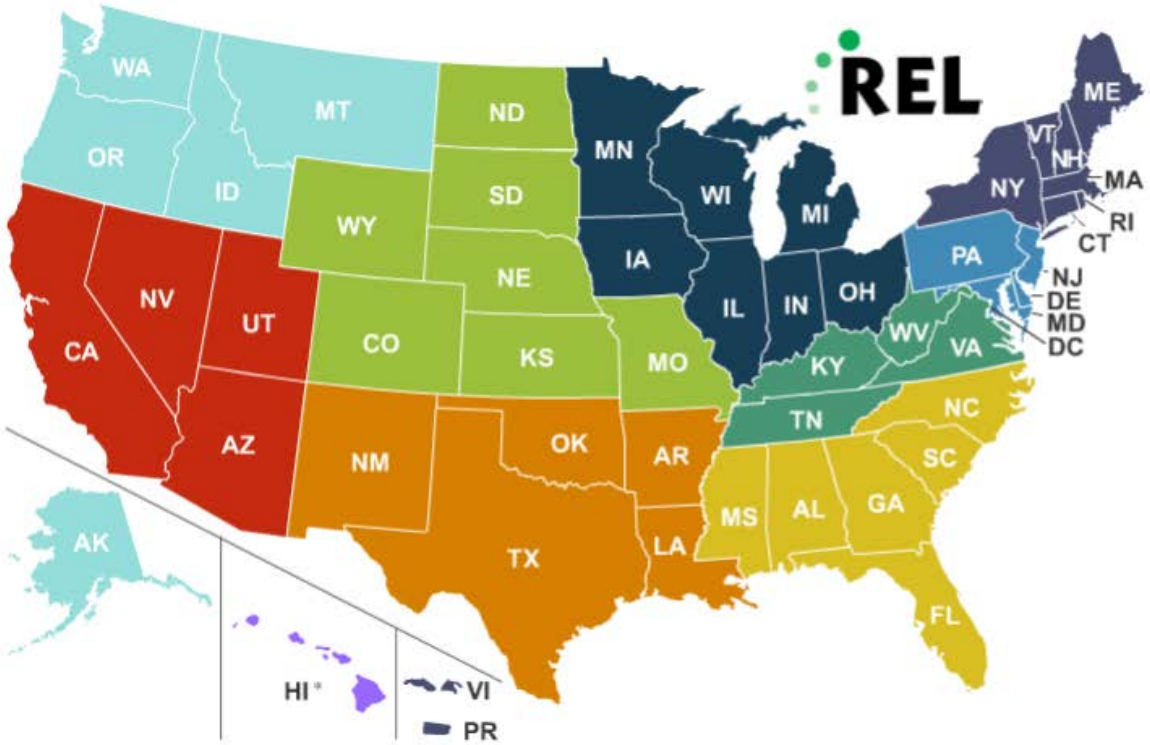
SRI Education

Algebra for All: Focus on Visual Representations

October 17, 2018

REL APPALACHIA
Regional Educational Laboratory At SRI International

The Regional Educational Laboratories



- Appalachia
- Central
- Mid-Atlantic
- Midwest
- NE & Islands
- NW
- Pacific*
- SE
- SW
- West

* The Pacific Region contains Hawaii pictured on the map and American Samoa, the Commonwealth of the Northern Mariana Islands, the Federated States of Micronesia (Chuuk, Kosrae, Pohnpei, & Yap), Guam, the Republic of the Marshall Islands, & the Republic of Palau not pictured on the map

The 10 Regional Educational Laboratories (RELs) work in partnership with stakeholders to conduct applied research and trainings.

The REL mission is to support a more evidence based education system.

Administered by the U.S. Department of Education, Institute of Education Sciences (IES)

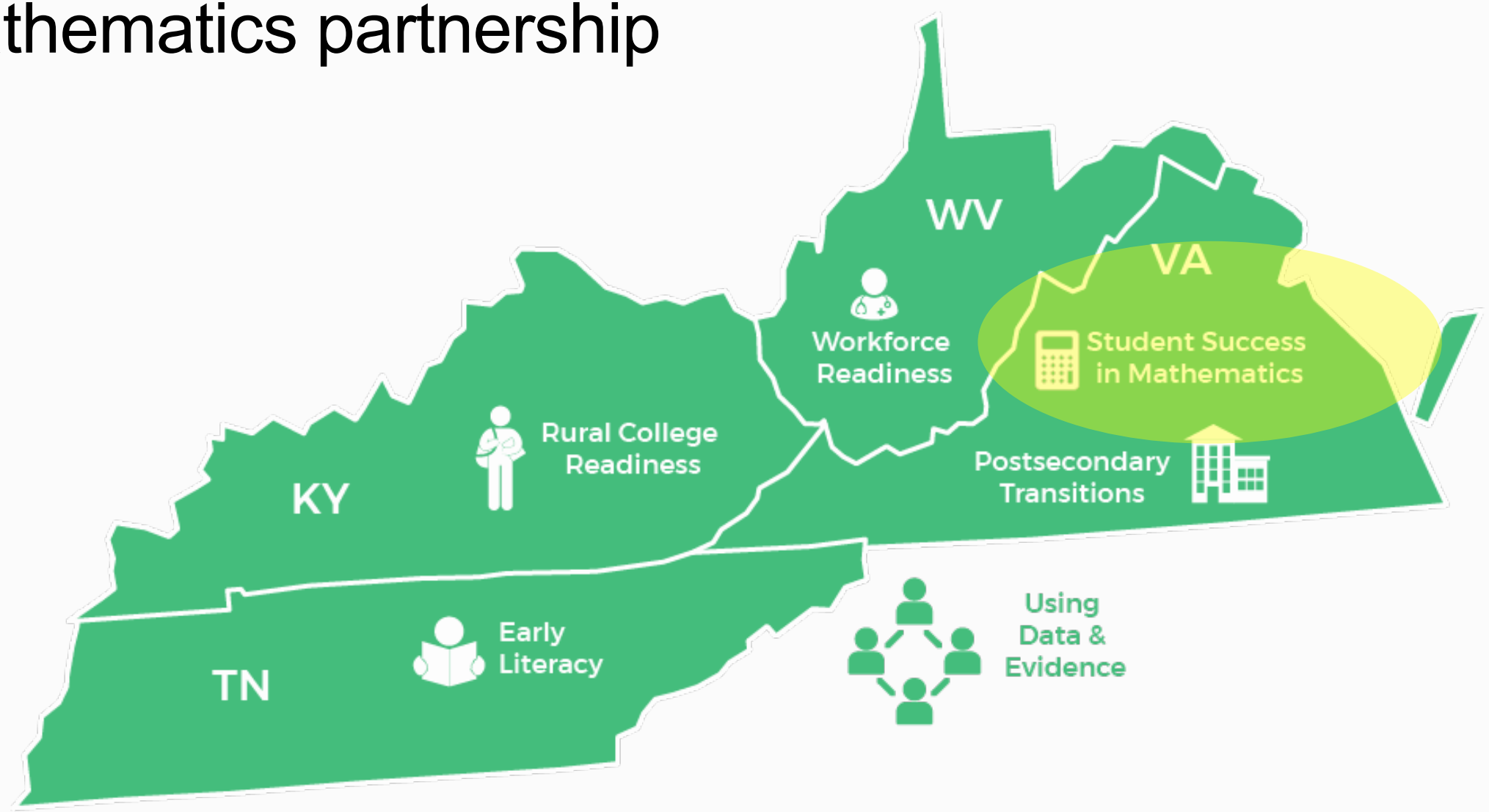
Find us on the web! https://ies.ed.gov/ncee/edlabs/regions/appalachia/

REL APPALACHIA

Working with the REL Program

- Sustain **partnerships** that use research to address high-leverage issues.
- Complete coherent and cumulative **research agendas.**
- Use REL AP as **key resource** for credible research and support.
- **Increase capacity** to access, understand, interpret, apply, and conduct research.
- **Increase use** of research findings in education decisionmaking.

