#### Student Success in Mathematics Partnership Meeting *October 5, 2021*

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#### Welcome



Laura Kassner Partnership Liaison



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2

#### Student Success in Mathematics partnership: REL Appalachia staff



#### Pam Buffington **Partnership Lead**





#### Laura Kassner **Partnership Liaison**





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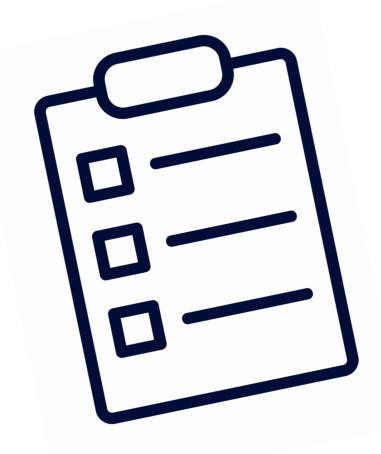


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#### Agenda

- Welcome
- Goals and objectives
- Supporting students with mathematics disabilities and difficulties
- Student Success in Mathematics (SSM) partnership products and resources –Professional learning model (PLM) compendium
  - -Resource collection
- Closing



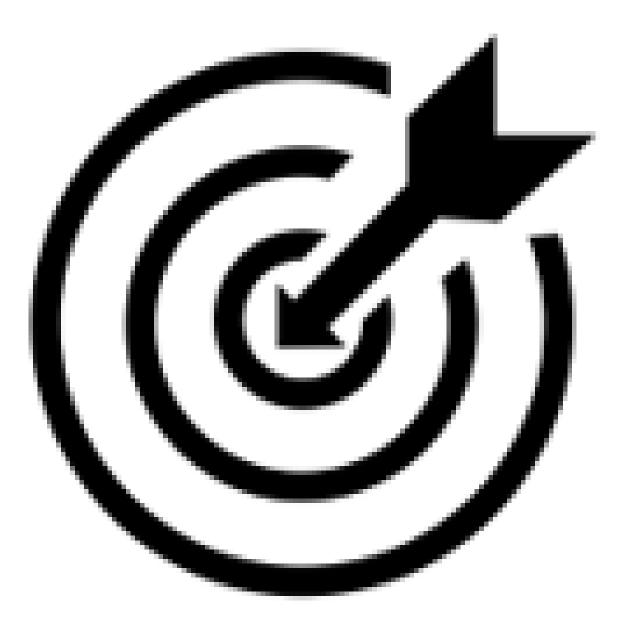




## Meeting goals

- Develop an increased understanding of evidencebased instructional approaches to support students with mathematics disabilities and difficulties.
- Enhance access to and use of Student Success in Mathematics (SSM) partnership products and resources.







#### Meeting objectives

- Increase knowledge of Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades practice guide recommendations.
- Provide strategies to assist mathematics educators in designing their instructional approaches to meet the needs of students struggling in mathematics.
- Engage in a process for planning high-quality, accessible mathematics lessons.
- Discuss the use of SSM products and resources, including the compendium of resources for the professional learning models (PLM) project and the partnership collection.





# Supporting students with mathematics disabilities and difficulties



**Amy Brodesky** Mathematics Education Expert



**REL Appalachia at SRI International** 

7

#### Presentation goals

- Provide suggestions and examples of professional learning activities to help teachers support students with mathematics disabilities and difficulties.
- Make connections to Virginia Department of Education (VDOE) resources on students with disabilities.
- Discuss ways to implement the professional learning activities in your districts.
- Build knowledge of the new IES Practice Guide, Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades (2021).
- Take a deeper dive into the material from the March 2020 session (Brodesky, 2020).







8



- 1. Background information on students with mathematics disabilities and difficulties
- 2. High-quality, accessible mathematics instruction
- 3. Research-recommended instructional practices
- 4. Mathematics accessibility lesson planning process



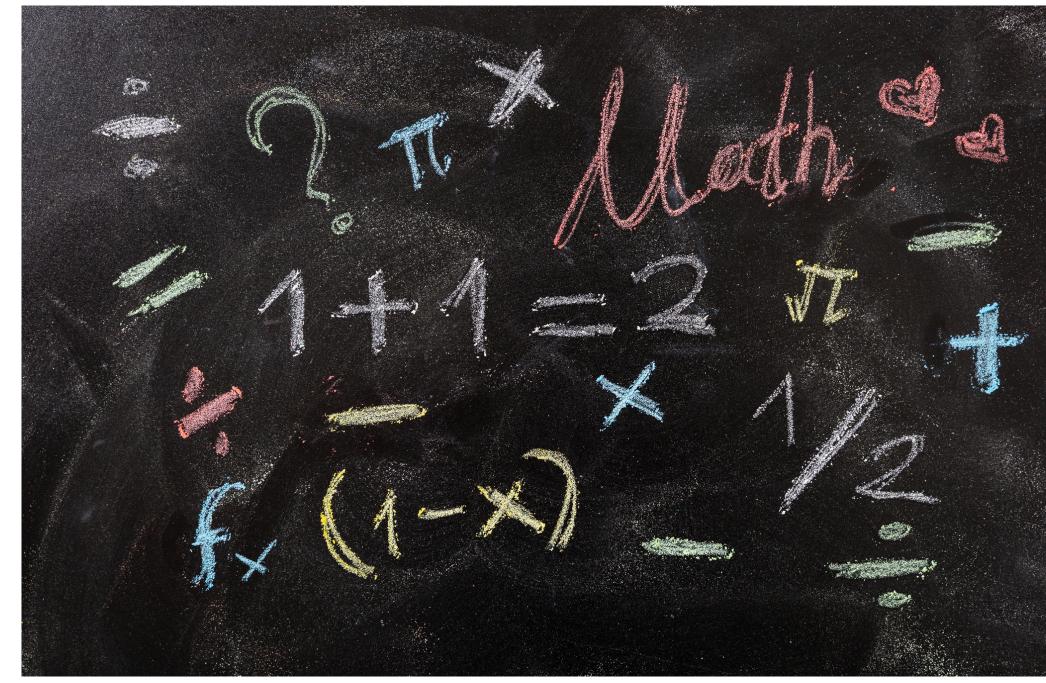


Professional learning approaches to build teachers' knowledge & practices for these topics:



## Section 1: Students with mathematics disabilities and difficulties









#### Discuss teachers' professional learning needs

- difficulties in mathematics?
- What are ways to increase teachers' motivation and buy-in to improve instruction for students with mathematics disabilities and difficulties?
- with disabilities and difficulties?







• What is important for teachers to know about students with disabilities and

• What are ways to help teachers build strengths-based approaches for students

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11

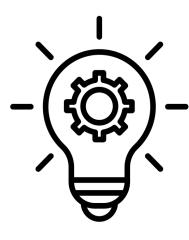
#### Professional learning (PL) suggestions for this section

PL #1. Bring in the voices of students with disabilities: use video, quotes, and examples.

PL #2. Provide background information about mathematics disabilities and difficulties.

PL #3. Convey positive messages and promote strengths-based approaches.





(*Lampert et al. 2019*; *Kobett & Karp, 2020*)





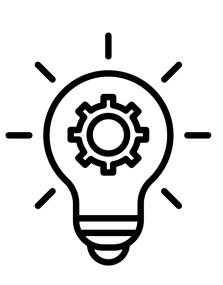
## PL #1. Bring in the voices of students with disabilities -

Elementary Example: <u>Dear Teacher video</u>

After watching the video, discuss:

- What messages from this video resonate with you?
- While the video does not focus specifically on mathematics, what messages do you think are particularly important to keep in mind for mathematics teaching?





(Brain Highways, 2020)



## PL #1. More ways to bring student voices into professional learning

#### A. Videos of students: <u>Through Your Child's Eyes</u>: Math Challenges – Alex, Grade 6 – Sam, Grade 10

- See examples on the next two slides.



# B. Quotes from students with disabilities & STEM professionals with disabilities

(Lampert et al. 2019; Lewis & Lynn, 2018; Understood for All, 2021)

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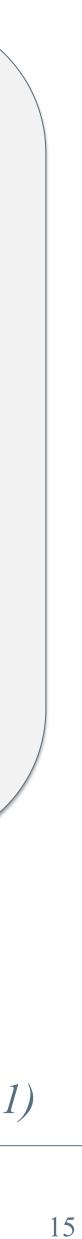
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"As I sat in that [gen. ed. math] class, something magical happened to me. I could understand what he was teaching. I was learning. I even started participating in the class, raising my hand and answering questions. I was LD. But then again, I wasn't. I still couldn't multiply or divide very well, and I had to use elaborate ways to come up with the answer. But I wasn't memorizing, I was thinking, and I was figuring out the answer. I was learning. This was one of the experiences that shot a pinhole in the bubble that trapped me in my LDness."

Student with a learning disability



(Rodis et al., 2001, p. 21)



"I found visual things to click with me a little bit better, if someone uses manipulatives. When I'm struggling with a concept, if I can give it a really concrete foundation, it will stick with me much better."

#### **Dylan Lynn, PhD Statistician with Dyscalculia**



"The reason why it's funny that I did statistics, but it makes a lot of sense, is that if I could take one of these concepts and apply it to something that was actually tangible to me, so like the percentage of people that are over the age of 55 that have a landline phone, if I come up with these stories or narratives, then I could actually recall how to do an operation, but I had to have some sort of narrative that was actually based in the real world."

(Lewis & Lynn, 2018)



#### PL #2. Provide background information on math disabilities and difficulties

#### Purpose:

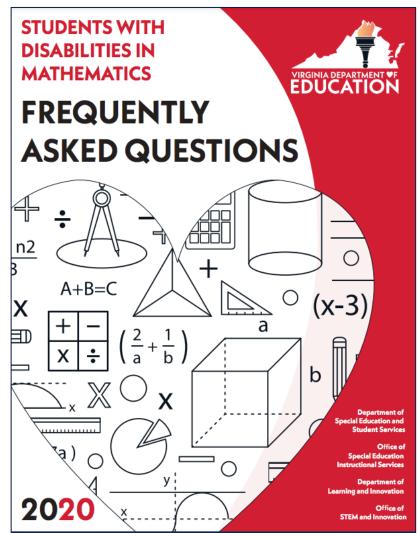
• Build teachers' knowledge of the characteristics of math disabilities/disabilities.

#### **Resources**:

- VDOE (2020a) Students with Disabilities in Mathematics: Frequently Asked Questions
- Example slides in this deck (18–24)













## Specific learning disabilities

- 14 percent of U.S. public school students are served by Individuals with Disabilities Education Act (IDEA): 7 million students
- 34 percent have a "specific learning disability"

#### A disorder in one or more psychological processes involved in:

- Listening
- Speaking
- Basic reading
- Reading comprehension

(Learning Disabilities Association of America, 2021; U.S. Department of Education, 2018)



• Written expression • Arithmetic calculation • Mathematical reasoning





## Mathematics learning disabilities (MLD)

- Students with MLD tend to:
  - Have persistent difficulties with math facts \_\_\_\_
  - Use problem-solving procedures that younger students use \_\_\_\_
  - Make frequent errors when executing procedures \_\_\_\_\_
  - Have a poor understanding of foundational concepts. \_\_\_\_
- Possible causes:
  - connect new information with prior knowledge
  - Memory deficits related to working memory \_\_\_\_



• *Prevalence*: About 5–8 percent of public school students in the United States have MLD

Underdeveloped cognitive structures, which are the mental processes necessary to

(Geary, 2004, 2011)









## **MLD** research issues Lack of consensus about where to draw the line between:

## Mathematics Disabilities



# Mathematics Difficulties

(*Lewis & Fisher, 2016*)





#### Mathematics difficulties



- Number sense
- Fluency with math facts
- Problem solving
- Word problems
- Moving from concrete to abstract
- Making generalizations
- Applying strategies to new situations
- Reflecting on thinking—metacognition

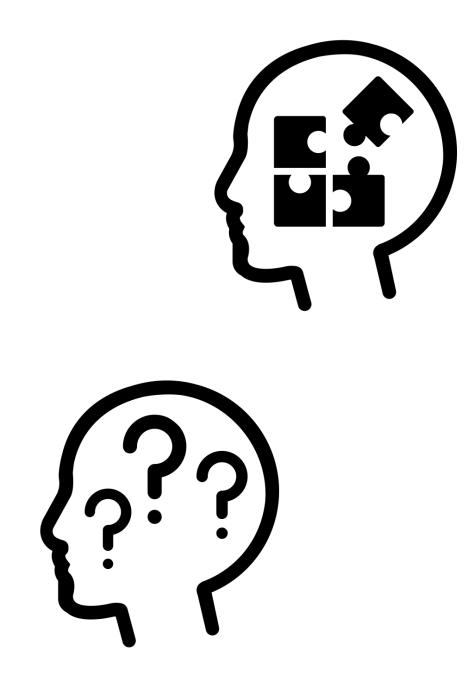


(Allsop et al., 2018)





#### Students who struggle with mathematics over time may:



- Experience mathematics anxiety
- Have learned helplessness • Take a passive approach to learning • Become discouraged easily • Not know how to ask for help

- Try to hide that they are having difficulty.



(Allsop et al., 2018)





#### Research on dyslexia and mathematics

- Common difficulties: memorization, math facts
- Areas of strength: 3-D spatial thinking, complex reasoning, patterns, making connections, thinking in narratives – related to mathematical proofs



(Attree et al., 2009; Eide & Eide, 2012; Everatt et al., 2008; Trott, 2015)





"As a dyslexic, I've never been good at calculations or recalling rote facts like times tables. Here's the thing: beyond a certain point in mathematics, it's not really about calculations... Geometry class was when math became interesting, and easier for me. Suddenly I was in a world, not of strands of symbols to be processed, but of shape, space, lines, angles, concepts, and narrative-like proofs. Suddenly everything made sense."

PhD mathematician in topology who has dyslexia and ADHD

Source: https://toomai.wordpress.com/2014/09/17/dyslexic-mathematician/.



(Lewis & Lynn, 2018)







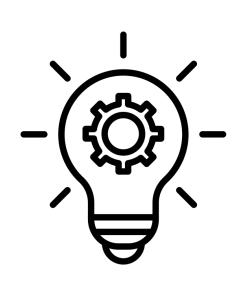
## PL #3. Promote strengths-based approaches to math teaching for students with disabilities

In professional learning with teachers, it's helpful to:

- Shift mindsets away from deficit models of students with disabilities.
- Communicate the importance of strengths-based approaches.
- Address myths about learning math and students with disabilities, such as:
  - Myth: some people are "math people" and some are not.
  - Myth: students with learning disabilities do not benefit from multiple strategies and should be shown only one way to learn.

*Resource:* Example slides 26–28





(Lambert, 2018; Kobett & Karp, 2020)





#### Neurodiversity of learners

- We all learn differently.
- Students with disabilities are part of a **continuum** of learners.
- Move away from viewing disabilities as deficits.



