for improving math achievement among rising eighth grade students, but most targeted students will need additional support in order to ensure success in algebra. The following are appended: (1) Data, outcomes, and methodology; and (2) Sensitivity analyses."

Sorensen, N. (2014). *Supplementary learning strategies to support student success in algebra I*. Promoting Student Success in Algebra I, Research brief. Washington, DC: American Institutes for Research. Retrieved from https://www2.ed.gov/programs/dropout/learningsupports092414.pdf

From the abstract: "This research brief is one of five that summarize the literature in different topic areas related to helping struggling students in Grades 6–9 succeed in algebra. The five topic areas are Curricular Alignment, Instructional Practices, Supplementary Learning Supports, Professional Development, and Instructional Coaching. The research briefs are part of the Promoting Student Success in Algebra I (PSSA) project funded by the U.S. Department of Education's High School Graduation Initiative (HSGI). The PSSA project at American Institutes for Research is designed to provide actionable information for educational program developers/administrators in three ways. First, these research briefs together will summarize research on five strategies being implemented by HSGI grantees that help struggling students succeed in Algebra I, a critical gateway course for high school graduation and enrollment in college. Second, the project includes a forum for practitioners—district program developers/administrators and teachers—to make connections between the findings from the research briefs and their daily work, with the results of these discussions published in a series of perspective briefs. Third, the project includes profiles of practices that provide an in-depth look at implementation of these five strategies.

This brief describes what is known about supplementary learning opportunities for struggling students in Algebra I, focusing specifically on double-dose algebra and expanded learning opportunities (ELOs). Many districts now require students to successfully complete Algebra I in order to graduate from high school, and the course is often a prerequisite for subsequent mathematics courses that are essential for college admission. Yet, as districts encourage or require success in the course for all students, some will be underprepared and will struggle to master the course content. Failing Algebra I places these struggling students at high risk of dropping out, particularly in urban districts (e.g., Oriheula, 2006; Silver, Saunders, & Zarate, 2008)."

Star, J. R., Caronongan, P., Foegen, A., Furgeson, J., Keating, B., Larson, M. R., Lyskawa, J., McCallum, W. G., Porath, J., & Zbiek, R. M. (2015). *Teaching strategies for improving algebra knowledge in middle and high school students* (NCEE 2014-4333). Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of

Education Sciences, U.S. Department of Education. Retrieved from the NCEE website: http://whatworks.ed.gov

From the abstract: "Mastering algebra is important for future math and postsecondary success. Educators will find practical recommendations for how to improve algebra instruction in the What Works Clearinghouse (WWC) practice guide, 'Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students.' The methods and examples included in the guide focus on helping students analyze solved problems, recognize structure, and utilize alternative approaches to solving algebra problems. Each recommendation includes the level of supporting research evidence behind it, examples to use in class, and solutions to potential implementation roadblocks. Teachers can implement these strategies in conjunction with existing standards or curricula. In addition, these strategies can be utilized for all students learning algebra in grades 6–12 and in diverse contexts, including during both formative and summative assessment. Administrators and professional development providers can use the guide to implement evidence-based instruction and align instruction with state standards or to prompt teacher discussion in professional learning communities."

What Works Clearinghouse. (2013). WWC review of the report "The effects of POWERSOURCE intervention on student understanding of basic mathematical principles." What Works Clearinghouse single study review. Retrieved from https://eric.ed.gov/?q=powersource&id=ED541917

From the abstract: "The study examined the impact of 'POWERSOURCE'©, an intervention consisting of formative assessments, instructional resources, and professional development designed to help teachers provide individual instruction to their students in Algebra I. This study took place in seven districts in Arizona and California during the 2007–08 school year. Researchers assessed the effectiveness of 'POWERSOURCE'© by comparing student performance on a mathematics achievement test. Results were presented for a composite score and for the three subscales that comprise the main achievement outcome. The outcomes assessed dimensions of student performance in algebra and were constructed from a range of national, state, and local assessments. The study reported that students of teachers who were assigned to use the 'POWERSOURCE'© intervention performed better on questions about properties of arithmetic. The What Works Clearinghouse (WWC) did not confirm this finding to be statistically significant after adjusting for the clustering of students within teachers as the unit of assignment. The research described in this report for the sample that randomly assigned teachers to condition meets WWC evidence standards without reservations. Appended are: (1) Study details; (2) Outcome measures for the mathematics achievement domain; (3) Study findings for the mathematics achievement domain; and (4) Supplemental findings by domain. A glossary of terms is included."