REL Appalachia's Virginia Student Success in Mathematics partnership



Tweet with us!

- To tweet at us during this event,
  - @REL_Appalachia

- Use hashtag
  - #AlgebraforAll

Please share this information in the chat:

- Name
- Affiliation
- Role

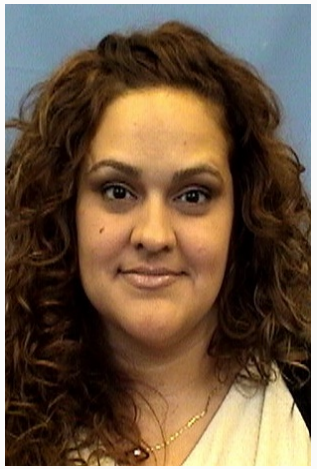
Meeting agenda

- Welcome and Introductions
- Session Objectives
- Framing the Research: Content and Practices for Algebra Readiness
- Purpose: Using Visual Representations as a Strategy to Support Algebra Readiness and Success for All
- Connecting Research to Practice: Using Mathematics Tasks
- Accessing Webinar and REL Appalachia Resources
- Closing

Meet your presenters

Pam Buffington, PhD
Partnership Lead
Education Development Center

Jill Neumayer DePiper, PhD
Partnership Member
Education Development Center



Carmen Araoz
Partnership Liaison
SRI Education

Session Objectives

- Increase knowledge about research on algebra I completion and future student success.
- Increase understanding of how visual representations can support students' algebraic problem solving, especially those who are English learners or who have low literacy skills.
- Increase awareness of the role of ratio and proportion skills and concepts in supporting algebra readiness through engagement with selected ratio and proportion tasks.

Framing the Research: Content and Practices for Algebra Readiness

Algebra: Critical to future success

- Completing Algebra I by grade 9 is *key* to preparing students for on-time graduation and life after high school *(Tierney, Bailey, Constantine, Finkelstein, & Hurd, 2009).*

Algebra: Critical to future success

- Algebra knowledge and skills are important:
  - For success in future mathematics courses, including geometry and calculus (Star et al., 2015), and for postsecondary success
  - In creating a skilled workforce for scientific and technical careers (Katz, 2007; National Mathematics Advisory Panel, 2008).

Algebra: Critical to future success

- Algebra readiness begins in elementary grades (Empson, Levi, & Carpenter, 2011; Silver & Stein, 1996):
  - Number and operations, such as missing number problems
  - Fraction problem-solving
  - Comparing relationships

Proportional reasoning is foundational

- Proportionality is the “cornerstone of higher mathematics and the capstone of elementary concepts” (Lesh, Post, & Behr, 1988, p. 98).
- Proportional thinking and reasoning in upper elementary/middle grades is:
  - Foundational to students’ algebraic understanding and to Algebra I course readiness (Empson et al., 2011; Siegler, Fazio, Baily, & Zhou, 2012).
  - A prerequisite to higher level mathematics because relationships between quantities are key to functions and variation (Siegler et al., 2010).

Activity: Read and solve the following task

In the fifth grade, there are 5 times as many students in the choir than in the band.

There are 48 grade 5 students in the choir and band combined. How many grade 5 students are in the choir?

What are the proportional relationships that you would use to solve this task?

Activity: Read and solve the following task

In the fifth grade, there are 5 times as many students in the choir than in the band.

There are 48 grade 5 students in the choir and band combined. How many grade 5 students are in the choir?

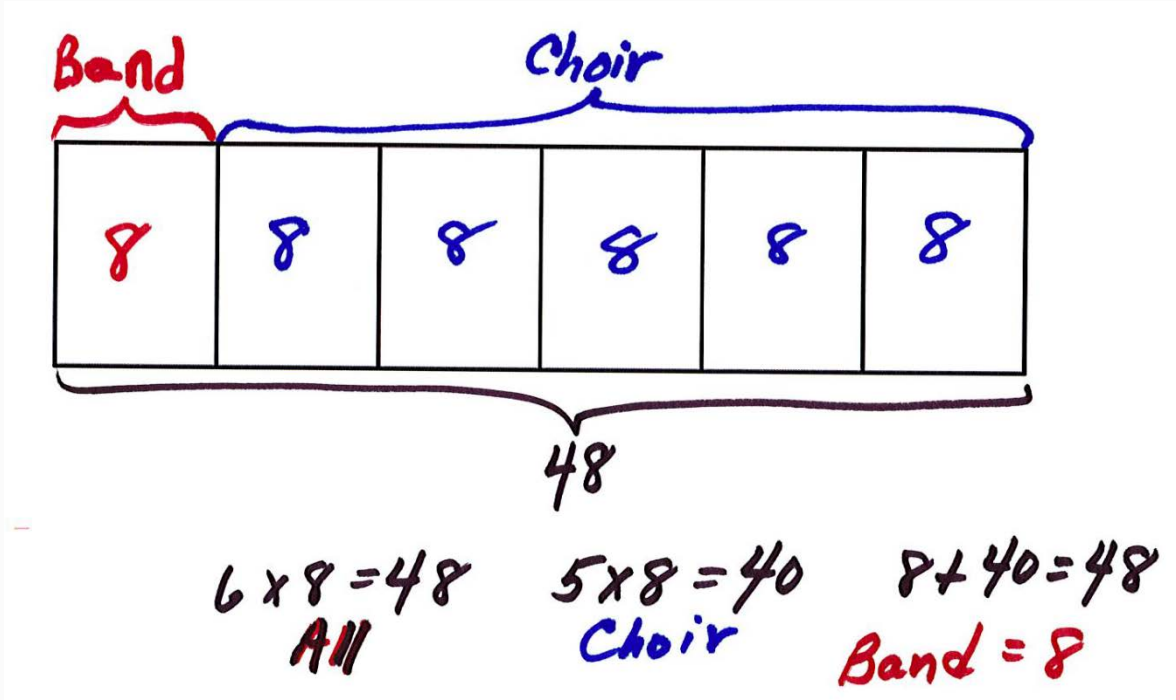
- *What are the proportional relationships that you would use to solve this task?*
- *Possible relationships:*
  - **5 to 1** (choir to band students)
  - **5 to 6** (choir students to total students)
  - **40 to 48** (choir students to total students).
- *What are other possible relationships that may relate to Algebra 1 coursework?*

Share in the Chat!

Using Visual Representations to Support Reasoning

In the fifth grade, there are 5 times as many students in the choir than in the band.

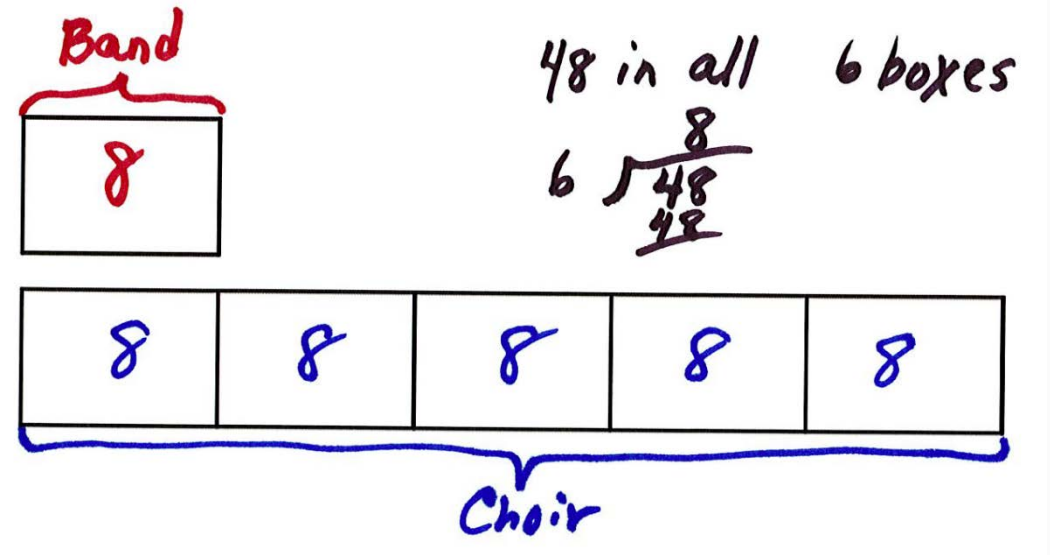
There are 48 grade 5 students in the choir and band combined. How many grade 5 students are in the choir?