# 

(*Lambert*, 2018)









#### **Essential shifts**

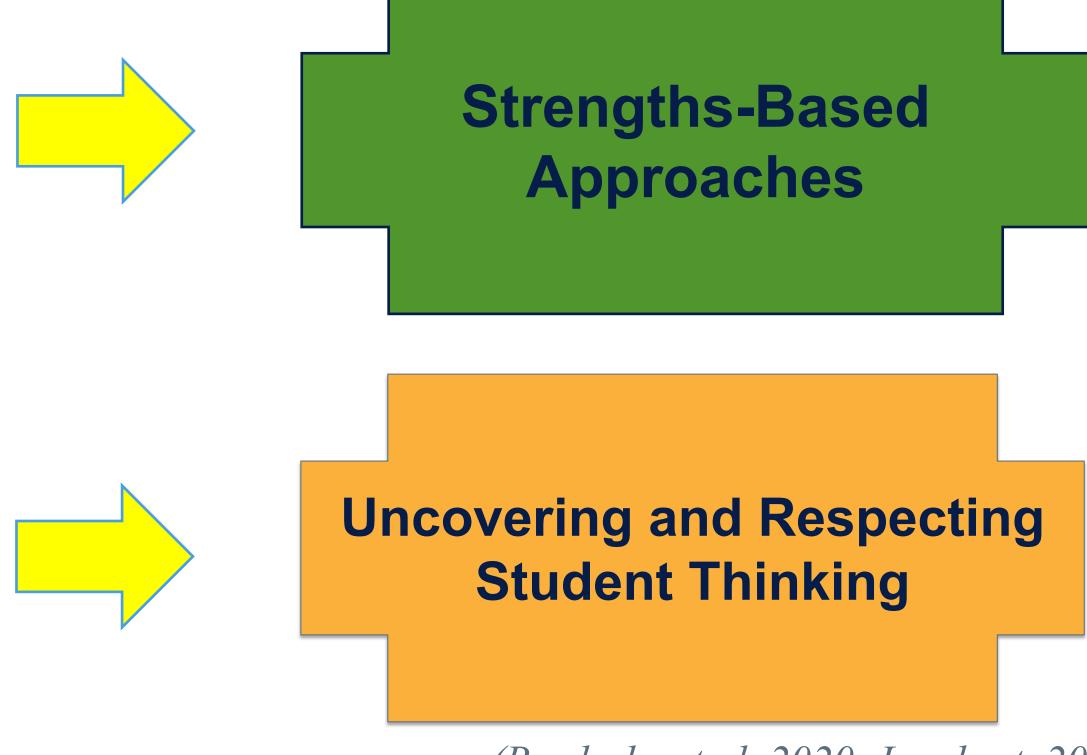


#### **Deficit Models** of Students with Disabilities

#### **Making Assumptions** that Students Cannot Do Mathematics





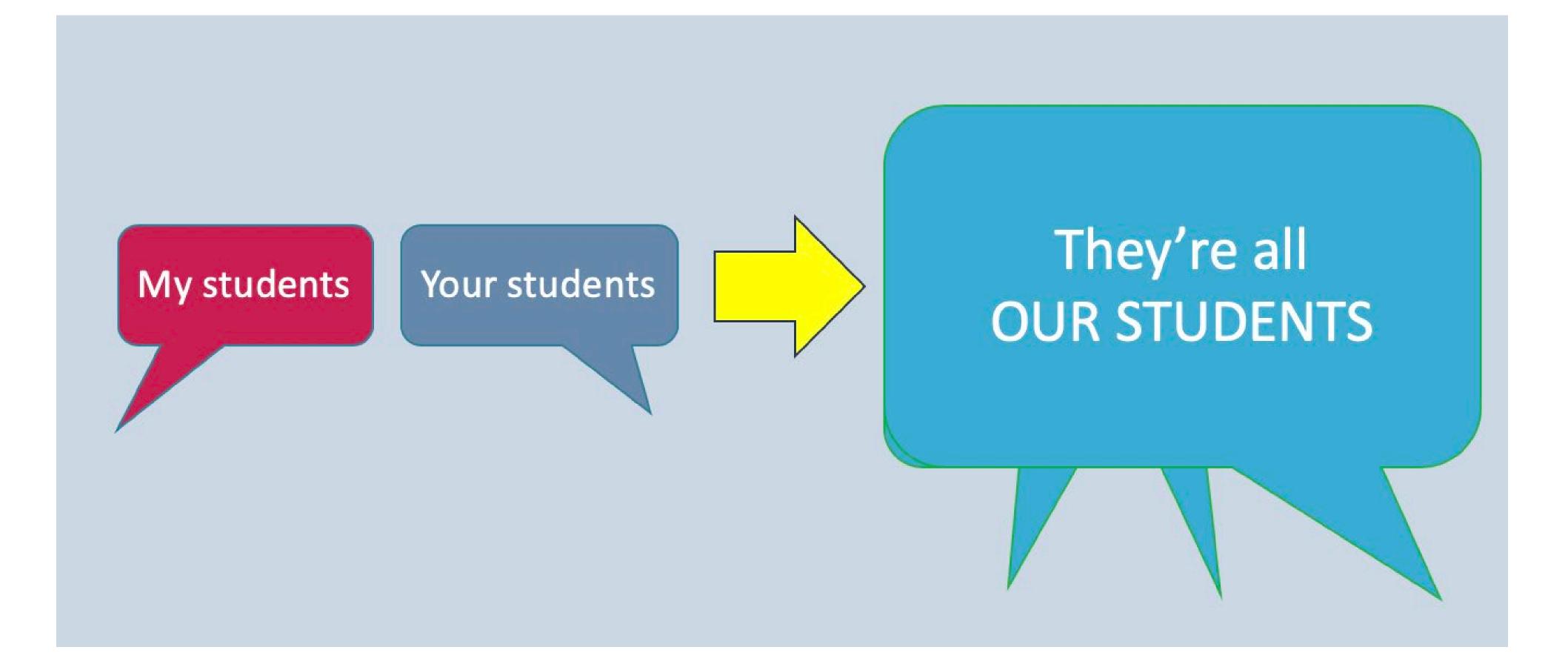


(Brodesky et al. 2020; Lambert, 2018)





#### General educators & special educators need to work together



(Bottge et al., 2015; Brodesky, 2020; Brodesky et al. 2020; Geary et al., 2012; Hourcade & Bauwens, 2001; Pashler et al., 2007)







## Section 2. High-quality, accessible mathematics instruction





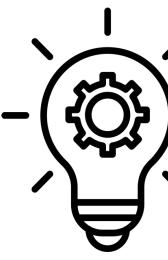


#### Professional learning (PL) suggestions for this section

PL#4. Build a shared vision of high-quality, accessible, and inclusive mathematics instruction.

PL#5. Use common language and clarify terms. 5A. Modifications 5B. Accommodations 5C. Scaffolding vs. Accommodations 5D. Scaffolding Suggestions





(Allsopp et al., 2018; Brodesky, 2016)





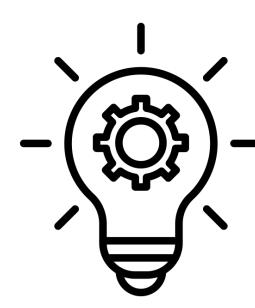
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# What would you look for as evidence of high-quality accessible mathematics instruction?

#### Write ideas for each prompt on a sheet of paper.

- What would you hope to see? Teachers... Students...
- What would you hope to hear? Teachers.... Students...
- What would the classroom environment **feel** like?





**Example professional learning activity:** Imagine that you are visiting a mathematics class.



## High-quality, accessible mathematics instruction

- Meets the needs of a range of learners while maintaining the integrity of the math.
- Teaches math in a way that builds conceptual understanding, reasoning, and sense-making
- Provides opportunities to experience joy in doing math and to struggle productively.
- Helps build positive identities as math thinkers and doers.
- Sets high expectations for student learning.
- Provides accommodations and support while helping to build independence.
- Uses research-based instructional strategies for supporting students with math disabilities/difficulties.



(Allsopp et al., 2018; Brodesky, 2016)







#### PL #4. Build a shared vision by discussing recommendations from leaders in the field

Equity Principle

"Excellence in mathematics education requires equity—high expectations and strong support for all students."

"Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students."





(National Council of Teachers of Mathematics, 2000)







"Teachers so want their students to succeed that they sometimes fall into the trap of doing the work, and the thinking, for their students. However, as tempting as it is to show students how to perform the mathematics before releasing them to try it on their own so they do not have to struggle, it is imperative that teachers allow all learners, including those who struggle, to make sense of and reason about the mathematics on their own first."





(Dixon et al., 2019, p. 11)



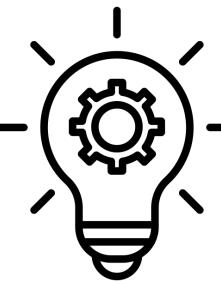


#### PL #5. Use common language & clarify terms

- A modification changes what a student is taught or expected to learn.
  - -Changes the mathematics goals
- An <u>accommodation</u> changes *how* a student learns the material.
  - -Keeps the mathematics goals; provides support to help students access mathematics tasks and reach goals
- Professional learning suggestions:
  - Clarify terms
  - Provide and discuss relevant examples and non-examples

Understood for All (2021). Difference between accommodations and modifications. Understood.org. https://www.understood.org/articles/en/the-difference-between-accommodations-and-modifications





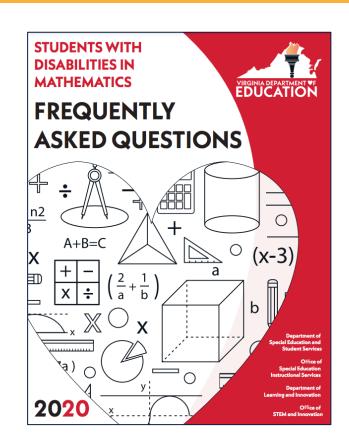


#### PL #5A. Modifications

Example points to review with teachers:

- Modifications are typically made for students with significant disabilities.
- Modifications are focused on the individual student's IEP goals.
- Student may be working on objectives below grade level, but the subject should be the same as for the rest of the class. For example:
  - If a student is working on developing knowledge about addition and the class is practicing addition and subtraction facts, have the student focus solely on addition.
  - If a fourth-grade class is solving word problems but a student is at a second-grade level, modify the word problem's content to align with the second-grade standards. Use the Mathematics Vertical Articulation Tool (MVAT) as a resource.





(VDOE, 2020a, 2020b)





## PL #5B. Accommodations

<b>Student Difficulty</b>	Math Tasks: Potential Barriers	Accommodations
Reading difficulties	Word problems	<ul> <li>Read aloud</li> </ul>
Fine motor difficulties	Manipulatives and diagrams	<ul> <li>Use a larger manipulative</li> </ul>
Difficulty staying focused	Student handout with multiple problems on page	<ul> <li>Give fewer problems at a time</li> </ul>
Auditory processing difficulties	Verbal directions	<ul> <li>Give visual and verbal directions</li> </ul>



(Brodesky et al., 2002)





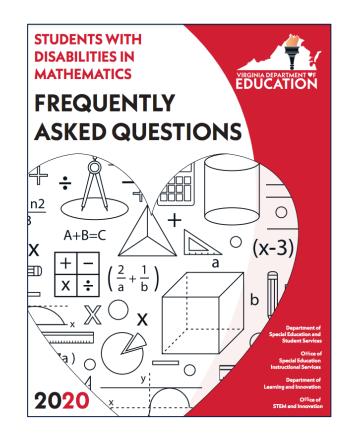
## Resource: Examples of accommodations

Table of Accommodations with examples for different student difficulties

Challenge Areas: Language & Reading; Memory; Processing Speed & Response Time, Social

	SPECIFIC DIFFICULTIES		ACCOMMODATIONS	_	INSTRUCTIONAL STRATEGIES
ŀ	Following and understanding verbal directions; giving directions	:	Read text aloud to students Create digital versions of pencil/	•	Present instructions both orally and in written form with visual cues
ŀ	Using and understanding vocabulary (multiple meanings, similarities/		paper documents Use an audio recorder	•	Keep oral and written instructions short and simple





Visual-Spatial Processing, Fine Motor Skills, Executive Functioning Skills; Attention, Psycho-

Based on EDC's Addressing Accessibility in Mathematics project and other sources (e.g., Brodesky et al., 2002)



## PL #5C. Accommodations vs. scaffolding

**Accommodations:** Provide *access* to mathematics tasks, such as larger print. Accommodations are specified by student's IEPs.

**Scaffolding Strategies:** Provide tailored supports based on students' current level of understanding with the goal of advancing their learning; scaffolding provides support without giving too much away. Scaffolding strategies are useful for *all* students.

#### **Examples of Scaffolding:**

- Help students get started and connect to what they already know
- Help students stay on track rather than showing them how to solve the problem
- Nudge students toward working independently



Ask students questions to help them access the problem at their level of understanding

(*Dixon et al., 2019*)





## PL #5D. Suggestions for scaffolding

- Use Just-in-Time scaffolding: Start by encouraging students to problemscaffolding much more precise and targeted to students' needs.
- Avoid Just-in-Case scaffolding: A "just-in-case" strategy is suggesting a strategy *before* providing students with the time to discover strategies for solving the task on their own.

(Dixon et al., 2019, p. 13)



solve and reason about the mathematics to see what they already know. Once teachers have a clear idea of what students understand, they can make the

• Anticipate potential difficulties but keep strategies in your back pocket.

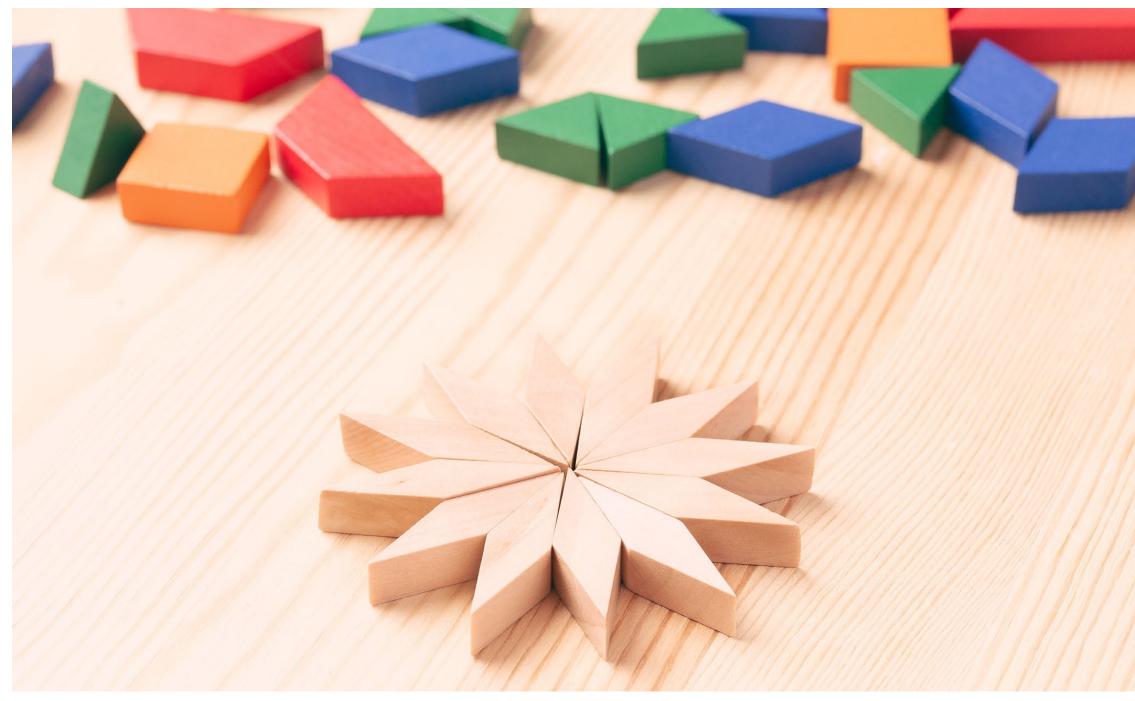






## Section 3. Research-Recommended Instructional Practices









# Guiding questions for this section

## Learn about recommendations

- Why are they important for students with mathematics disabilities and difficulties?

## Identify focus practices for future professional learning • **Reflect on current practices**: To what extent are teachers using these

- evidence-based practices?
- professional learning with teachers?



# • What are the evidence-based recommendations in the IES Practice Guide?

• Which evidence-based practice(s) might you want to focus on in future

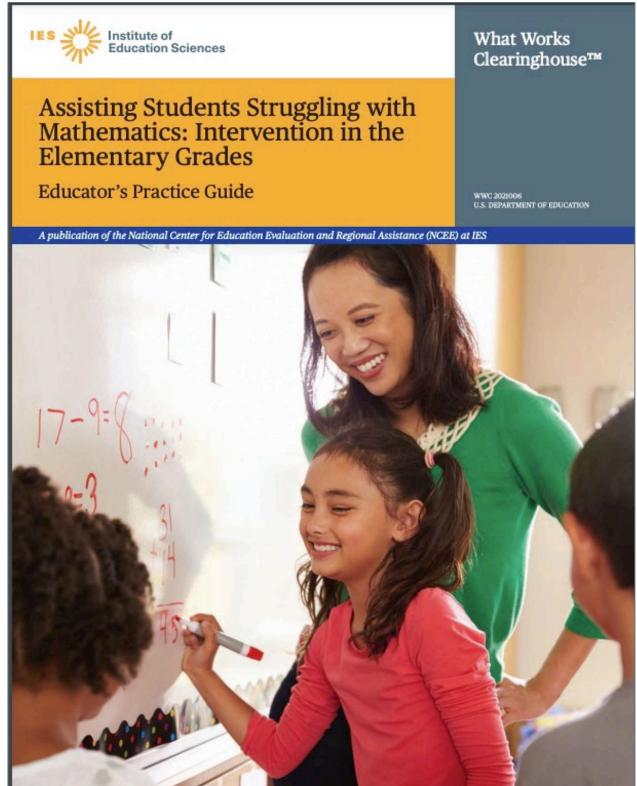


## IES practice guide: Assisting Students Struggling with Mathematics

Evidence-Based Recommendations

- 1. Systematic Instruction
- 2. Mathematical Language
- 3. Representations
- 4. Number Lines
- 5. Word Problems
- 6. Timed Activities





(Fuchs et al., 2021)