Additional Organizations to Consult

- Precision Education: The Virtual Learning Lab: https://ies.ed.gov/funding/grantsearch/details.asp?ID=1804

From the website: "The Virtual Learning Lab will bring together expertise in informatics (the study of information processing in the context of computer systems), mathematics education, and teacher professional development to launch the emerging field of precision education, in which prior students' data are used to support decisions about the learning opportunities provided to future students. Precision education is now becoming possible due to the availability of large data sets accumulated as students work with technology-based systems, combined with advances in learning analytics, an approach to data collection and analysis that seeks to identify optimal learning sequences for individual students. The Lab's program of research is organized around the goal of improving the impact of *Algebra Nation*, a free online learning platform for students enrolled in algebra and their teachers, which is designed to promote mastery of basic algebra. *Algebra Nation* is currently in use throughout the state of Florida, with plans for expansion to other states."

- National Council of Teachers of Mathematics: http://www.nctm.org/

  From the website: "Founded in 1920, the National Council of Teachers of Mathematics (NCTM) is the world's largest mathematics education organization, with 60,000 members and more than 230 Affiliates throughout the United States and Canada. The National Council of Teachers of Mathematics is the public voice of mathematics education, supporting teachers to ensure equitable mathematics learning of the highest quality for each and every student through vision, leadership, professional development, and research. The National Council of Teachers of Mathematics is the global leader and foremost authority in mathematics education, ensuring that each and every student has access to the highest quality mathematics teaching and learning. We envision a world where everyone is enthused about mathematics, sees the value and beauty of mathematics, and is empowered by the opportunities mathematics affords." The NCTM provides information on Algebra Readiness for Every Student page: http://www.nctm.org/alg16/

Additional Ask-A-REL Responses to Consult

Ask-A-REL Southeast at Florida Center for Reading Research, Florida State University. (2016). *What research is available on differentiated instruction for algebra 1 or algebra in general?* Retrieved from http://rel-se.fsu.edu/_/ask-a-rels/11-16/Ask%20A%20REL%20-%20Differentiated%20instruction%20-%20Algebra%20I.pdf

Ask-A-REL West at WestEd. (2013). *Research on algebra 1 as an 8th grade math offering*. Retrieved from https://relwest.wested.org/system/documents/pdfs/193/original/REL_West_Website_Algebra_I_in_8th_grade.09.2013.pdf?1400004073

Methods

Keywords and Search Strings

The following keywords and search strings were used to search the reference databases and other sources:

- Mathematical knowledge for algebra
- Mathematical skills for algebra
- Predictors of algebra success
- Algebra readiness
- Algebra readiness strategies
- Algebra readiness intervention
- Algebra readiness intervention middle school
- Algebra readiness indicators
- Algebra readiness indicators middle school
- Algebra success predictor variables

Databases and Resources

We searched ERIC, a free online library of over 1.6 million citations of education research sponsored by the Institute of Education Sciences (IES), for relevant resources. Additionally, we searched the academic database ProQuest, Google Scholar, and the commercial search engine Google.

Reference Search and Selection Criteria

In reviewing resources, Reference Desk researchers consider—among other things—these four factors:

- Date of the publication: Searches cover the most current information (i.e., within the last ten years), except in the case of nationally known seminal resources.
- Search priorities of reference sources: Search priorities include IES, nationally funded, and certain other vetted sources known for strict attention to research protocols. Applicable resources must be publicly available online and in English.
- Methodology: The following methodological priorities/considerations guide the review and selection of the references: (a) study types—randomized controlled trials, quasi experiments, surveys, descriptive data analyses, literature reviews, policy briefs, etc., generally in this order; (b) target population, samples (representativeness of the target population, sample size, volunteered or randomly selected), study duration, etc.; (c) limitations, generalizability of the findings and conclusions, etc.
- Existing knowledge base: Vetted resources (e.g., peer-reviewed research journals) are the primary focus, but the research base is occasionally slim or nonexistent. In those cases, the best resources available may include, for example, reports, white papers,

guides, reviews in non-peer-reviewed journals, newspaper articles, interviews with content specialists, and organization websites.

Resources included in this document were last accessed on (6/13/2017). URLs, descriptions, and content included here were current at that time.

This memorandum is one in a series of quick-turnaround responses to specific questions posed by education stakeholders in the Appalachia region (Kentucky, Tennessee, Virginia, and West Virginia), which is served by the Regional Educational Laboratory Appalachia (REL AP) at SRI International. This Ask-A-REL response was developed by REL AP under Contract ED-IES-17-C-0004 from the U.S. Department of Education, Institute of Education Sciences, administered by SRI International. The content does not necessarily reflect the views or policies of IES or the U.S. Department of Education, nor does mention of trade names, commercial products, or organizations imply endorsement by the U.S. government.