Using Visual Representations to Support Reasoning

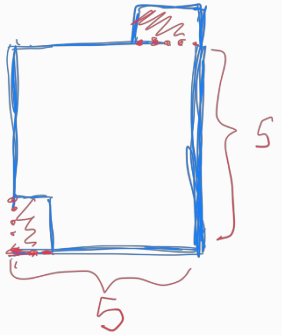
In the fifth grade, there are 5 times as many students in the choir than in the band.

There are 48 grade 5 students in the choir and band combined. How many grade 5 students are in the choir?



Using Visual Representations as a Strategy to Support Algebra Readiness and Success for All

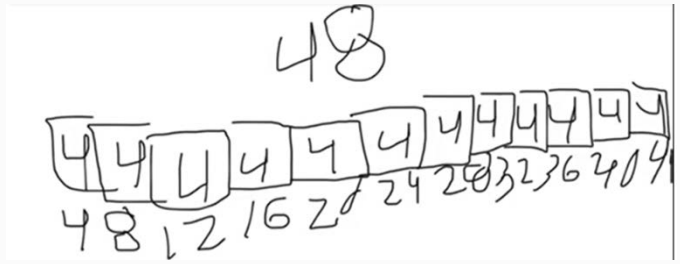
What are mathematical visual representations (VRs)?

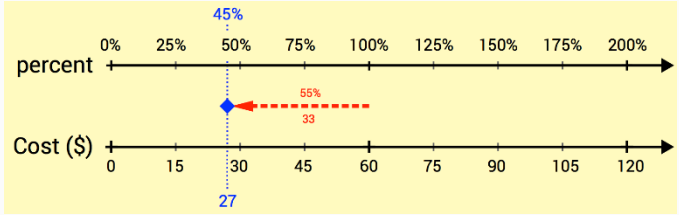


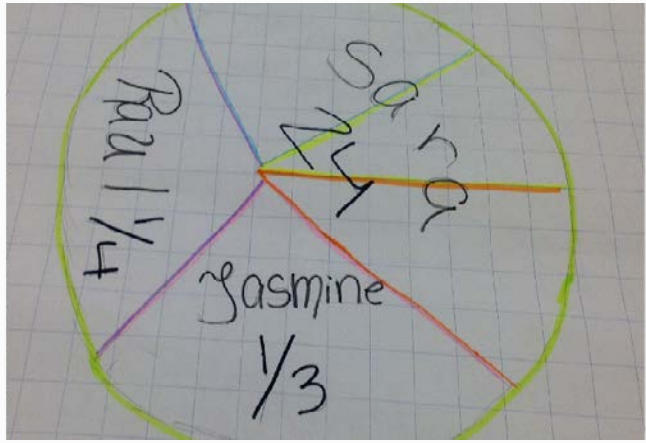
- **Drawing** (e.g., enhancing figures in geometry tasks)
- **Diagramming** (e.g., tape diagrams, number lines, double number lines, area models related to word problems and other quantitative tasks)

1/3 + 1/6 = 1/2

8

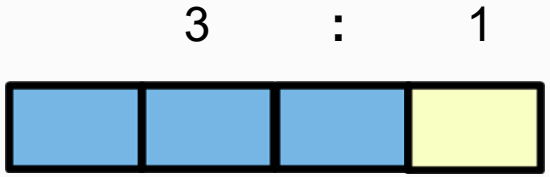






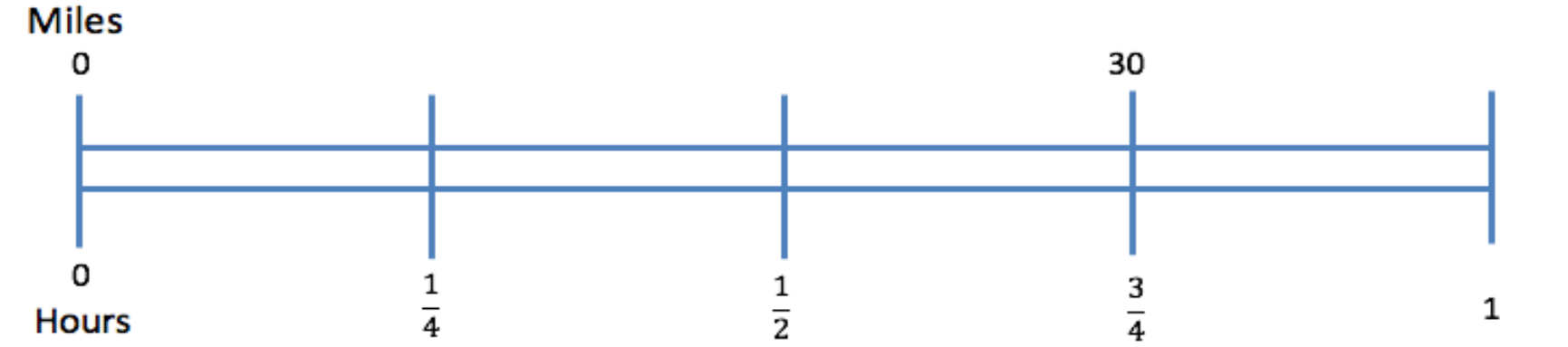
Why use mathematical visual representations?

- Competent mathematical thinkers use mathematical visual representations flexibly in problem solving *(Stylianou, 2002; Stylianou & Silver, 2004).*
- Mathematical visual representations can reinforce students' conceptual understanding of rational number *(Gersten et al., 2009; Siegler et al., 2010).*
- Understanding how to select the representations most appropriate for solving a task from a variety of visual representations provides more access to solving the task. *(Woodward et al., 2012)*



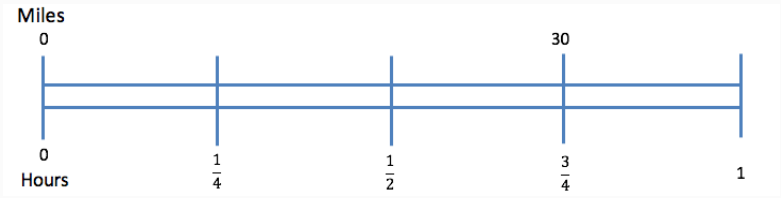
What relationships does the double number line show?

Sam bought a used motorcycle. It was on sale because it could not go very fast. Sam was able to go 30 miles in $\frac{3}{4}$ of an hour.

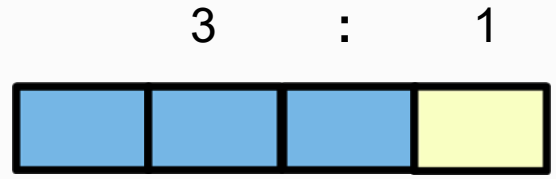


Visual representations present relationships

- A double number line can show relationships, such as a rate, and present proportionality.



- Visual representations, such as tape diagrams and double number lines, can show the mathematical structure in problems and are different from pictures about the problem context *(Diezmann & English, 2001).*

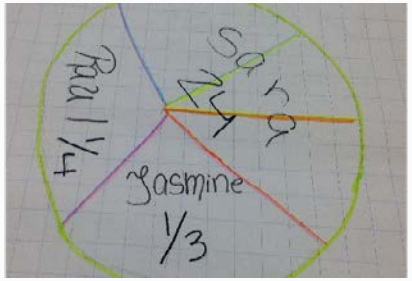


Why use visual representations with English learners?