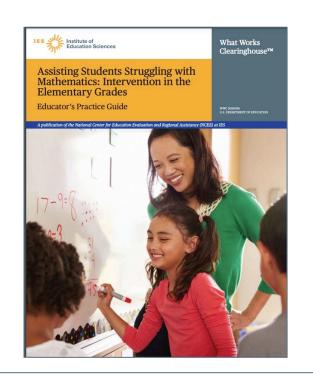




## Crosswalk of recommendations

## **IES Practice Guide**

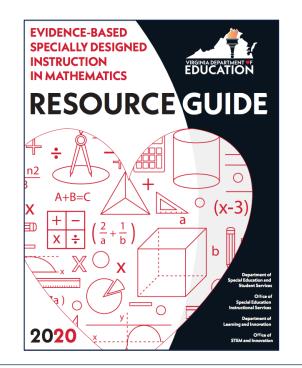
- 1. Systematic Instruction
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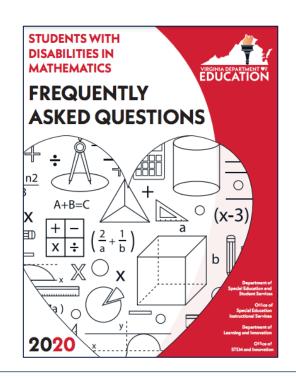




## **VDOE Resource Guides**

- 1. Explicit Instruction
- 2. Formal Mathematical Language
- 3. Concrete, Representational, & Abstract Connections
- 4. Word Problem Solving
- 5. Fact and Computational Fluency





(Fuchs et al., 2021; VDOE, 2020a, 2020b)





# Professional learning (PL) suggestions for this section

- PL #6. Strengthen teachers' mathematical language practices
- PL #7. Strengthen teachers' use of manipulatives
- PL #8. Build teachers' practices for using representations (concrete, semi-concrete, abstract)
- PL #9. Engage teachers in number line activities
- PL #10. Strengthen understanding of word problems types
  - 10A. Explain why key word approach should be avoided
  - 10B. Explore different ways to help students with word problems



(Fuchs et al. 2021)

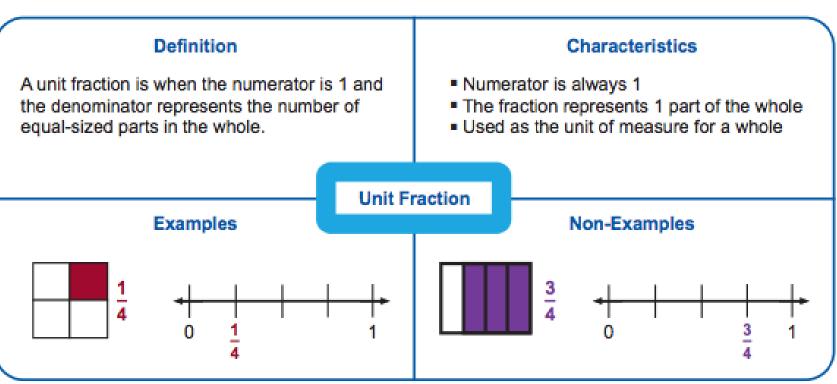






## Practice Guide Recommendation 2: Mathematical language

1. are learning.

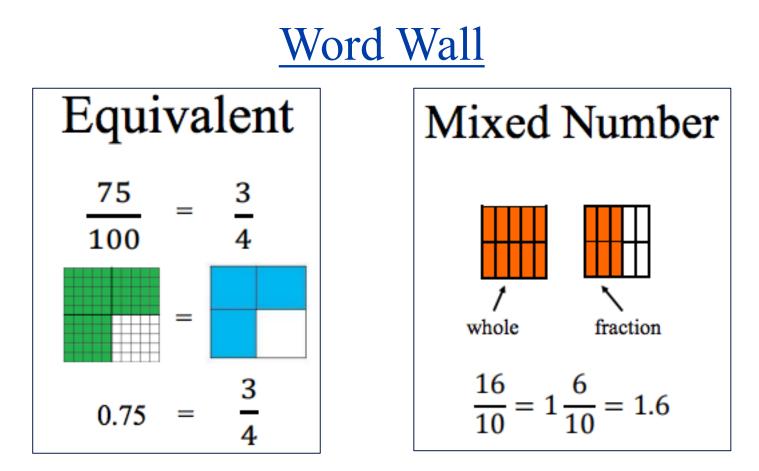


#### Graphic Organizer

- 2. of important mathematical vocabulary words.
- 3. problem solving.



Routinely teach mathematical vocabulary to build students' understanding of the mathematics they



Use clear, concise, and correct mathematical language throughout lessons to reinforce students' understanding

Support students in using mathematically precise language during their verbal and written explanations of their

(Fuchs et al., 2021)







# Complexities of mathematics vocabulary

## Some terms...

- Are shared with everyday English but have distinct meanings in mathematics *– right, volume, expression*
- Sound like everyday English words - sum and some
- Have more than one meaning in mathematics *– square, round*
- Are related and often confused *– mean and median; factor and multiple*

What mathematical terms do your students find confusing?



(Rubenstein, 2007)



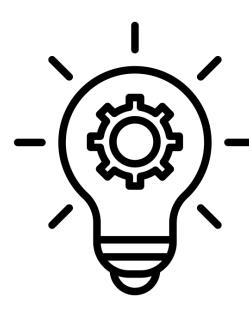


## PL #6. Strengthen teachers' mathematical language practices

## Suggestions for teachers:

- 1. Use formal mathematical terms that are consistent across grade levels. 2. Be precise and specific when using mathematical terms.
- 3. Plan for language use in lessons.
  - Incorporate vocabulary activities.
  - Provide opportunities for students to speak and write about mathematics using formal mathematical language.
- 4. Hold students accountable for using formal mathematical language correctly. 5. Listen for students' use of mathematical language and provide feedback. (Fuchs et. al, 2021; Powell et al., 2018; VDOE, 2020)









## Resource: VDOE Guide

## • VDOE Guide's table shows formal mathematical terms to use instead of informal ones.

#### INSTEAD OF THAT.....

#### Numbers in the Fraction

Problem: Language suggests that each part of a fraction (i. numerator, denominator) is a separate and independent number instead of a digit ( or series of digits) that comprise fraction.

#### Top number and bottom number

Problem: This suggests that the numerator and denomination are separate and independent numbers.

#### Two over three

Problem: This communicates the location of the digits but the actual number and magnitude.



	SAY THIS
.e., e a	This fraction is a number Solution: A fraction is a number in itself and has a magnitude on a number line. A fraction is not two separate numbers.
tor	Numerator and Denominator Solution: A fraction is a number with a specific magnitude that can be represented on a number line. While a fraction may have different parts, these parts do not work in isolation but rather contribute to one number – the fraction.
not	<b>e.g., Two-Thirds</b> Solution: This is accurate and communicates the magnitude of the number.

(VDOE, 2020a)



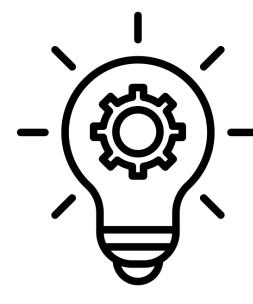


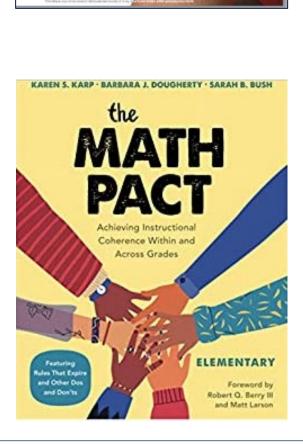
## PL #6. Strengthen teachers' mathematical language practices

Examples of professional learning activities 1. Article Discussions

- -Discuss a "Rules that Expire" article for your grade level (elementary, middle, or high school) from the NCTM Journals.
- 2. Whole School Agreement (Math Pact): - Facilitate collaborative sessions with teachers to identify consistent math terms to use across grade levels.









## PL #6. Strengthen teachers' mathematical language practices, cont.

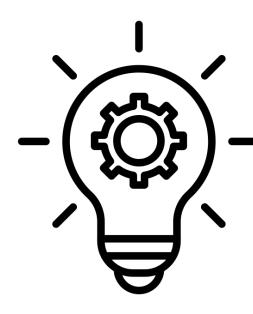
## **Focus on Common Pitfalls**

- Vocabulary terms are taught in isolation
- Word Walls become wallpaper
- but then they are not used

## **Brainstorm and Discussion Suggestions**

- Set expectations for using mathematical language, "I'm going to listen for..."
- Plan and teach lessons to incorporate active use of vocabulary tools





### • A lot of time is spent on creating vocabulary tools, such as student-created dictionaries,

(Brodesky et al., 2019; Fuchs et al., 2021)





## Discuss: Why is it important to promote student communication?

When students share their ideas ...

- They solidify mathematical understandings
- They find mistakes and correct them
- Teachers gain insight into their mathematical thinking
- It helps build a learning community

Discuss: Why might students with mathematics disabilities and difficulties be hesitant to share their ideas in class?











Low-achieving students tend to participate less in class discussion and make fewer mathematical contributions than their higher-performing peers.

Equitable participation can be promoted through teachers' use of classroom structures, high expectations, resources, such as concrete materials, and other supports.



## Student participation is an issue of equity

(Baxter et al., 2001; Foote & Lambert, 2011)





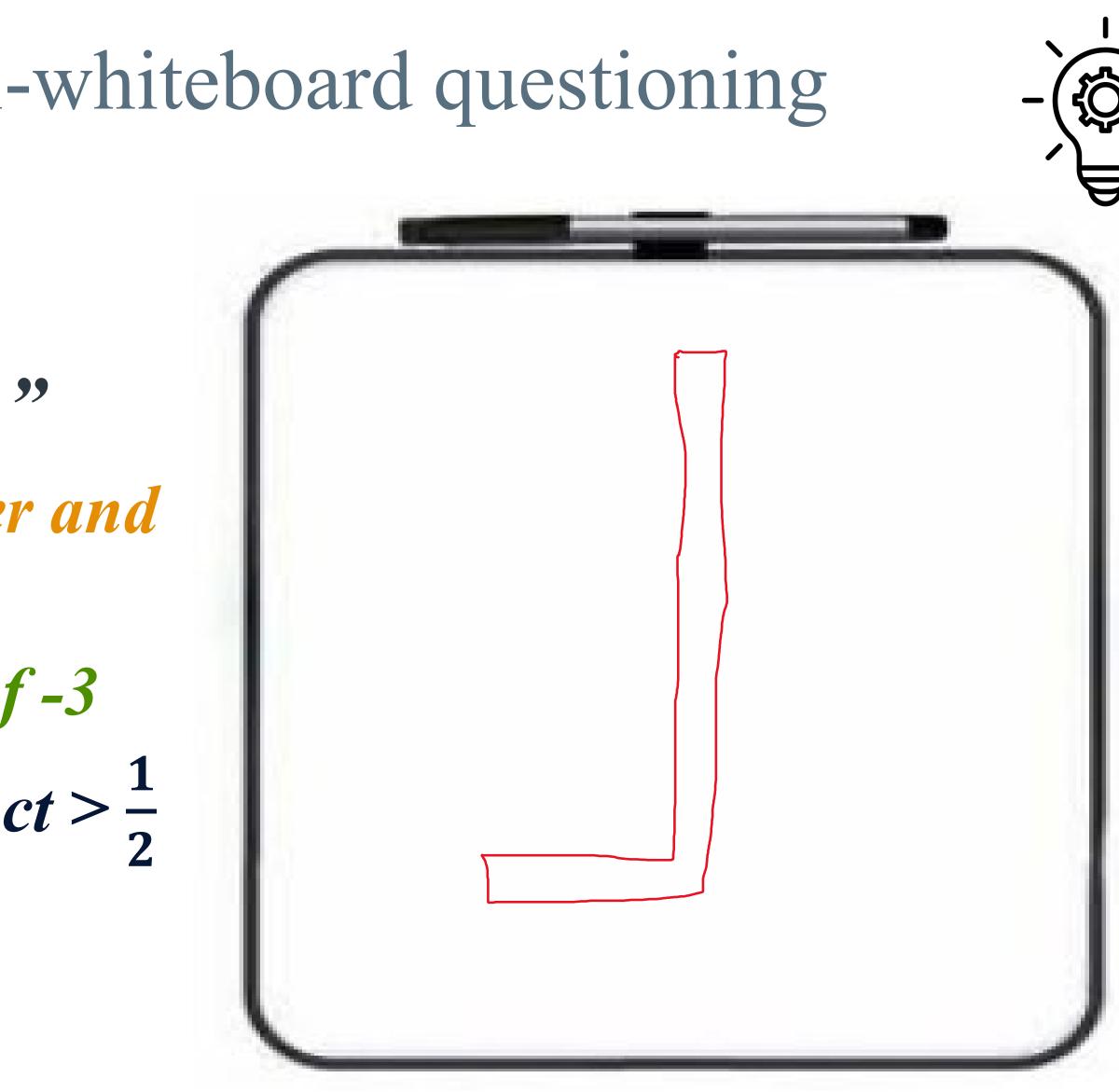
## Example PL #6 activity: Mini-whiteboard questioning

## "Show me an example of ... "

- A shape that has a large perimeter and a small area
  - Two integers that have a sum of -3
- Two fractions that have a product  $>\frac{1}{2}$

(Swan, 2005)









### **Sentence Starters and Stems**

- "First, I did..."
- "I agree because..."
- "I know this makes sense because..."

#### **Sentence Frames**

"I chose the fraction because it is than  $\frac{1}{2}$ ."

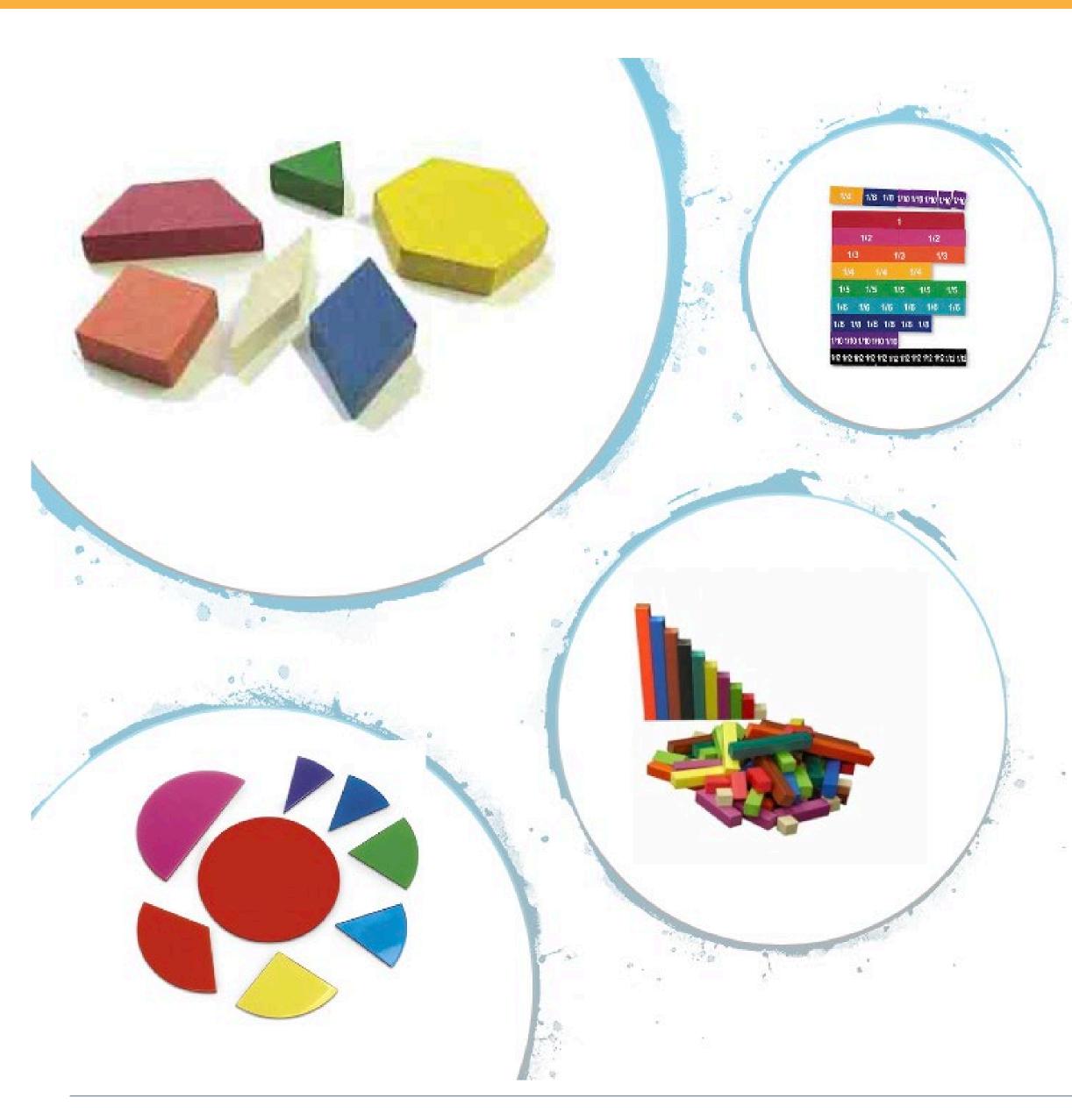




(Brodesky et al., 2019)









# Recommendation 3: Representations

Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.

(Fuchs et al., 2021)

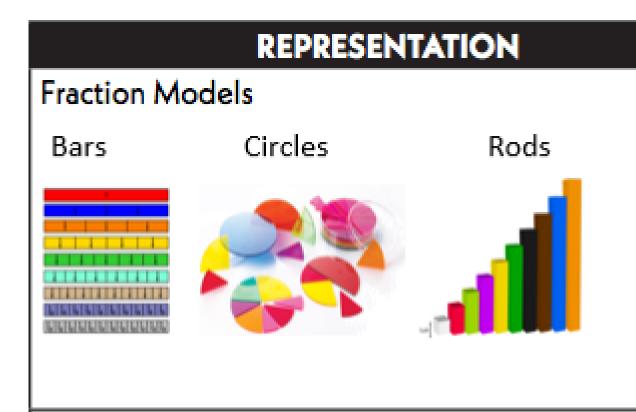




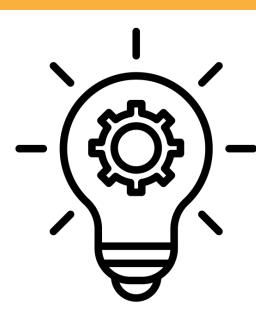
## PL #7. Strengthen teachers' use of manipulatives

- During professional learning sessions, engage teachers in using different types of manipulatives- start with concrete, physical manipulatives and then move to virtual manipulatives.
- Helpful resources for manipulatives: -VDOE (2020a), Appendix C: Mathematics Instructional Connections for Physical and Visual Representations

- Virtual Manipulatives, such as Math Learning Center Free Apps







#### CONTENT CONNECTIONS

- Represent fractions (area or length model)
- Equivalent fractions
- Compare and order fractions
- **Operations with fractions**

Challenges or limitations could include:

Modeling certain fractions

(Brodesky et al., 2019)





## PL #7. Strengthen teachers' use of manipulatives

## **Professional learning activity suggestions**

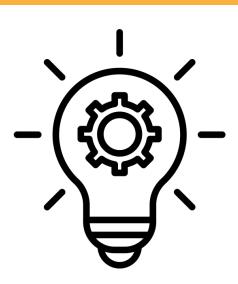
- Provide rich tasks for teachers to solve by using manipulatives and then discuss the mathematical ideas that the manipulatives help to bring out
- Discuss strengths and limitations of specific manipulatives for a chosen math topic. Compare manipulatives by having teachers using different ones for the same task.

#### **Address common concerns from teachers about manipulatives**

- "Takes too much time"
- "What if students start throwing the manipulatives..." "Middle grades students may think the manipulatives are for younger kids"







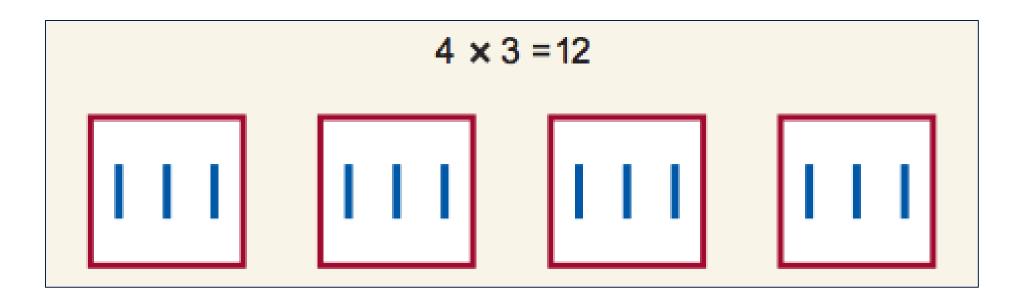
Let's discuss ways to address these concerns. What would you suggest to teachers?

(Brodesky et al., 2019)





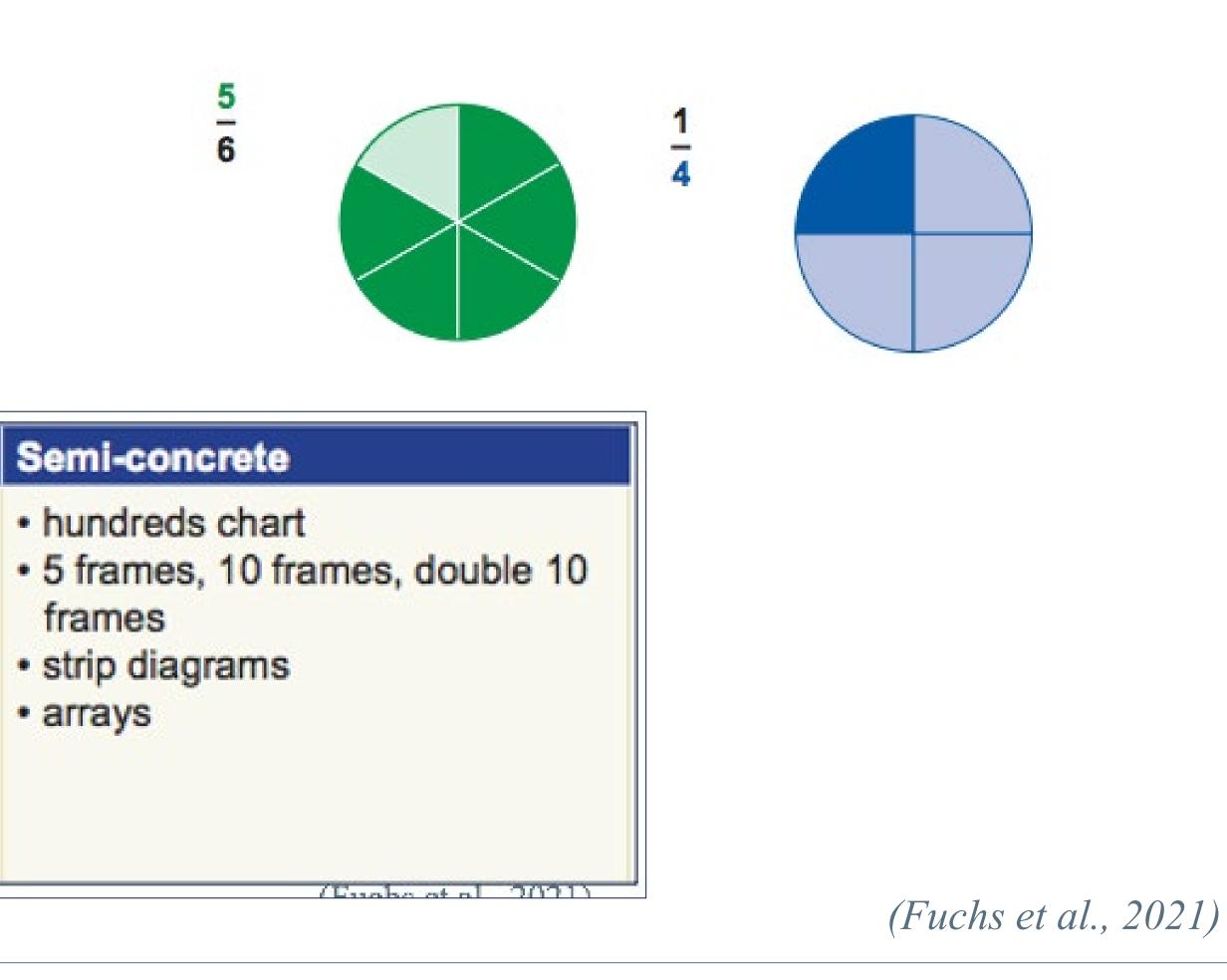
## Recommendation 3, cont.: Use semi-concrete representations



#### Concrete

- base 10 blocks
- connecting cubes
- Cuisenaire rods<sup>®</sup>
- · beads
- two-colored counters
- beans and cup
- 1-inch tiles
- balances









## Concrete—Semi-Concrete—Abstract (CSA) approach

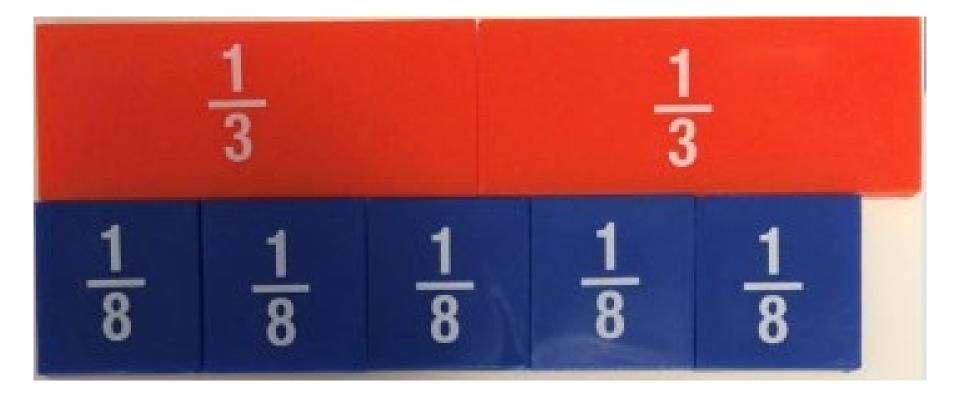
Example for comparing fractions: Which is greater 2/3 or 5/8?

• Concrete: fraction bar manipulatives

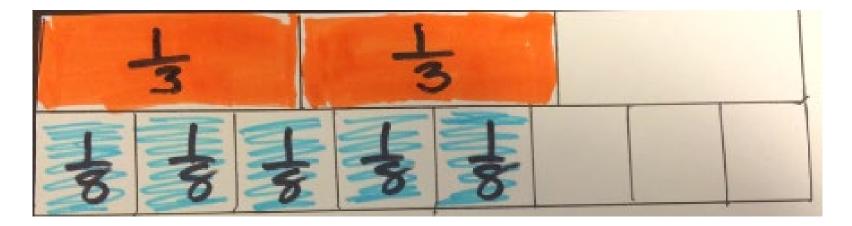
• Semi-Concrete: drawing of fractions

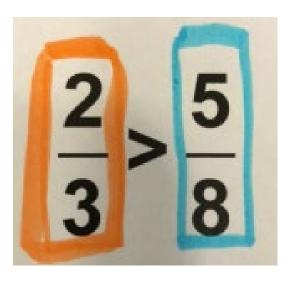
• Abstract: numeric











(Brodesky et al. 2016)





# PL #8. Build teachers' practices for using the concrete—\_\_\_\_\_ semi-concrete—abstract (CSA) approach

### **Suggestions for teachers:**

- Help students build meaningful understanding of each level.
- Have students work with and connect the representations: Use of CSA does *not* need to be sequential.
- example with the consistent use of orange for thirds and blue for eighths.
- Make strategic use of color to connect representations, as shown in the prior • Use consistent language across the representations.
- Incorporate concrete and semi-concrete tasks in assessments (not just abstract). For example, students use manipulatives to solve a problem and take a photo to document their work or make a drawing of this solution.



(Brodesky et al., 2016; Allsopp et al.)



61

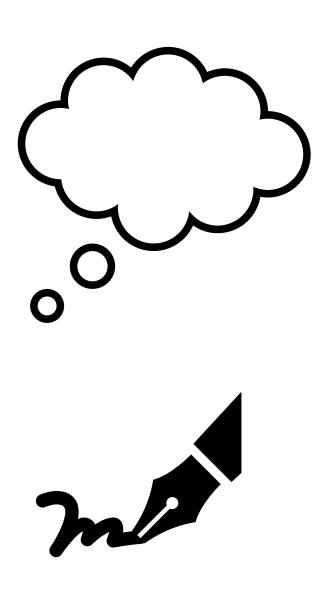
## Recommendation 4: Number lines

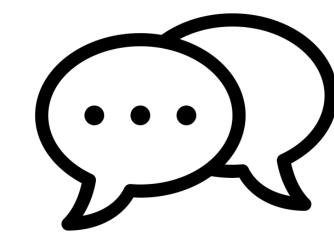
**Example professional learning introductory activity** Let's start by connecting to our experiences with number lines: 1. *Think about*: What are ways that you use number lines in mathematics? What are number lines particularly helpful for?

2. List ideas on a sheet of paper.

3. Share what you wrote.









## Number lines are an important tool for...

- Magnitude and operations for both whole numbers and fractions
- Elapsed time problems
- Displaying and analyzing data
- Graphing coordinates on coordinate grids

#### **Research highlights**

- improve their overall mathematics performance across a variety of mathematics content.
- larger gains for at-risk learners than did a program that focused primarily on the part-whole approach.



Temperature and how to read thermometers, linear spring scales, or depth charts [vertical]

• Consistent use of number lines can help students build understanding of the number system and

• An intervention program that strongly emphasized the number line representation for fractions found

(Fuchs et al., 2021; Fuchs et al., 2013; Siegler et al., 2010)







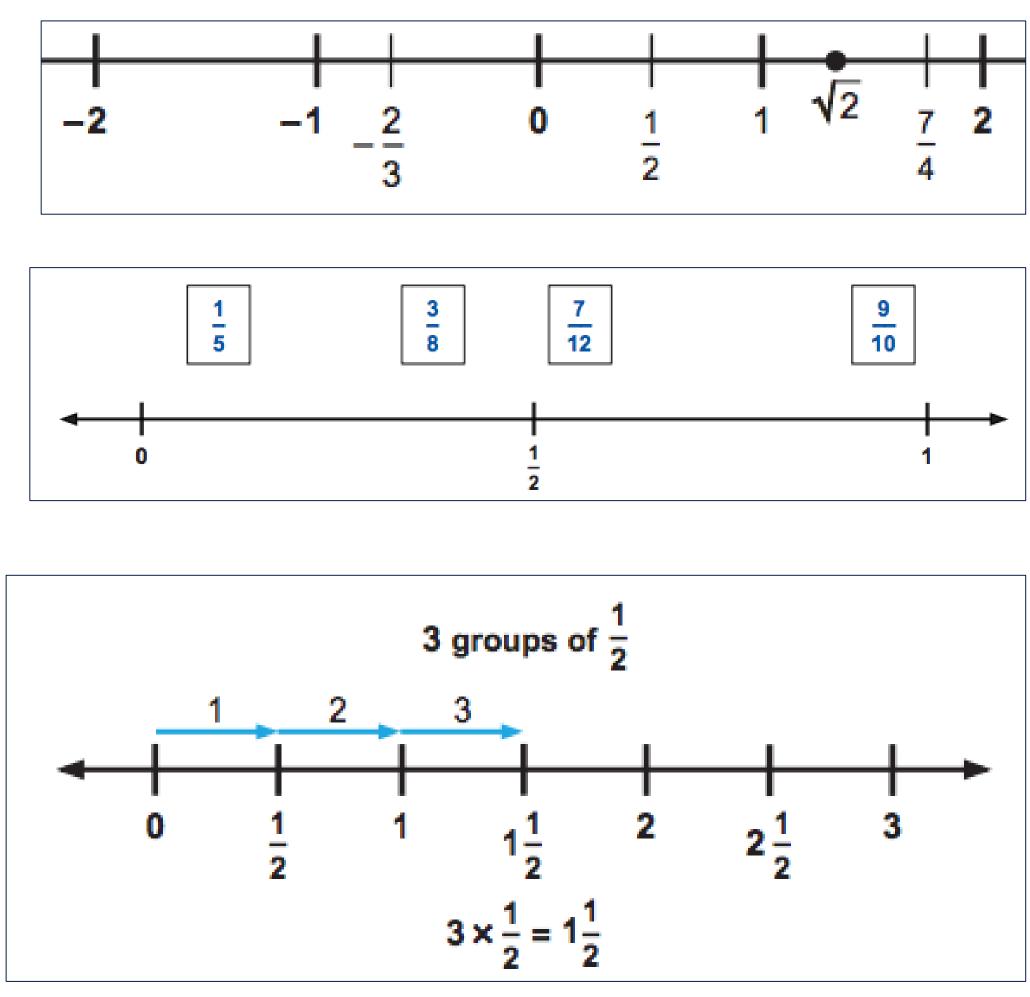
## Recommendation 4: Number lines

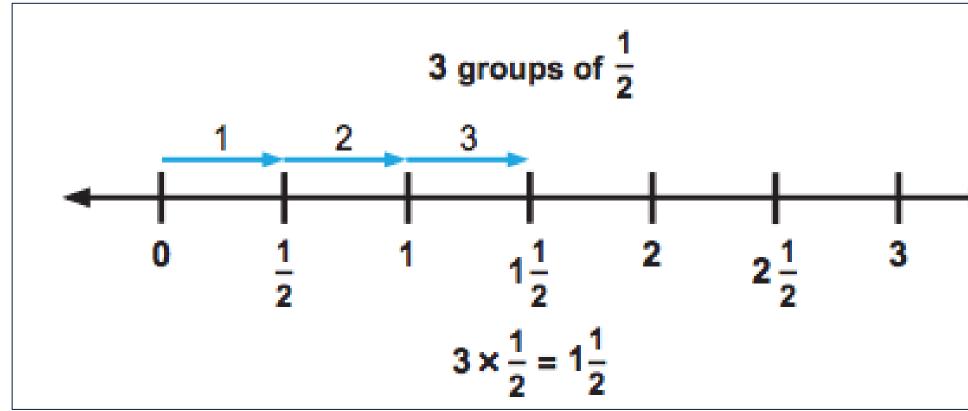
*How to implement the recommendation:* 

- Represent whole numbers, fractions, and decimals on a number line to build students' understanding of numerical magnitude.
- 2. Compare numbers and determine their relative magnitude using a number line to help students understand quantity.
- 3. Use the number line to build students' understanding of the concepts underlying operations.