- Diagrams can reinforce learning of mathematical concepts, processes, language, and norms of mathematical communication *(Chval & Khisty, 2001; Goldin-Meadow, 2000; Moschkovich, 2002; Woodward et al., 2012).*

- Using diagrams to represent the structure of the mathematics can engage English learners in the mathematics of a task while also addressing linguistic challenges *(Schleppegrell, 2007).*

Why use visual representations with English learners?



Presenting visual representation to students provides English learners much-needed opportunities to respond to questions and communicate mathematically about ideas, arguments, and conclusions, using both academic and nonacademic vocabulary (Moschkovich, 1999; Schleppegrell, 2007; Téllez & Waxman, 2006).

Questions

What questions do you have about how visual representations support English learners' engagement in mathematics or their mathematical communication?

Connecting Research to Practice: Using Mathematics Tasks

Sharing jelly beans

- Explore visual representations (VRs) together while engaging with a mathematical task.
- Experience language strategies in the context of problem solving.

Sharing jelly beans: Three reads

1. What is the problem about?
2. What do you need to find out?
3. What important information is given?

Sharing jelly beans: Three reads

Hector had a bag of jelly beans.

He gave $^1/_4$ of the jelly beans to Susan.

Then Hector gave $^1/_6$ of the jelly beans he had left to Pepita.

After giving jelly beans to Susan and Pepita, Hector had 20 jelly beans left in his bag.

How many jelly beans did Hector have at the beginning?

1. What is the problem about?

Sharing jelly beans: Three reads

Hector had a bag of jelly beans.

He gave $^1/_4$ of the jelly beans to Susan.

Then Hector gave $^1/_6$ of the jelly beans he had left to Pepita.

After giving jelly beans to Susan and Pepita, Hector had 20 jelly beans left in his bag.

How many jelly beans did Hector have at the beginning?

2. What do you need to find out?

Sharing jelly beans: Three reads

Hector had a bag of jelly beans.

He gave $^1/_4$ of the jelly beans to Susan.

Then Hector gave $^1/_6$ of the jelly beans he had left to Pepita.

After giving jelly beans to Susan and Pepita, Hector had 20 jelly beans left in his bag.

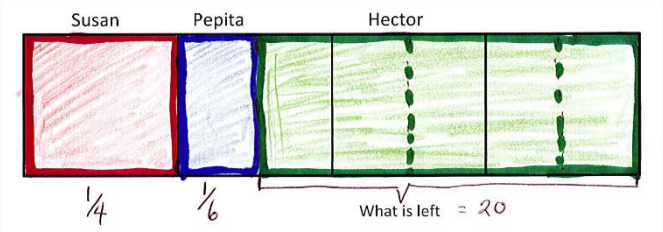
How many jelly beans did Hector have at the beginning?

3. What important information is given?

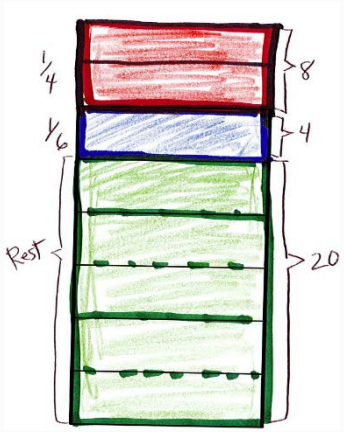
Poll

Which one of these representations does your VR resemble?

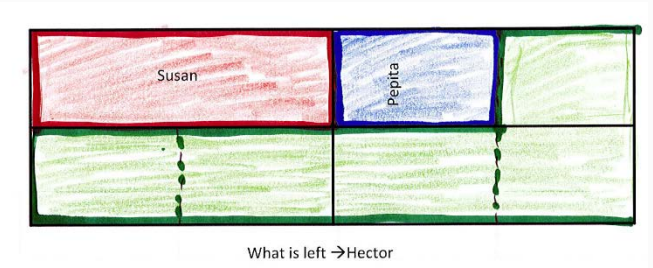
A.



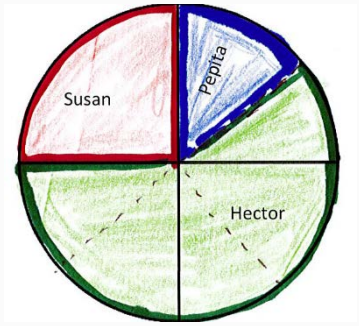
B.



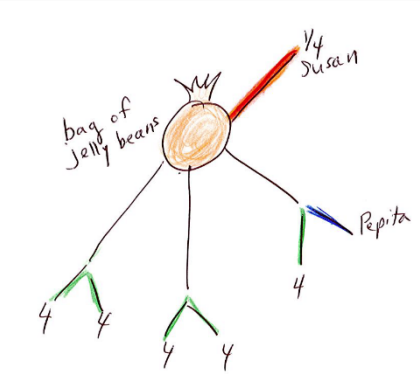
C.



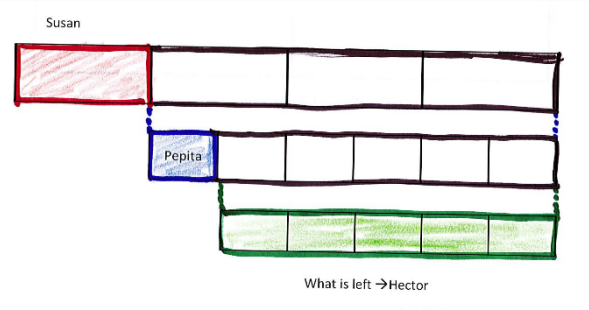
D.



E.



F.



G. None of the above

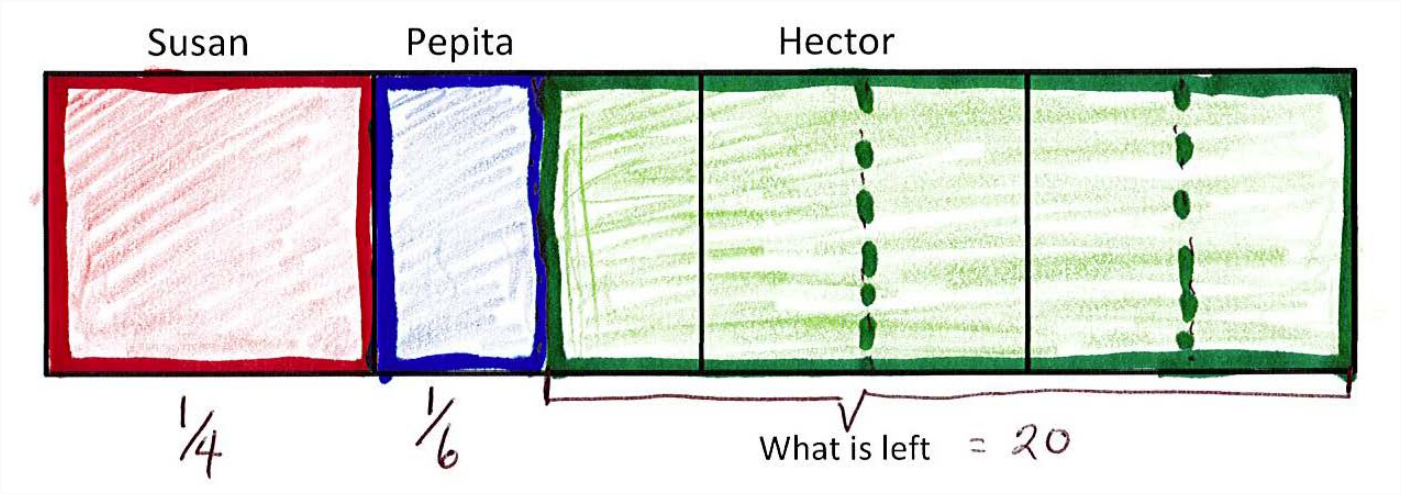
Sharing jelly beans: Diagram

In pairs:

- Share the diagrams you started.
- If you finish discussing your diagrams, you can:
  - Create additional diagrams that could help solve the problem.
  - Discuss what other questions could be answered using the diagrams the student created.