(Fuchs et al., 2021)



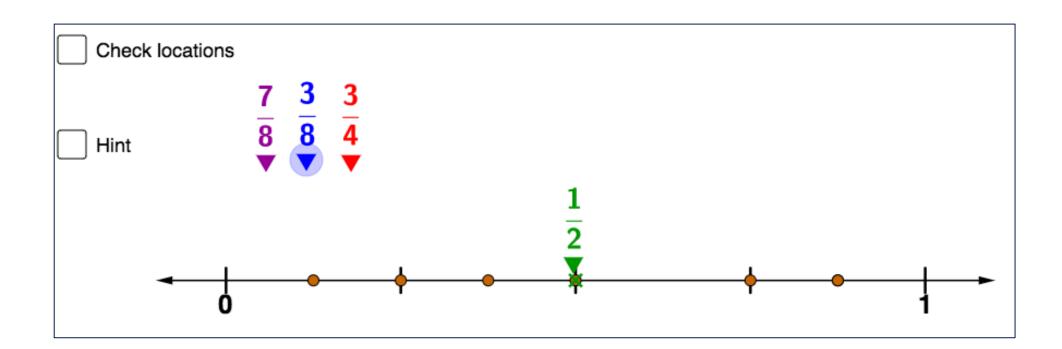




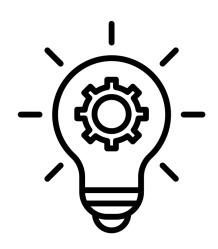


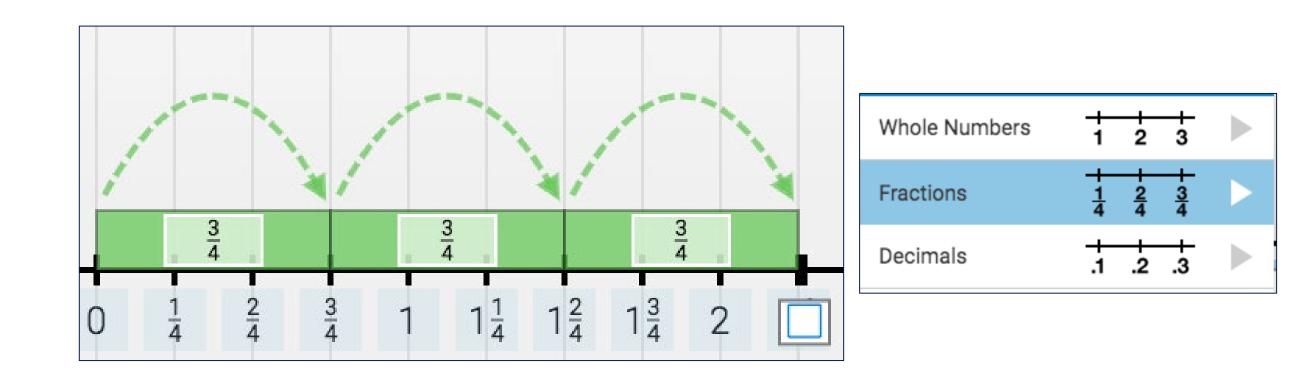
## PL #9. Engage teachers in number line activities

- Provide opportunities for teachers to use number lines to represent and solve problems for specific math topics (to build their knowledge & teaching practices)
- Model and discuss ways to build students' understanding of number lines and to provide support to address potential challenges
- Share resources for virtual number lines that can be used by teachers and students EDC: Locating Fractions on Number Line Math Learning Center: <u>Number Line App</u>













## Recommendation 5: Word problems

• Teach students to identify and solve word problem types.

Combine

(total) (part) (part)



- problem information differently.
- the problem.



• Expand students' ability to identify relevant information in word problems by presenting

• Teach vocabulary or language often used in word problems to help students understand

(Fuchs et al., 2021)







## What words might be unfamiliar to students in these problems?

- Unknown words or familiar words that may be confusing in context
  - A) There is a 2-mile relay race at Brown Elementary School. Each leg of the race is 12 miles. How many children are needed to run the race?
- Categories and the things that comprise them
  - B) Sasha has 6 pets. Four of the pets are turtles, and the others are puppies. How many puppies does Sasha have?
- Words that compare two quantities: more, less, fewer, older, younger, taller, shorter, bigger, smaller, hotter, and colder
  - C) Willa is 42 inches tall. Renaldo is 8 inches shorter than Willa. How tall is Renaldo?

word problems to help students understand the problem.



How to implement the recommendation: Teach vocabulary or language that is often used in (*Fuchs et al., 2021*)



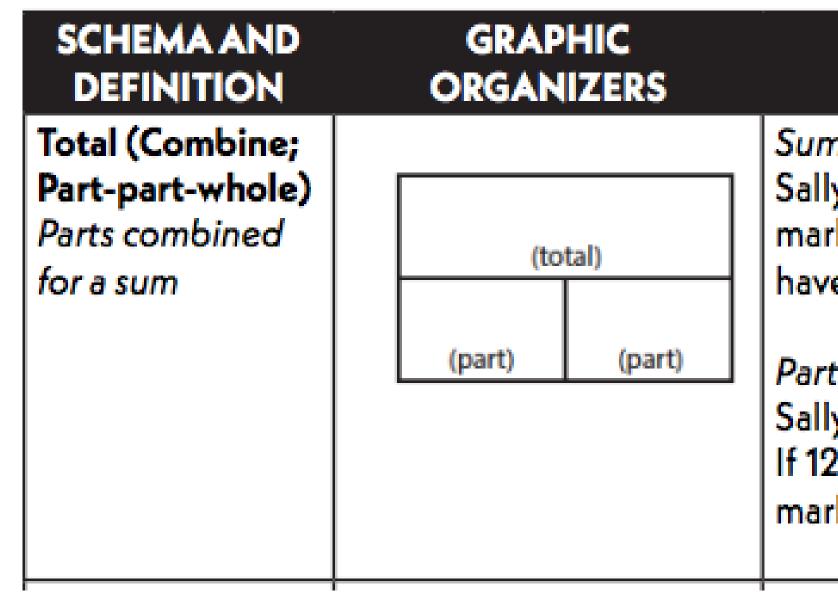


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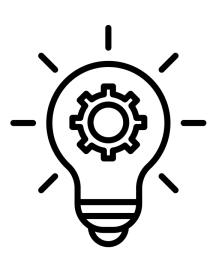
# PL #10. Strengthen understanding of word problem types

Example professional learning activities

- Sort word problems by type or other categories; do not solve them
- Write their own word problems
- Explore resources: VDOE (2020a) Guide, Appendix G & H







EXAMPLES	VARIATIONS
m unknown:	More than two
ly has 12 red markers and 13 purple	parts:
rkers. How many markers does Sally	Sally has 34
/e altogether?	markers. Of the
	markers, 12 are
rt unknown:	red, 13 are purple,
ly has 25 red and purple markers.	and the rest are
2 of the markers are red, how many	green. How many
rkers are purple?	green markers
and Mar	does Sally have?



IES Panel's Advice: Avoid using the key word approach

- Key word approach: Linking a word with a single operation, such as:
  - In All: +
  - Left: –
  - Each: X
  - Share: ÷

If students use the key word approach, what operation might they use for these problems?

(Fuchs et al., 2021)



PL #10A. Explain why key word approach should be avoided

#### **Example Problems:**

Amy bought 5 bags of apples. Each bag has 8 apples in it. How many apples does she have *in all*?

Pam got a bag of candy to *share* with friends. After she gave away 16 candies, she had 4 candies *left* over. How many candies were in the bag?



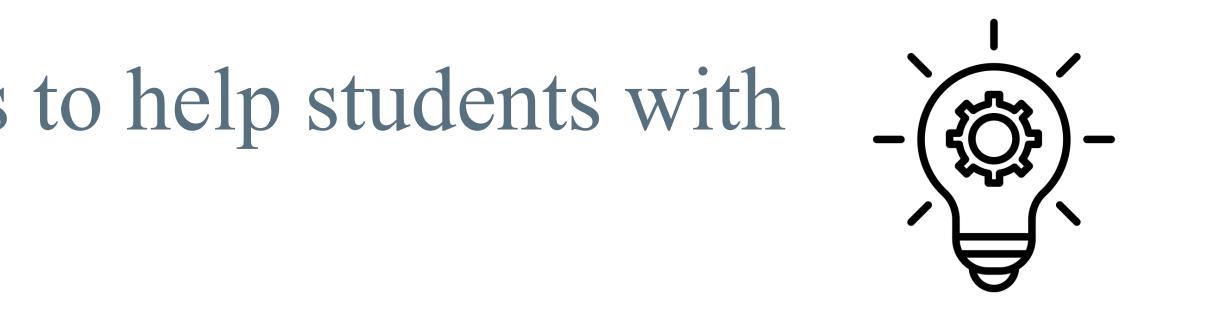


## PL #10B. Explore different ways to help students with word problems

#### **Suggestions for teachers:**

- Word problem contexts that are familiar, appealing, and personalized
- Improved readability of problems
- Problem-solving processes and graphic organizers
- Concrete and visual representations
- Acting out the situations
- Use schema-based instruction
- 3-Reads strategy
- Sorting activities
- Numberless word problems







(*Karp et al., 2020; Powell & Fuchs, 2018*)





## Discuss: Which practices might you focus on in professional learning with teachers?

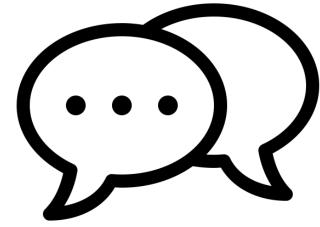
#### **Recommended** practices

Mathematical Language Representations

#### **Questions for planning professional learning**:

- 1. What are teachers' current strengths and challenges related to these practices?
  - Which practices are you more likely to observe teachers implementing? Less likely?
  - What's the quality of the implementation for the different practices?
  - What information would you like to find out about teachers' use of these practices?
- 2. Which one(s) do you want to focus on in your professional learning with teachers? Why?
- 3. What types of professional learning activities might you use with teachers?





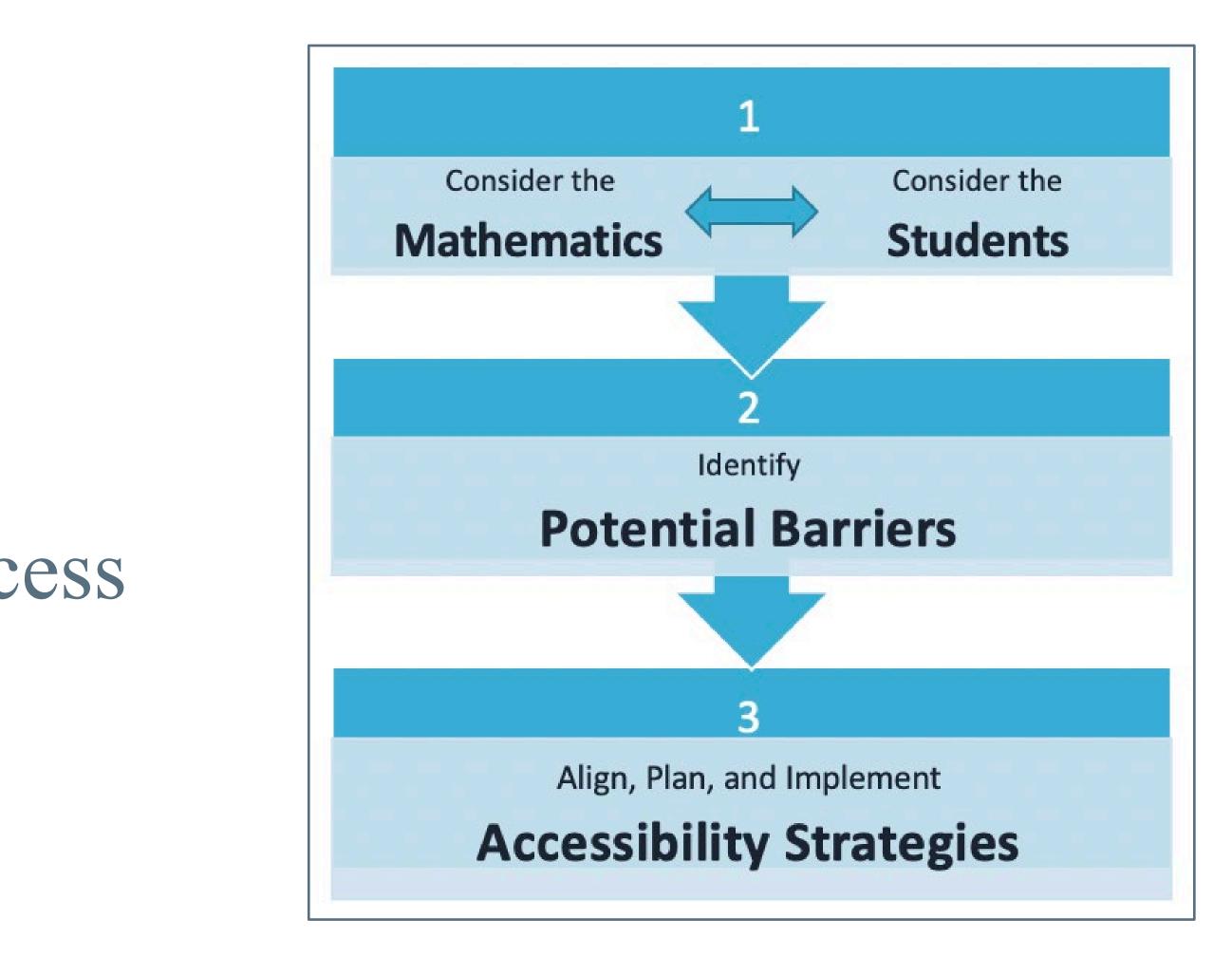
Number Line

Word Problems

71

## Section 4: Mathematics Accessibility Planning Process





## Handout, Page 1





## Professional learning (PL) suggestions for this section

PL #11. Engage teachers in a Mathematics Accessibility Planning activity 11A. Show example strategies 11B. Suggestions for using the planning process activity PL #12. Use a focus student approach





## Mathematics Accessibility Planning Goals

## Goals

- Build on students' strengths.
- Provide access to learning key mathematics content.
- Be proactive: plan strategies to have in your back pocket.
- Support students in being independent mathematics learners.

### What Not To Do

- Making too many adaptations to the lesson so the mathematics content is lost.
- productively
- Setting expectations too low.



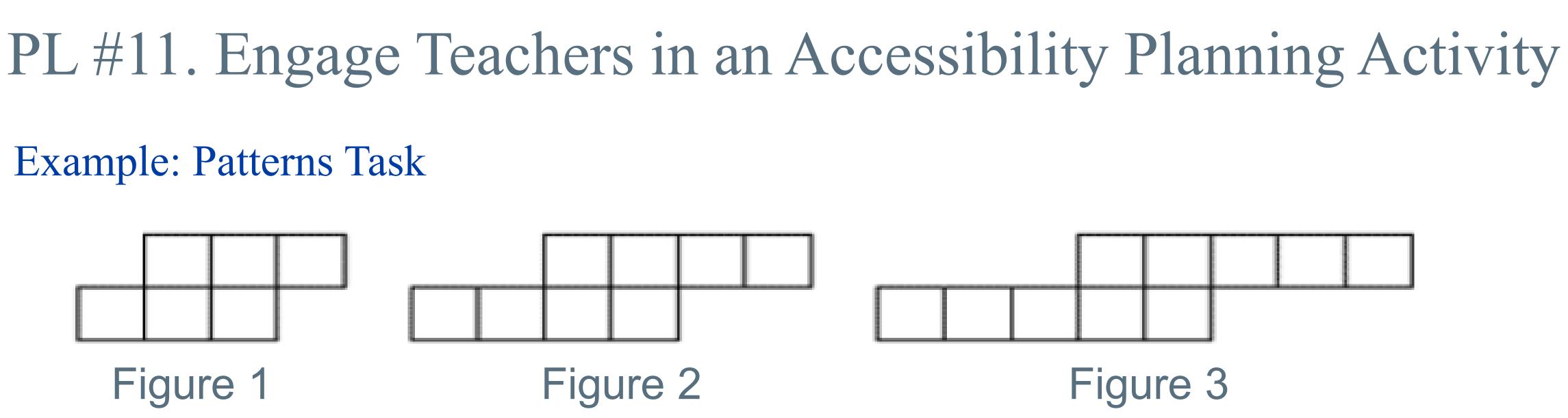
# • Taking away opportunities for students to do the mathematics themselves and struggle

(Brodesky et al. 2012)





Example: Patterns Task



Based on this pattern, how could you figure out the number of small squares in any figure? Write an expression to find the number of squares in Figure *n*.

#### **Directions**

- **1.** Get to know the problem by doing it yourself.
- **2.** Talk with a partner: How did you see the pattern? How did you move from the pattern to the expression?



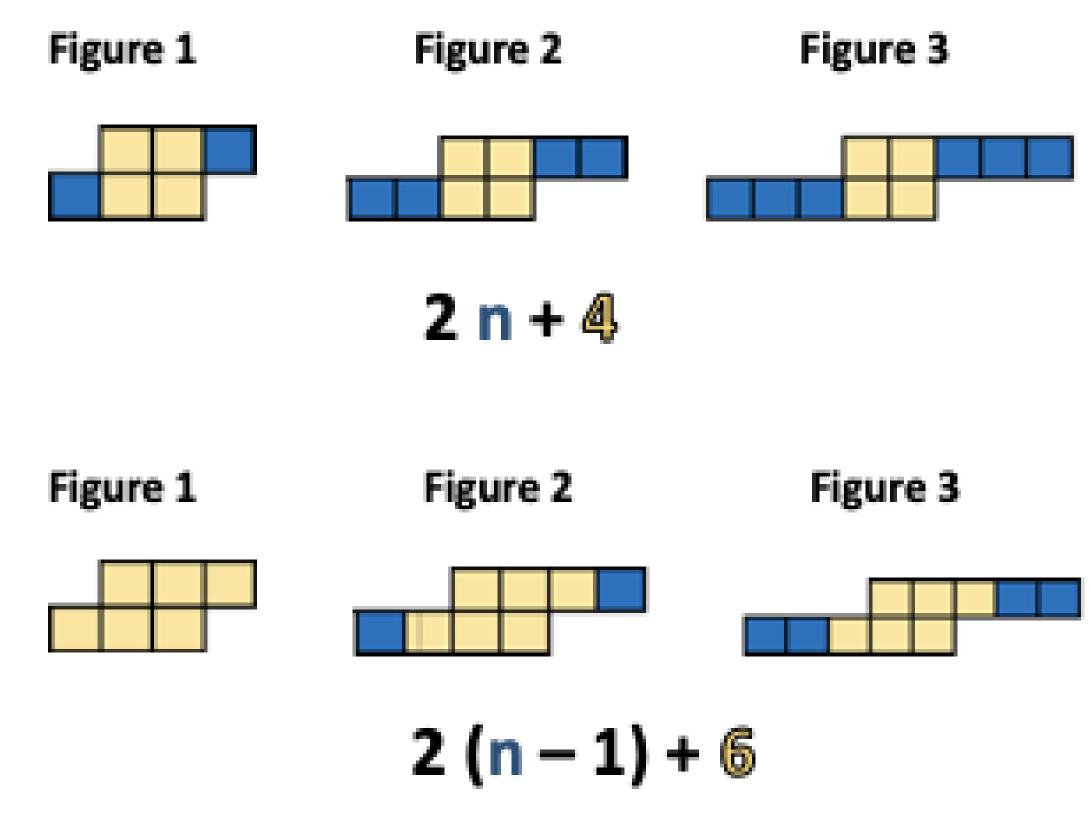
(Brodesky et al. 2012)





## How did you see the pattern?

Different ways of seeing the pattern connect to different expressions







(Brodesky et al. 2012)



## Part 1. Consider the mathematics goals

#### **Mathematical Goals of Task:**

- Analyze and extend a visual pattern.
- Represent a visual pattern by using an algebraic expression

#### **Relevant Standards**

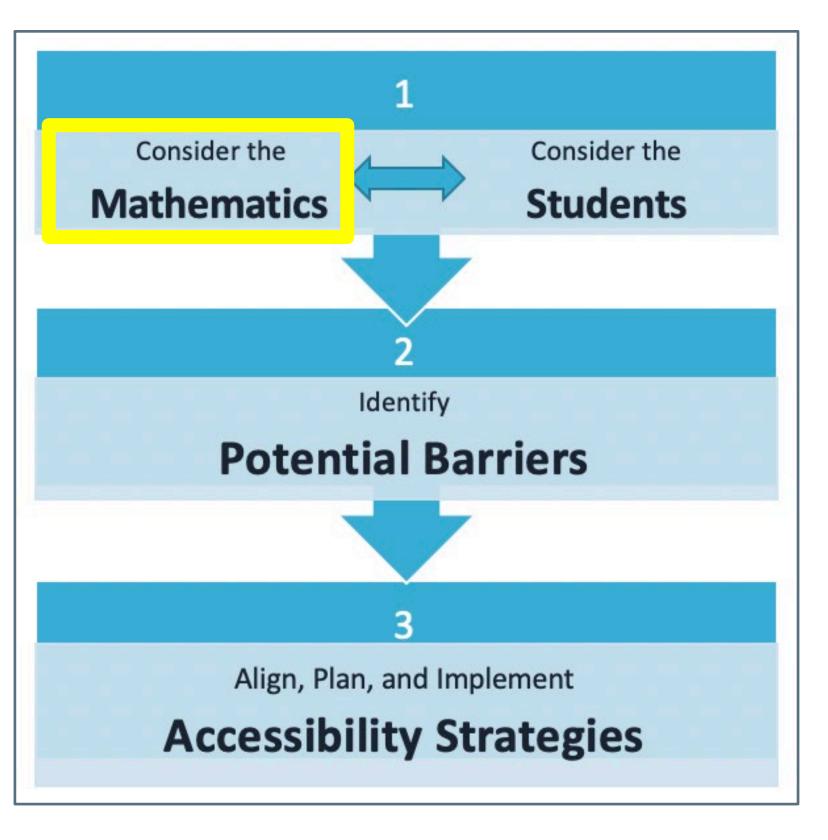
<u>6.EE.B.6</u> Use variables to represent numbers and write expressions when solving a real-world or mathematical problems.

MP1. Make sense of problems and persevere in solving them. MP7. Look for and make use of structure.

(Common Core State Standards, 2010)







Handout, Page 1

(Brodesky et al. 2012)







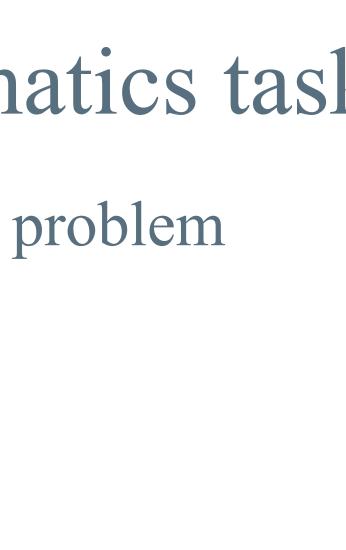
## Part 1. Consider the mathematics task

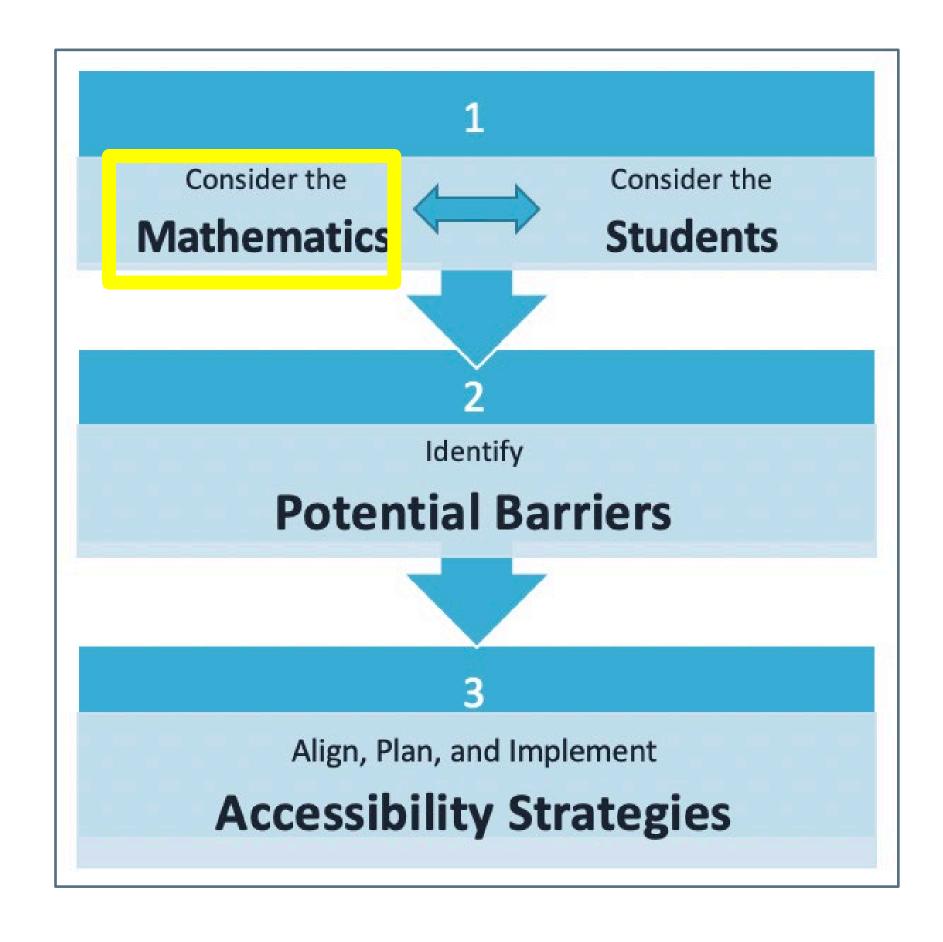
A. What kinds of task demands does the problem place on students?

- Conceptual?
- Language?
- Visual-spatial?
- Other?

**B.** Consider: Which task demands are essential to the goals?









(Brodesky et al. 2012)





### Part 1. Consider the students

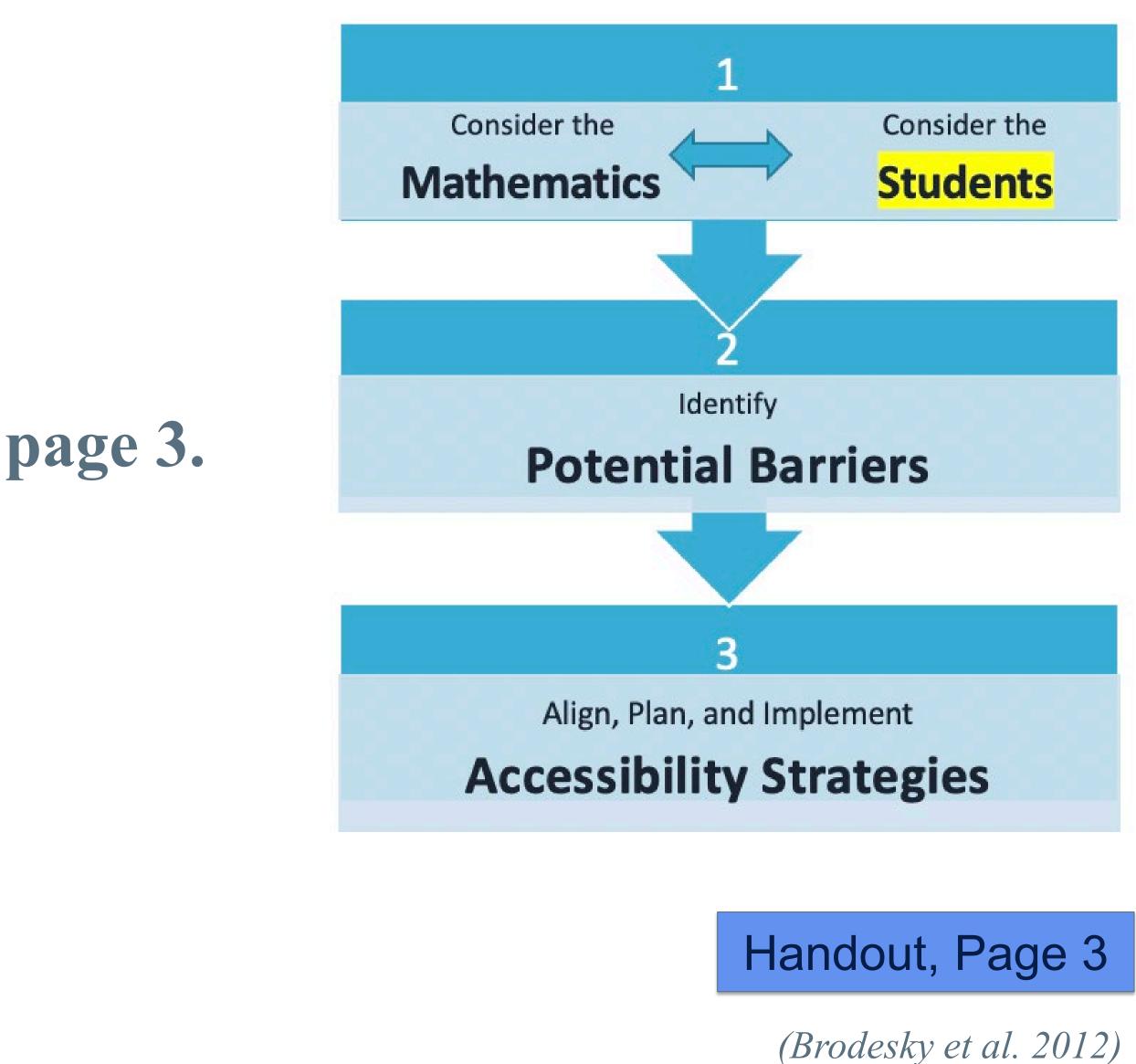
#### Directions

A. Read about the sample students on page 3.

B. Each person picks a student.

C. List your student's strengths and difficulties on the Planner, page 1.







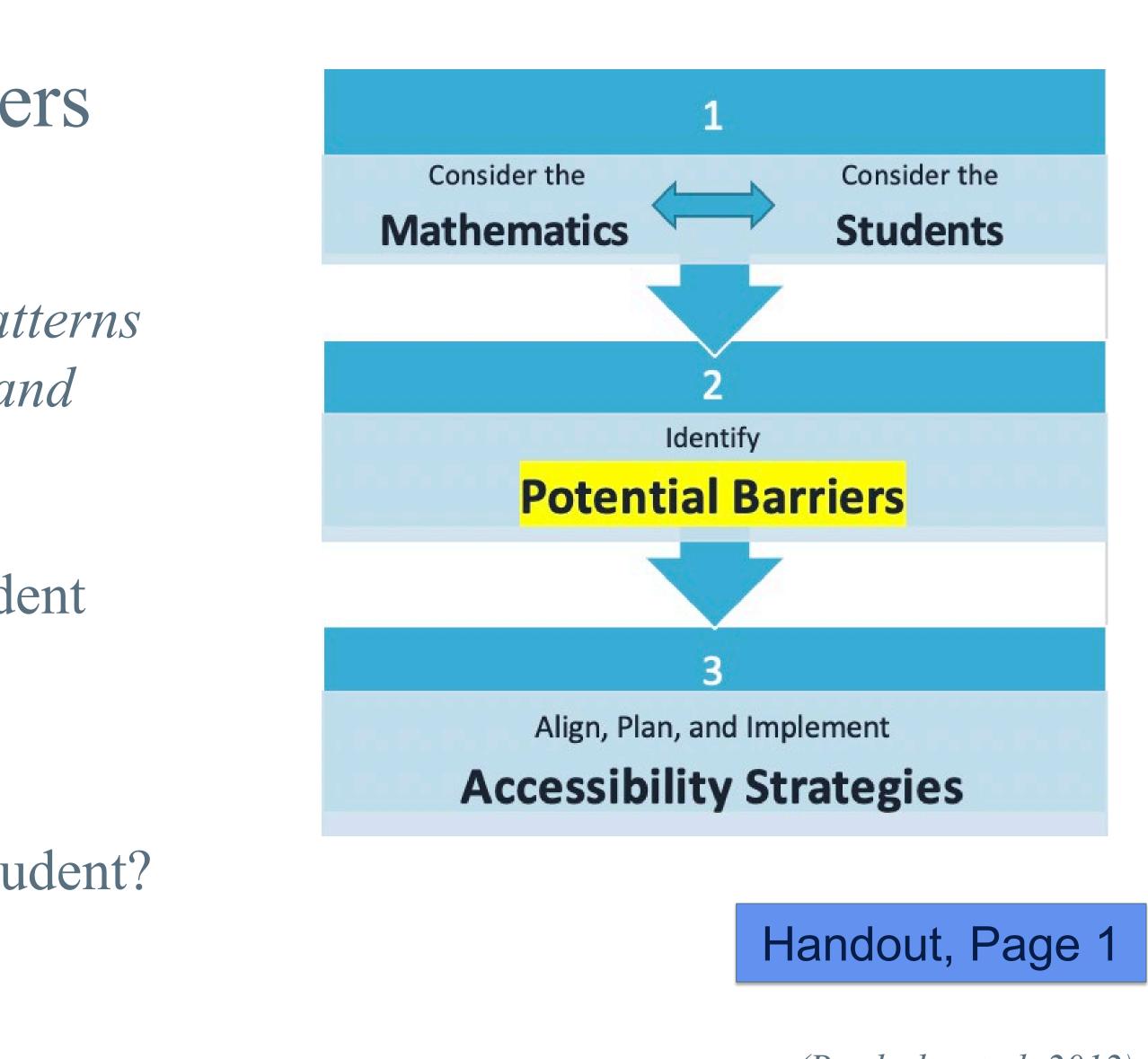


## Part 2. Identify potential barriers

What is the match or mismatch of the Patterns task with your focus student's strengths and difficulties?