- *I represented the candies Pepita had by...*
- *I represented the candies Hector had left by…*
- *I see a relationship between… and … in the diagram.*

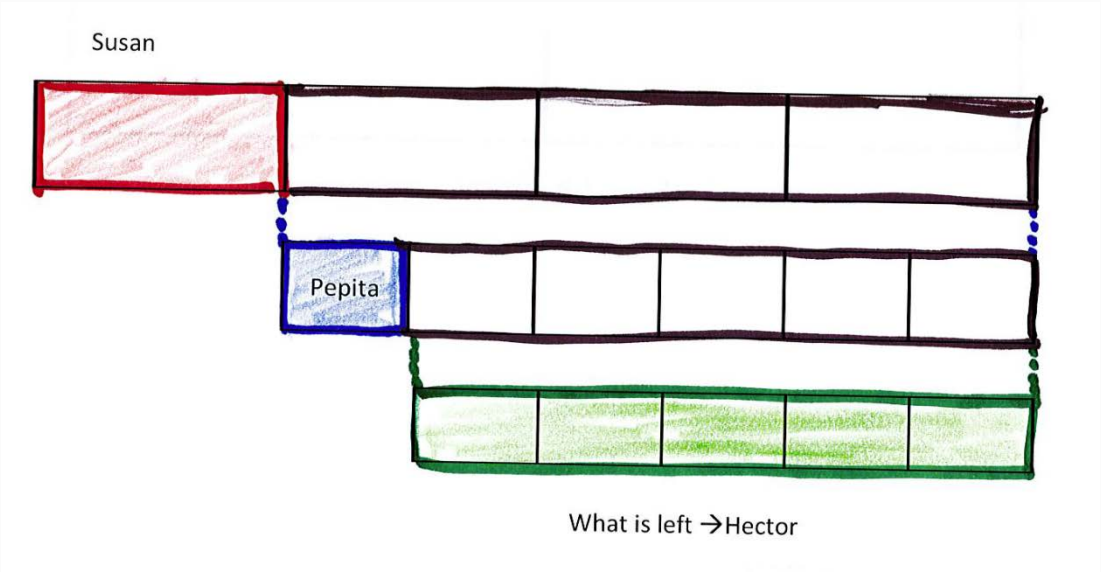
Sharing jelly beans: Example representation



This diagram represents the candies [Hector/Susan/Pepita] had by...

I see a relationship between… and … in this diagram.

Sharing jelly beans: Example representation



This diagram represents the candies [Hector/Susan/Pepita] had by...

I see a relationship between… and … in this diagram.

Sharing jelly beans experience: Debriefing

- In what ways did or could the sharing jelly beans experience support students to understand and use mathematical language?
- How can visual representations support students' *problem-solving* and *mathematical structure* in the mathematics classroom?
- What do you notice about how visual representations can support students who are English learners specifically?

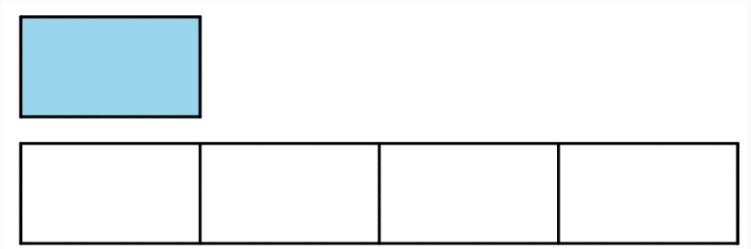
Comparing driving

- Tara and Sam's combined driving distance this week was 60 miles.
- Sam drove 4 times as far as Tara.
- **How many miles did Tara drive?**

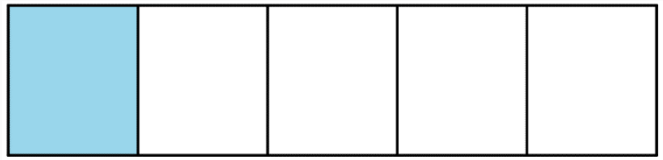
  - Use **<u>two different methods</u>** to find the answer and show your thinking.
  - Use a diagram for at least one method.

Sharing comparing driving: Diagram

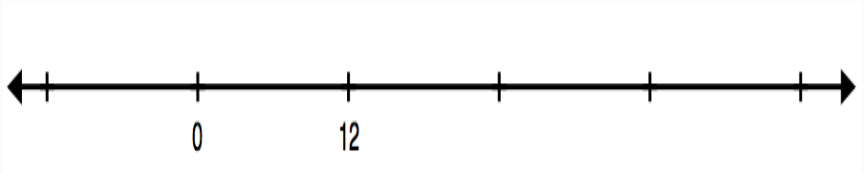
A.



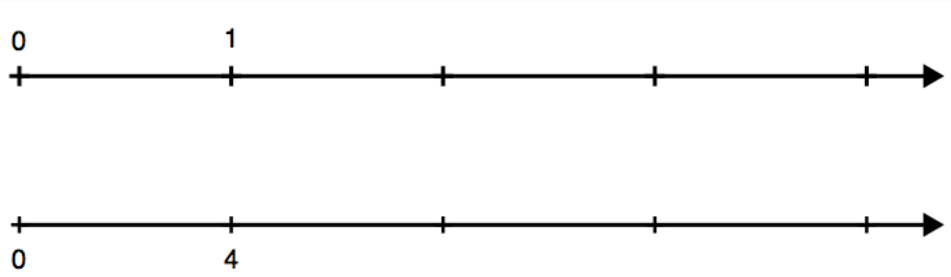
B.



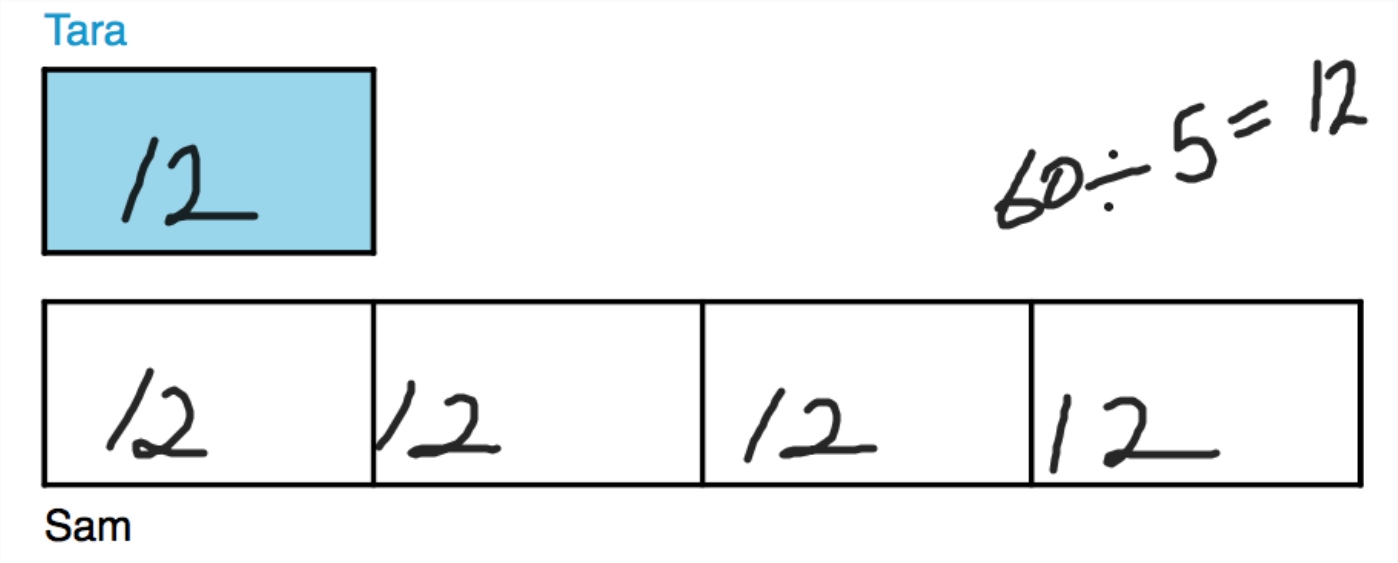
C.



D.



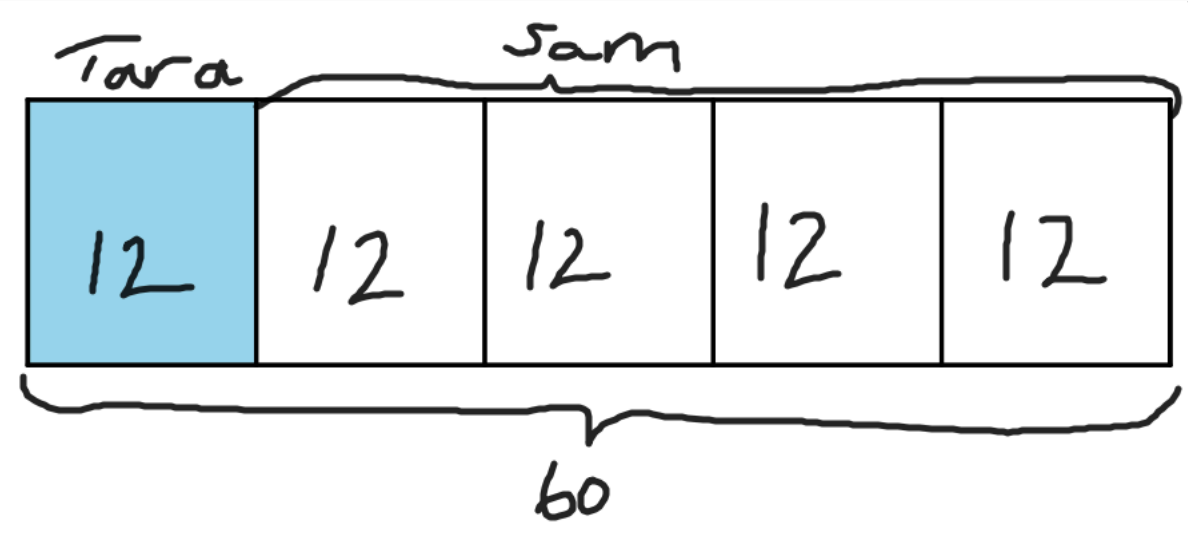
Comparing driving



This diagram represents the distance [Tara/Sam] drove by...

I see a relationship between… and … in this diagram.

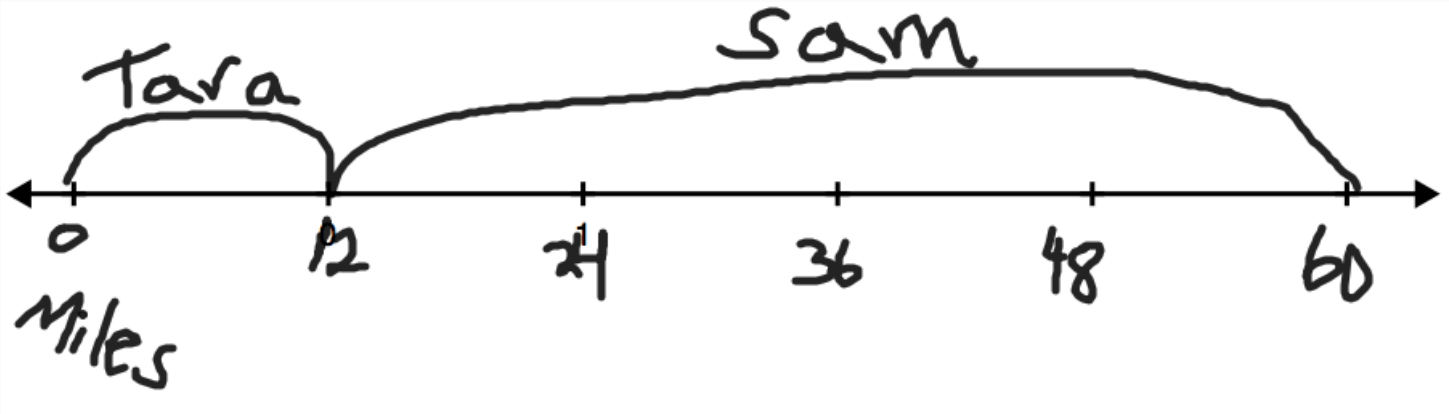
Comparing driving



This diagram represents the distance [Tara/Sam] drove by...

I see a relationship between… and … in this diagram.

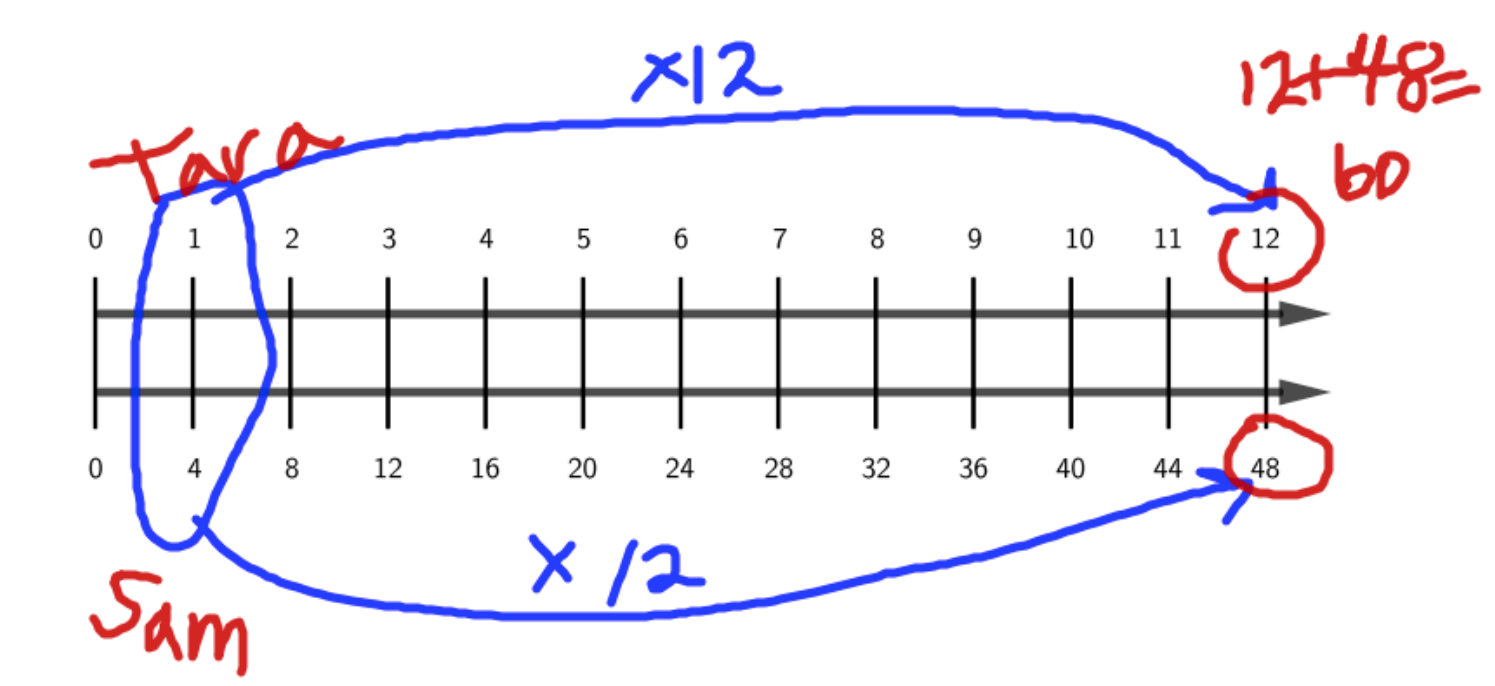
Comparing driving



This diagram represents the distance [Tara/Sam] drove by...