- 1. What **strengths** does your focus student bring to this task?
- 2. What potential barriers might thePatterns task pose for your focus student?





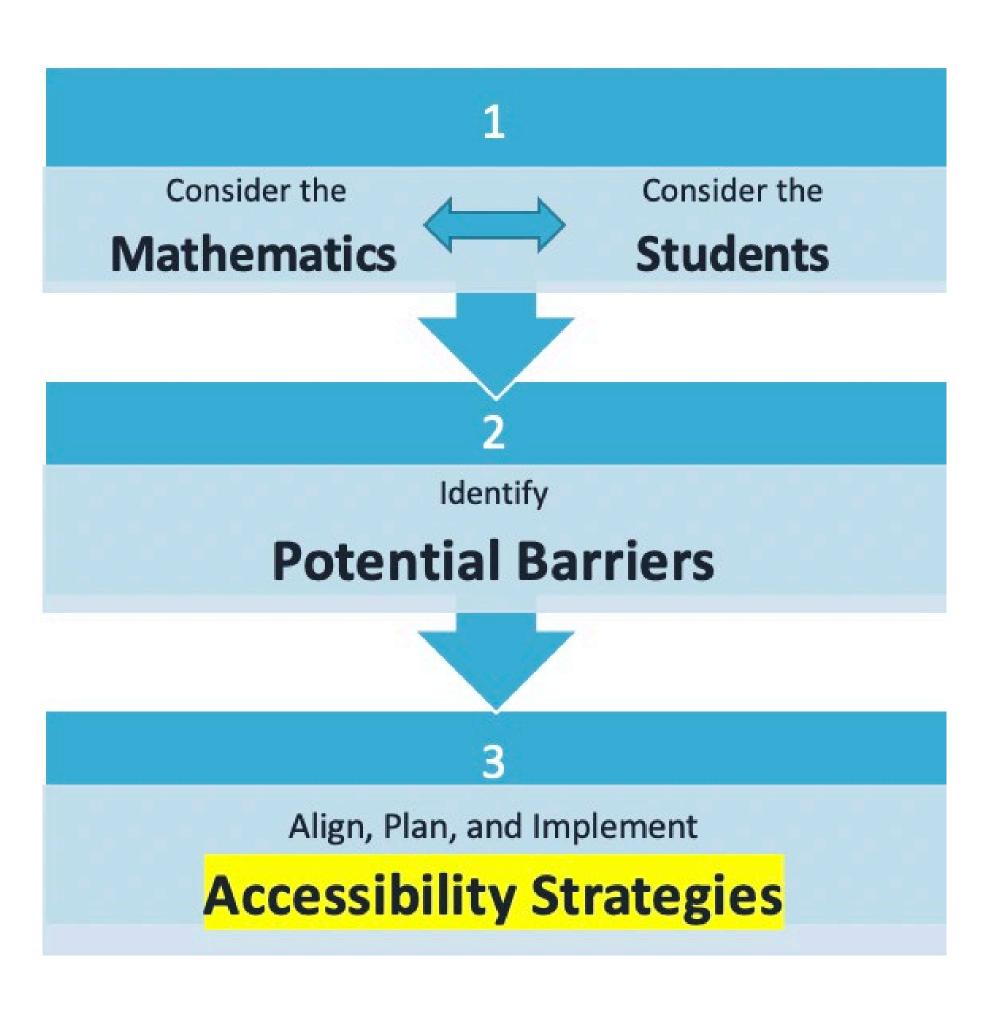




## Part 3. Align accessibility strategies

- What will you do to choose strategies: • Consider a wide variety of strategies by using Accessibility Strategy Cards.
- *Align* strategies with students' strengths and difficulties and the math goals.
- Use strategies to build on students' strengths.









## Strategy card activity

#### **Directions**

- **1.** Each person gets four strategy cards to start.
- 2. Find two strategies that are a *particularly good match* to your student. - You can trade for new cards.
  - Place rejected cards face up in the middle so others can use them. - Work together to find good strategies for each student.
- **3.** After everyone has identified two strategies, add them to your Planner, page 1. Share with the group:
  - Which strategies did you select for your student? Why? - How would you use this strategy for the Patterns task?



Handout, Page 1

(Brodesky et al. 2012)









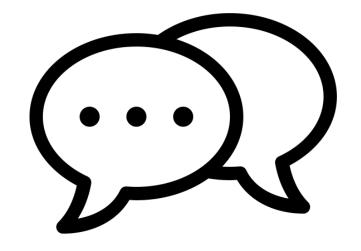
### Share strategies

- Which strategies did you select for your student? Why?
- How does your strategy align with the math goals and the student's strengths and needs?

#### Suggestion for organizing discussions with teachers:

- Group teachers based on the student they were focused on, so each group can discuss strategies for the same student.
- Ask a reporter from each group to share a few strategies with the whole group.





Handout, Page 4

(Brodesky et al. 2012)





### PL #11A. Show example strategies for each student

#### **Common Difficulty: Moving to the Abstract** It's a big leap from building and drawing the pattern to writing an algebraic expression! This is a difficulty for Celia and many students.





(Brodesky & Fagan. 2012)





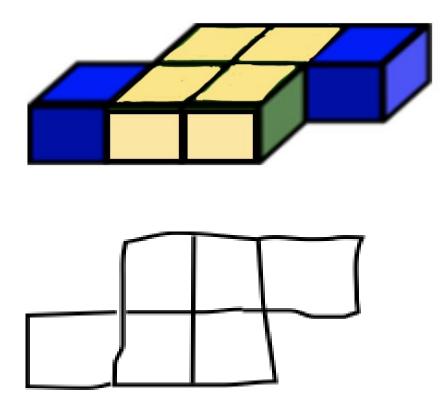
Example Strategy #1 Concrete, semi-concrete, abstract (CSA)

#### **Concrete**: manipulatives

#### Semi-concrete: drawings

2n + 4 Abstract: numerical, symbolic, table, graph, expression, and equation





(*Gersten et al., 2009*)





### Example Strategy #2 Use color-coding to connect representations

Figure 0



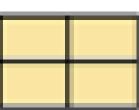




Figure #	Number of blue	Number of yellow	Total
	squares	<mark>squares</mark>	squares
0	0	<mark>4</mark>	4
1	2	<mark>4</mark>	6
2	4	<mark>4</mark>	8
3	6	<mark>4</mark>	10
4	8	<mark>4</mark>	12
n	<b>2n</b>	<mark>4</mark>	<mark>2n + 4</mark>

4 + 2n or 2n + 4



Figure 2 Figure 1

(Brodesky et al. 2012)



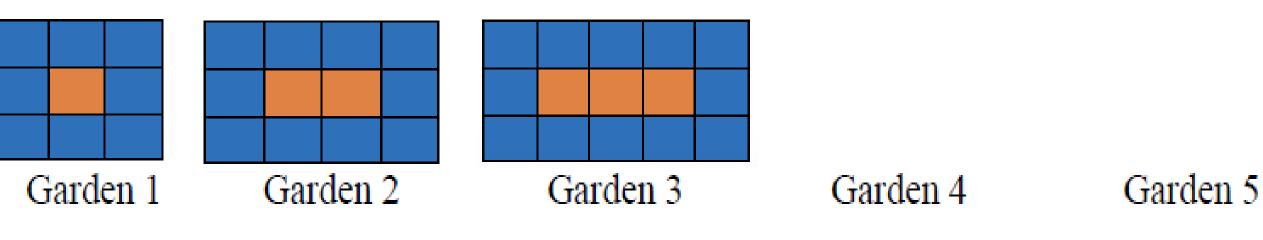


### Example Strategy #3 Graphic organizers: Link sheet or rule of 4

**Graphic Organizer** Graph Table Blue Orange 1 8 10 234 12 14 5 16 012345 Orange Words/Explanation Equation Let n = orange tiles Let b = blue tiles b = 20+6For every lorange tile there are 2 blue ones on top and one on the bottom and add 6 blue tiles three on each side



#### Tiling Garden Beds Task



(Brodesky et al. 2012)









# PL #11B. Suggestions for using the planning process activity

#### 1. Select a mathematics task:

- -From a high-quality lesson in your math program for the grade(s) of your target audience. -With a low floor, high ceiling that will be accessible to teachers with different math
- backgrounds.
- -That allows for multiple approaches so that teachers will be motivated to share different approaches.
- -That focuses on high-priority math goals/standards that pose challenges for students.

#### 2. Emphasize the importance of:

- -Aligning strategies with the student's strengths/needs and the mathematics goals.
- -Building on student's strengths.
- -Working collaboratively so that everyone in your small group finds good strategies



(Brodesky et al. 2012)







## PL #12. Focus student approach

#### For focusing professional learning on actual students:

- Teachers each select a focus student with mathematics difficulties.
- They collect and examine the student's work to identify strengths and difficulties. They align strategies with the student's needs.
- professional learning discussions.

#### For accessibility planning:

- Consider the strengths and challenges of each focus student.
- individual accommodations.



Focusing on individual students helps to reduce "overgeneralizations" about students in

• Teachers choose three students to serve as proxies for the range of learners in their class.

Planning with those students in mind will help teachers address the range of learning needs. This process can serve as a first step in accessibility planning. Some students will also need

(Brodesky et al. 2012)







## Wrap-up: Summary of topics

1. Students with mathematics disabilities and difficulties

2. High-quality, accessible mathematics instruction

3. Research-recommended instructional practices

4. Mathematics accessibility lesson planning process





## Wrap-up: Summary of professional learning suggestions

- Bring in the voices of students with disabilities: use video, quotes, and examples.
- Provide background information about mathematics disabilities and difficulties. 2.
- Convey positive messages and promote strengths-based approaches. 3.
- Build a shared vision of high-quality, accessible, and inclusive mathematics instruction. 4.
- Build a common language and clarify terms (modifications, accommodations, and scaffolding). 5. Strengthen teachers' mathematical language practices 6.
- Strengthen teachers' use of manipulatives 7.
- Build teachers' practices for using representations (concrete, semi-concrete, abstract) 8.
- Engage teachers in number line activities 9.
- 10. Strengthen understanding of word problems types
- 11. Engage teachers in a Mathematics Accessibility Planning activity
- 12. Use a focus student approach





91

## Wrap-up: Post takeaways and discuss

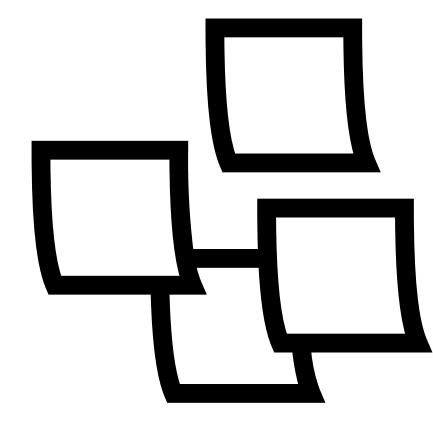
#### 1. Post your ideas by going to the Ideaboardz: <u>https://go.edc.org/WrapUpOct5</u> • Add sticky notes and type responses to one or more prompts.

- - *What ideas stood out for you?*
- What ideas do you want to apply in your professional learning with teachers?
- *What are you wondering about?*
- Read other participants' responses.
- Select the + to show your agreement with a response.

#### 2. Discuss the Posts











### Support and scaling resources *Professional learning model compendium*



**Pam Buffington** Partnership Lead





## Why is this compendium important?

- Research suggests that educators find their district-based professional learning fragmented and disconnected from their needs.
- The compendium provides resources to support the design and implementation of a cohesive model of teacher PD to address identified problems of practice.





(Bill & Melinda Gates Foundation, 2014; Jensen et al., 2016)





## Who is the audience for the PLM compendium?

- Mathematics teacher leaders and coaches, curriculum leaders.
- School administrators who create and provide mathematics professional development for elementary and secondary teachers of mathematics.







## What is in the PLM compendium?

- Slide decks
- Talking points
- Facilitator notes

- Handouts/tools
- References
- Background research



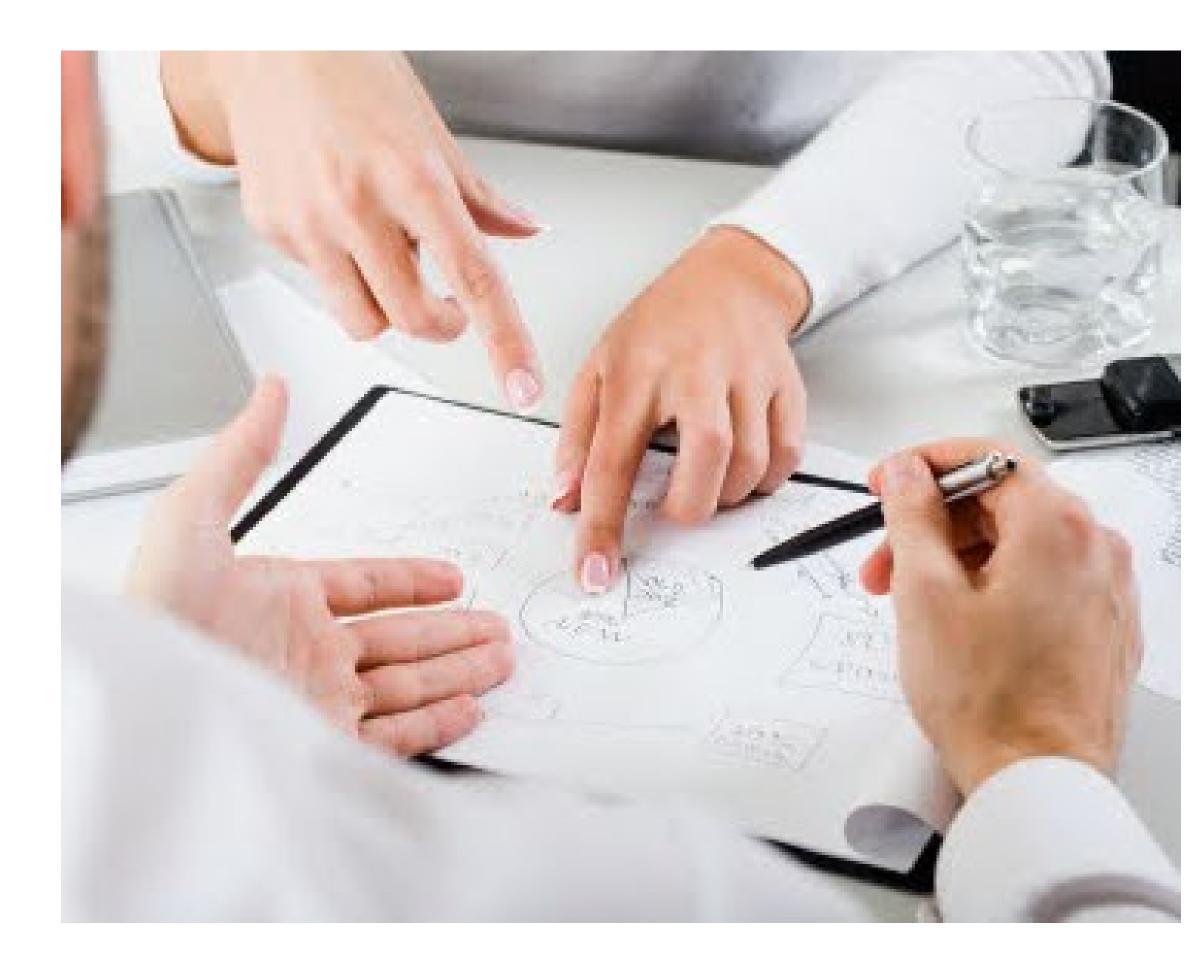


- Reflection/discussion questions
- Insights from practice





### How can the PLM compendium be used?





- For individual learning and reflection.
- For action planning.
- To extend practitioners' understanding of evidence-based strategies.