I see a relationship between… and … in this diagram.

Comparing driving



This diagram represents the distance [Tara/Sam] drove by...

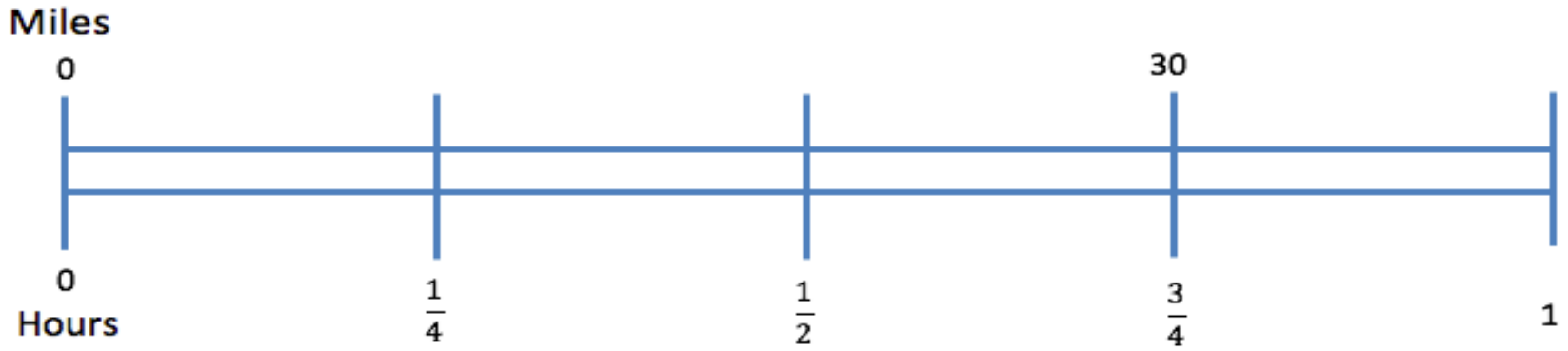
I see a relationship between… and … in this diagram.

Exploring double number lines

Sam's Motorcycle

Sam bought a used motorcycle. It was on sale because it could not go very fast. Sam was able to go 30 miles in $\frac{3}{4}$ of an hour.

a) How far can he go in 1 hour? Use a double number line to help solve this problem. Explain your solution.



Exploring double number lines (cont.)

Sam's Motorcycle

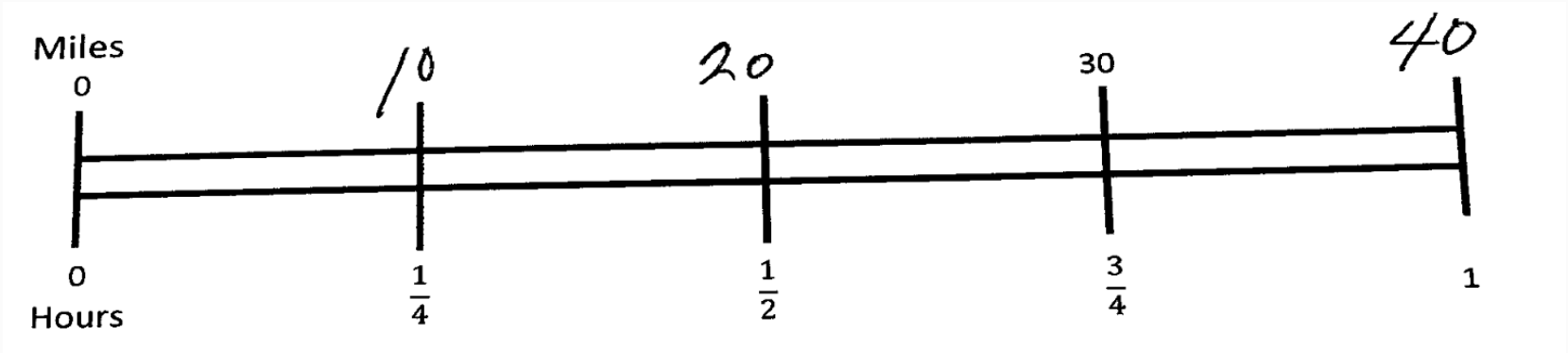
b) How far can he go in $3\frac{1}{2}$ hours?

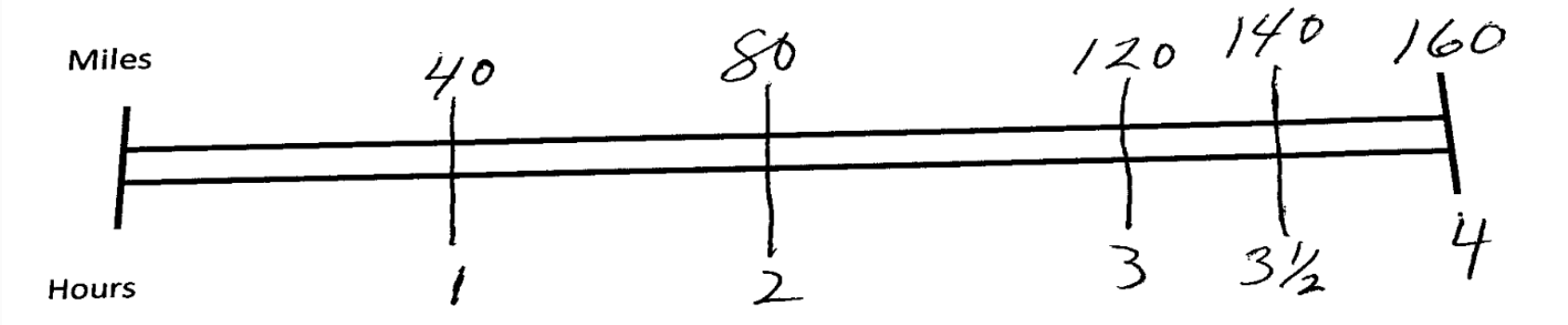
Use a double number line to help solve this problem.
Explain your solution.

Miles

Hours

Exploring double number lines (cont.)





Key Take-aways and Summary:

Visual representations can:

- Provide access to mathematics word problems
- Be used as thinking tools ***and*** as representation tools
- Enhance mathematical communication
- Provide opportunities to reinforce Standards of Mathematical Practices, such as:

  MP1. Make sense of problems and persevering in solving them

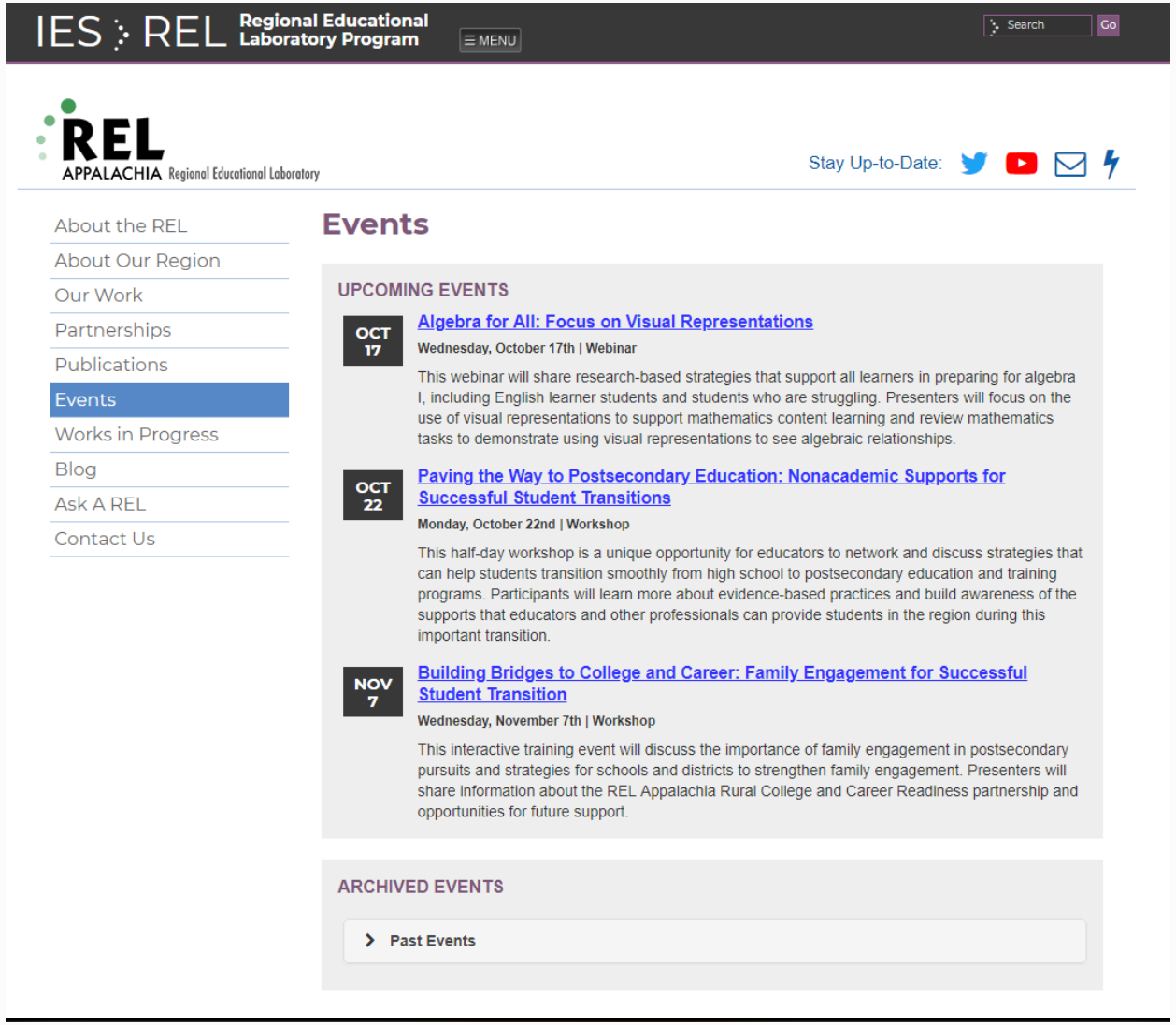
  MP3. Construct viable arguments and critiquing the reasoning of others

  MP7. Look for and make use of structure

Accessing Webinar and REL AP Resources

Webinar resources are available at: https://ies.ed.gov/ncee/edlabs/regions/appalachia/events.asp

- PowerPoint Slides
- Task Handouts
- Webinar Slide References



Ask A REL

Ask A REL is a collaborative reference desk service provided by the 10 RELs that, by design, functions much in the same way as a technical reference library. It provides references, referrals, and brief responses in the form of citations to research-based education questions.
https://ies.ed.gov/ncee/edlabs/regions/appalachia/ask-a-rel.asp

Example:
What type of mathematical skills and knowledge predict success in algebra I? What does the research say about strategies or interventions to improve algebra readiness (particularly in middle school)? https://ies.ed.gov/ncee/edlabs/regions/appalachia/askarel/aar05.asp

Ask A REL

Ask A REL Instructions

To ask an education-focused question, please complete the question submission form below:

1. Include your name and email address
2. Select your state from the drop-down menu
3. Type your question in the box
4. To receive a copy of your question, check the box "I would like to receive a copy of my question sent to my e-mail."

Note: The questions you submit are sent directly to the REL selected and not stored on this site or by the Institute of Education Sciences. To ask a question or to provide a comment about the Regional Educational Laboratory Program or the Institute of Education Sciences, select the "Contact" button at the top of this page.

---

* **Full Name:**

* **Email:**

* **Confirm Email:**

* **State:** Select One

* **Question**

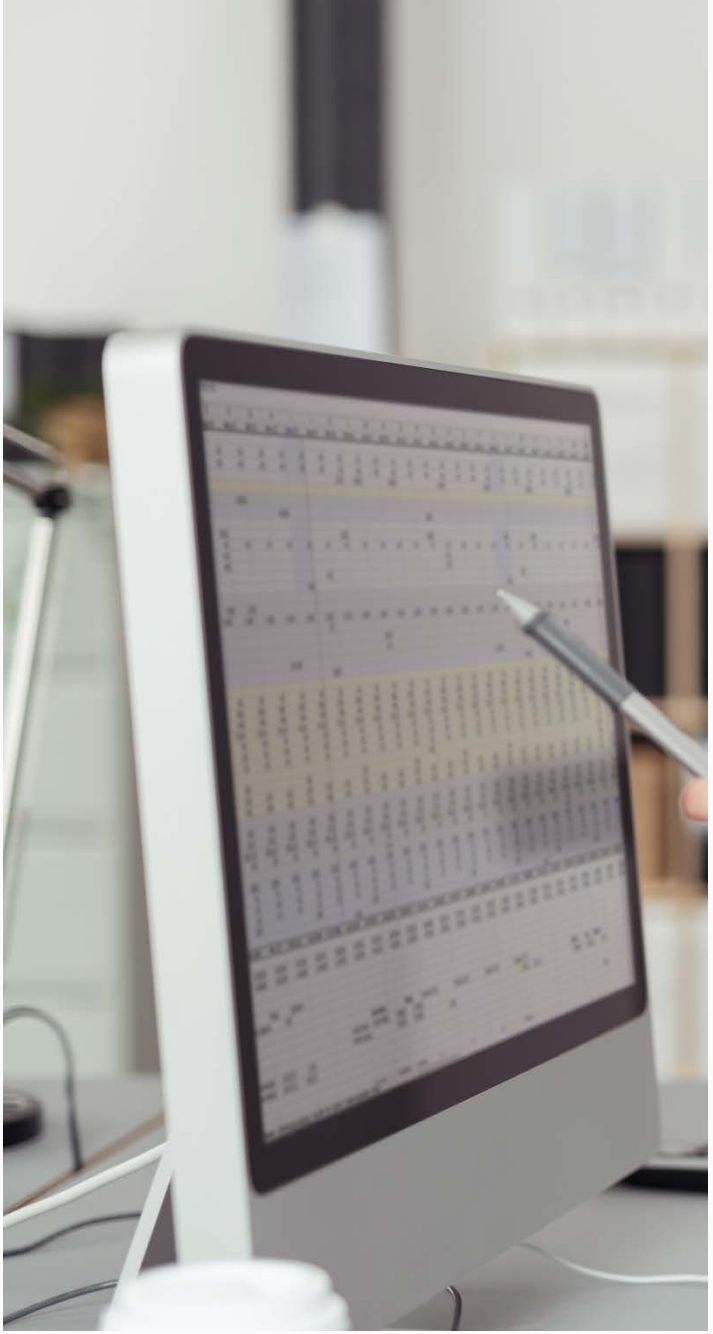
I would like to recieve a copy of my question sent to my email.

Submit

REL Appalachia Newsletter

Sign up for the REL Appalachia Newsletter!

https://ies.ed.gov/ncee/edlabs/regions/appalachia/newsletters.asp



Please complete the Stakeholder Feedback Survey.

This survey will be sent to all participants.

REL APPALACHIA
Regional Educational Laboratory At SRI International

Thank you!

Presenters

Pamela J. Buffington
pbuffington@edc.org

Jill DePiper
jdepiper@edc.org

Carmen Araoz
carmen.araoz@sri.com

REL Appalachia

https://ies.ed.gov/ncee/edlabs/regions/appalachia/

@REL_Appalachia

RELAppalachia@sri.com