### Review the PLM compendium

• Document can be found at: [link]



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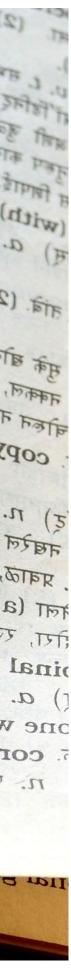
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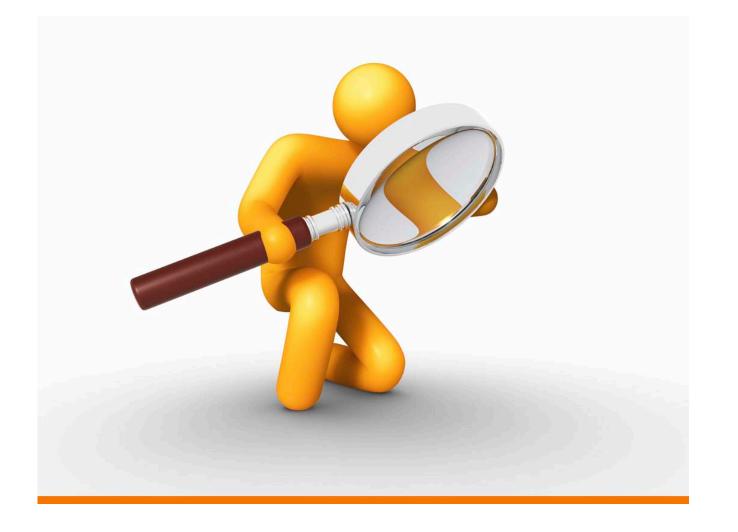




## Resources for support and scaling

Reflect and Share:







• What aspect of the compendium do you think is the most useful? Why?

• How will you use the compendium?

• Whom will you share it with?



## Support and scaling resources *Partnership collection*



**Ryoko Yamaguchi** Research Lead



**Jill Neumayer DePiper** Partnership Staff

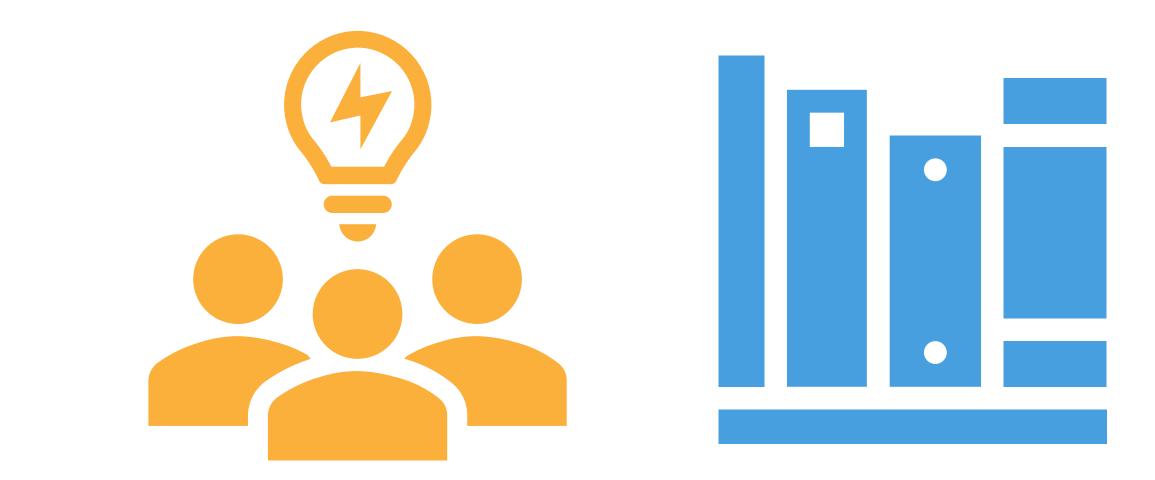




### What is a resource collection?

A set of products and practices that supported your work throughout this partnership and could inform others as they set division-wide goals for mathematics achievement.



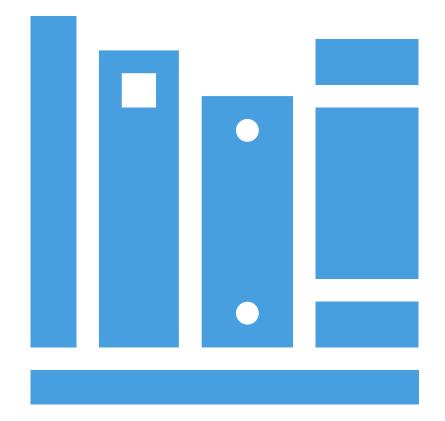




## What is included in the SSM resource collection?

- Resources for planning and sustaining school division initiatives
- Implementing a Professional Learning Model (PLM)
- Resources to build content knowledge, practices and processes to embed in your PLM
  - Materials on research-based algebra readiness instructional strategies
  - Instructional resources to support mathematics coaches Materials to support monitoring profess
  - Research studies related to mathematics teaching and learning
  - -Research literatures summaries
  - -Brief readings and infographics







## Example format for resources for planning and sustaining school division initiatives

Table 1. Resources for planning and sustaining school division initiatives

#### Description Resource

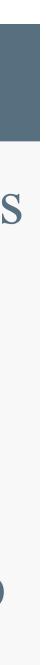
Why Build a Logic Model?

This REL Appalachia blog post introduces what a logic model is and provides an overview of its elements. It details the benefits of a logic model and provides examples of logic models.



#### How to use this resource

District administrators and curriculum leaders can review this short article to learn more about logic models and their importance. They can use this additional information on logic models as they study progress from the SSM partnership, revise the SSM partnership logic model, or embark on new initiatives.























## Example format for resources to build content knowledge, practices, and process to embed in your PLM

## teacher learning

Activity/ resource	Description
"How-to" analysis	This analysis memo provides step
memo: A review of	by-step instructions for analyzing
high school	local data, specifically outlining
mathematics	the steps to answer the following
coursetaking	questions among students who
pathways among	completed Algebra I in grade 7: 1
students who	When did they complete their
completed Algebra I	highest-level mathematics course
in grade 7	and 2) How did they perform on
	the geometry and Algebra II
	assessments?



Table 5. Resources to support monitoring progress of professional development and student and

#### How to use this resource

- The analysis memo provides a guide to using school-level data related to coursetaking pathways. Database managers 2 and analysts in SSM divisions can use the accompanying Excel template to enter data related to student outcomes. The template automatically creates bar graphs from the data.
- With information produced from the "how-to" memo, database managers and analysts can support school division leaders and mathematics content experts in using data to e,inform and improve decisions about policy and practice related to mathematics coursetaking pathways.





## Final input for the SSM resource collection

- Reflect and share:
  - -Are additional resources from our partnership work, beyond those discussed in the meeting in July, that they think would be useful to include?
  - -Are there any questions or input you would like to provide specific to the planned format for the resources to be included in the collection?



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## Closing



Pam Buffington Partnership Lead





### Next steps

- Questions, concerns, things you are still wondering about
- Follow-up coaching support







### Thank you!



#### https://ies.ed.gov/ncee/edlabs/regions/appalachia



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- edition). Paul H. Brookes Publishing Co.
- *Psychol. Behav. 12*, 163–168.
- classrooms. *Elementary School Journal*, 101(5), 529–547.
- learning difficulties and disabilities. Paul H Brookes Publishing.
- Bill and Melinda Gates Foundation. (2014). Teachers know best: Teachers' views on professional development. Author.
- differentiating between correct and incorrect examples. *Learning and Instruction*, 25, 24–34.
- Brain Highways. (2020). Dear Teacher [Video]. Youtube. <u>https://www.youtube.com/watch?v=lTMLzXzgB\_s</u>
- Student Success in Mathematics Partnership Meeting. Harrisonburg, VA.



Allsopp, D.H., & Lovin, L., & Vaningen, S. (2018). Teaching mathematics meaningfully: Solutions for reaching struggling learners (2nd

Attree, E.A., Turner, M.J., & Cowell, N. (2009). A virtual reality test identifies the visuospatial strengths of adolescents with dyslexia. *Cyber* 

Baxter, J. A., Woodward, J., & Olson, D. (2001). Effects of reform-based mathematics instruction on low achievers in five third-grade

Berch, D. B., & Mazzocco, M. M. M. (Eds.). (2007). Why is math so hard for some children? The nature and origins of mathematical

Booth, J.E., Lange, K.E., Koedinger, K.R., & Newton, K. J. (2013). Using example problems to improve student learning in algebra:

Bottge, B. A., Toland, M. D., Gassaway, L., Butler, M., Choo, S., Griffen, A. K., & Ma, X. (2015). Impact of enhanced anchored instruction in inclusive math classrooms. *Exceptional Children*, 81(2), 158–175. https://doi.org/10.1177/0014402914551742

Brodesky, A. (2020, March 3). Improving mathematics instruction for students with disabilities & difficulties. [Presentation] REL Appalachia

Brodesky, A., Hunt, J., & Storeygard, J. (2020, June 4). Strengthening educators' practices for engaging and empowering students with disabilities and difficulties as mathematics learners [Webinar]. CADRE NSF DRK-12 Meeting, Washington, D.C.





Development Center.

Brodesky, A. (2016) Powerful accessibility strategies for building conceptual understanding of decimal standards [Presentation]. National Council Teachers of Mathematics Innov8 Conference: Engaging the Struggling Learner, St. Louis, MO. Brodesky, A. & Fagan, E. (2012, April 24). Powerful professional development strategies that help teachers improve mathematics instruction for students with learning disabilities [Presentation]. Annual meeting of the National Council of Teachers of Mathematics, Philadelphia, PA. Brodesky, A., Parker, C., Murray, E., & Katzman, L. (2002). Accessibility strategies toolkit for mathematics. Education Development Center. Dixon, J.K., Brooks, L.A., Carli, M.R. (2019). *Making sense of mathematics for teaching the small group*. Solution Tree Press. Dixon, J.K. (2018). Just-in-time vs. Just-in-case scaffolding: how to foster productive perseverance. Houghton Mifflin Harcourt. Eide, B., & Eide, F. (2012). The dyslexic advantage: unlocking the hidden potential of the dyslexic brain. Plume. Everatt, J., Weeks, S., & Brooks, P. (2008) Profiles of strengths and weaknesses in dyslexia and other learning difficulties. Dyslexia, 14, 16-41.

Foote, M.Q. & Lambert, R. (2011). "I have a solution to share": Learning through equitable participation in a mathematics classroom. Canadian Journal of Science, Mathematics, and Technology Education, 11(3), 247–260. Friend, M., Cook, L., Hurley-Chamberlain, D. & Shamberger, C. (2010) co-teaching: An illustration of the complexity of collaboration in special education. Journal of Educational and Psychological Consultation, 20(1), 9–27.



Brodesky, A., Fagan, E. & MacVicar T. (2019, May 23). Forum on mathematics intervention for the middle grades [Presentation] Education







110

- (2013). Improving at-risk learners' understanding of fractions. Journal of Educational Psychology, 105(3), 683–700.
- comorbid reading difficulties. *Journal of Learning Disabilities*, 35, 563–573
- persistent low achievement: A five-year prospective study. Journal of Educational Psychology, 104, 206-223. https://doi.org/10.1037/a0025398
- Journal of Developmental and Behavioral Pediatrics, 32, 250–263.

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities*, 37, 4–15.

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools (NCEE 2009-4060). Washington, DC: Retrieved from <a href="https://ies.ed.gov/ncee/wwc/PracticeGuide/2">https://ies.ed.gov/ncee/wwc/PracticeGuide/2</a>



Fuchs, L.S., Newman-Gonchar, R., Schumacher, R., Dougherty, B., Bucka, N., Karp, K.S., Woodward, J., Clarke, B., Jordan, N. C., Gersten, R., Jayanthi, M., Keating, B., and Morgan, S. (2021). Assisting students struggling with mathematics: Intervention in the elementary grades (WWC 2021006). Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, U.S. Department of Education. Retrieved from <u>https://ies.ed.gov/ncee/wwc/PracticeGuide/26</u>

Fuchs, L. S., Schumacher, R. F., Long, J., Namkung, J., Hamlett, C. L., Cirino, P. T., Jordan, N. C., Siegler, R., Gersten, R., & Changas, P.

Fuchs L.S., & Fuchs D. (2002). Mathematical problem-solving profiles of students with mathematics disabilities with and without

Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2012). Mathematical cognition deficits in children with learning disabilities and

Geary, D. C. (2011). Consequences, characteristics, and causes of poor mathematics achievement and mathematical learning disabilities.

National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.



- Hourcade, J. J., & Bauwens, J. (2001). Cooperative teaching: The renewal of teachers. *Clearing House*, 74, 242–247.
- National Center on Education and the Economy.
- Press.
- Press.
- 1 18.
- disabilities. *Education Sciences*, 8(2), 72–84.
- papers/what-are-learning-disabilities/

Lewis K.E., & Lynn, D.M. (2018). Against the odds: Insights from a statistician with dyscalculia. Education Sciences, 8(2), 63–75. Lewis, K.E., Fisher, M.B. (2016) Taking stock of 40 years of research on mathematical learning disability: Methodological issues and future directions. Journal for Research in Mathematics Education, 47(4), 338–371.

National Council of Teachers of Mathematics (NCTM). (2000). Principles and Standards for School Mathematics. NCTM.



Jensen, B., Sonnemann, J., Roberts-Hull, K., & Hunter, A. (2016). Beyond PD: Teacher professional learning in high-performing systems.

Karp, K.S., Dougherty, B. J., & Bush S.B. (2020). The math pact: Achieving instructional coherence within and across grades. Corwin

Kobett, B., & Karp, K. (2020). Strengths-based teaching and learning in mathematics: Five teaching turnarounds for grades K-6. Corwin

Lambert, R., Chun, M., Davis, J., Ceja, K. L., Aguilar, K., Moreno, P., & Manset, L. (2019). "My dyslexia is like a bubble": How insiders with learning disabilities describe their differences, strengths, and challenges. Learning Disabilities: A Multidisciplinary Journal, 24(1),

Lambert, R. (2018). 'Indefensible, illogical, and unsupported': Countering deficit mythologies about the potential of students with learning

Learning Disabilities Association of America (2021). What are learning disabilities? <u>https://ldaamerica.org/advocacy/lda-position-</u>



112

- Core State Standards: Mathematics. Retrieved from http://www.corestandards.org/assets/CCSSI Math Standards.pdf
- Pashler, H., Rohrer, D., Cepeda, N.J., and Carpenter, S.K. (2007). Enhancing learning and retarding forgetting: Choices and consequences. Psychonomic Bulletin & Review, 19, 187–193.
- Exceptional Children, 51(1), 31–42.
- Rodis, P., Garrod, A., & Boscardin, M. L. (2001). *Learning disabilities and life stories*. Allyn and Bacon.
- school, 13(4), 200–207.
- Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from https://ies.ed.gov/ncee/wwc/PracticeGuide/15

Swan, M. (2005). *Improving learning in mathematics: Challenges and strategies*. UK Department for Education and Skills. Trott, C. (2015). *The neurodiverse mathematics student*. The University of Birmingham with The Higher Education Academy. Understood for All (2021). *Math Challenges: Videos of Kids*. [Video]. Understood.org. <u>https://www.understood.org/articles/en/through-your-</u> childs-eyes#Math challenges: Videos of kids



National Governors Association Center for Best Practices, & Council of Chief State School Officers (NGA & CCSSO). (2010). Common

Powell, S.R, & Fuchs, L.S. (2018). Effective word-problem instruction: Using schemas to facilitate mathematics reasoning. *Teaching* 

Rubenstein, R. N. (2007) Focused strategies for middle-grades mathematics vocabulary development. *Mathematics teaching in the middle* 

Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., Thompson, L., & Wray, J. (2010). Developing effective fractions instruction for kindergarten through 8th grade: A practice guide (NCEE #2010-4039). Washington, DC: National Center for Education

113

U.S. Department of Education (2018). Office of Special Education Programs, Individuals with Disabilities Education Act (IDEA) database. Retrieved December 27, 2018. from https://www2.ed.gov/programs/osepidea/618-data/state-level-data-files/index.html#bcc
Virginia Department of Education. (2020). Evidence-Based Specially Designed Instruction in Mathematics Resource Guide. https://www.doe.virginia.gov/special\_ed/disabilities/learning\_disability/swd-mathematics-resources.pdf
Virginia Department of Education. (2020). Students with Disabilities in Mathematics: Frequently Asked Questions. https://www.doe.virginia.gov/special\_ed/disabilities/learning\_disability/swd-mathematics-faq.pdf
Virginia Department of Education (2020).
Vocabulary Word Wall Cards. https://www.doe.virginia.gov/instruction/mathematics/resources/vocab\_cards/index.shtml

Photos: www.istock.com

Icons: https://thenounproject.com/



