

# Webinar Transcript: Language, Discussion, and Questions in Early Math

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**David Yanoski:** Today’s webinar is “Language, Discussion, and Questions in Early Math.” And it will be presented by Doug Clements, from the University of Denver, and our facilitator today will be Crystal Day-Hess. Crystal will introduce Doug and tell us a little bit more about both of them in just a second. But before we do that, I just want to talk briefly about REL Central.

If you’re familiar with the REL program, you know that the Regional Education Labs are a federally funded program, and REL Central, specifically, serves the applied education research needs of Colorado, Kansas, Missouri, Nebraska, North Dakota, South Dakota, and Wyoming. Obviously, this particular webinar is going out across the country and we have people from across the country joining us today, but REL Central, specifically, works to meet those applied education research needs by providing technical assistance and coaching and then helping stakeholders within our region participate in research studies related to education. With that very brief introduction to REL Central, I’m going to turn this over to our presenters for today, Crystal and Doug.

**Crystal Day-Hess:** Hi, hello everyone. This is Crystal Day-Hess, and I’m happy to introduce Dr. Douglas Clements. He is the Co-Executive Director of the Marsico Institute for Early Learning at the University of Denver. He’s also the author and principal investigator of multiple IES and NSF-funded studies on early childhood math development and learning, as well as numerous publications in this area. So without further ado, here’s Dr. Clements.

**Douglas H. Clements:** Hi, everybody. Thanks so much for joining us for this webinar. We’re hoping that today you’re going to learn about the importance of language for developing early math thinking and learning, and the contributions that high-quality language instruction and experiences can make to language development in general, as well. So most of these recommendations come from the practice guide, “Teaching Math to Young Children.”

And so I hope you’ll learn about the five basic recommendations. And we’re going to be concentrating on mostly one and then a little bit the others. So these multiple strategies for implementing the one we’re going to emphasize—Recommendation 4—you’ll learn about that and the adjustments necessary to make these relevant to the classroom and the populations with whom you work.

So let’s get started with the practice guide. So this practice guide is interesting. All the IES practice guides share certain attributes. They all cover a wide range of research and

recommendations. We're going to focus as I said on Recommendation 4. But it's important to note that every one of these practice guides, they're not just a few good ideas. They're also designed by experts.

So the important thing about them is that what happens is that these guides are the—National Research Council Reports gathered together panels of experts that include teachers, researchers, policy people and the like—gather them together to examine the body of literature and determine what recommendations they should have and what we should say. And importantly, they're all free online. So all these resources you can go to these links and see these things and download these recommendations online.

So let's overview—a brief overview—of the relevant recommendations. And I'm going to skip through one through three pretty quickly, but they are all so interesting and all relevant. Recommendation 1 is to teach number and operations using a developmental progression. That notion of what is a developmental progression—levels of thinking through which kids pass as they learn particular ideas in number—and by operations they mean early developing arithmetic—two and two is four kind of thing. I'll be sprinkling those explanations of developmental progressions are at the heart of what we call a learning trajectory. So I'll be sprinkling those in through the entire time.

Recommendation 2 is to teach geometry, patterns, measurement, and data analysis also using a developmental recommendation. So number and operations probably the core important topic, but we neglect other topics such as geometry, patterns, measurement and data at our peril. They are important complements to that. Recommendation 3 is use progress monitoring to ensure that math instruction builds on what each child knows. So all that's saying is let's understand where the kids are in these various developmental progressions and how we can modify instruction and experiences that we give kids based on that.

Recommendation 4 is our big focus. I'll wait on that till we concentrate on that and likewise Recommendation 5 we'll be doing in a little bit. Well, let's look at 4. So Recommendation 4 is to teach children to view and describe their world mathematically. To encourage kids to use informal methods—so their own thinking. Mary Baratta-Lorton published a book a long time ago that I love called, *Mathematics Their Way*. And that's what we mean by informal methods. How do kids think about this? To help kids link the formal math of vocabulary in words.

And when they get to kindergarten and first grade, they're going to encounter a little more formal mathematical representation. How do we help them link that to this informal knowledge that they come with? How do we use open-ended questions, so that the creativity and joy in thinking about mathematics is ascendant? And the kind of notion that you and I might have experienced mathematics as a little more boring is not a part of our kids experience. And finally, to encourage kids to recognize and talk about math. Probably the most time we'll spend today will be on that last one.

Now, Recommendation 5—let’s look at that just very quickly. To dedicate time each day to teaching math, and integrate math instruction throughout the day. What do we mean by that? We need to plan daily instruction on specific math concepts and skills. So we want to have whole groups, we want to have small groups, maybe some individual work and the like, that’s intentional mathematics. To embed mathematics in classroom routines and activities, so when we’re lining up we talk about not just who’s first, but who’s first, second, third, and fourth to get at ordinal numbers, for instance.

To highlight math within topics of study across the curriculum. So no matter what we’re doing, literacy or the like—if we’re reading a book and the book has page numbers. More likely if we’re reading Blueberries for Sal and there’s ku-plink, ku-plank, ku-plunk, and then she puts three berries in there and then she eats them all. How many berries are there now? We can discuss a lot of things, even in literacy context. To create a math-rich environment. Maybe the most important thing we can do is to make sure kids bump into interesting mathematics everywhere they move in the classroom. And finally, to use games to teach math—and we’ll have several examples today.

So another source that we’re using for today’s talk is the report—it’s a National Research Council report—also a body of experts—called, “Mathematics Learning in Early Childhood,” which claims that mathematics learning is best done along paths, which is the exact same thing people as developmental progressions of learning trajectories. And the subtitle of that report—you can barely see it in black under the word childhood there—talks about paths towards excellence and equity. So using learning trajectories helps teachers and children not only achieve excellence, it helps teachers achieve equity and social justice in their work with children. So that’s an important resource as well. Oh, also, free online.

So I’d like you to take a few minutes now—at least three minutes—and skim the practice guide if you can. Go online, take a look. Look specifically at maybe the most interesting sections for you would be how to carry out this recommendation section for each recommendation—if you want to, shoot to 4. But if you’re scrolling through, I just wanted to point out that the section starts on page 15 for Recommendation 1. So you can take a look at that if you want to. And then if you have any questions or comments about it, put them in the chat box. Crystal’s got the chat box under control here and she’ll make sure either she or I will answer your questions. So I’m going to be quiet for a few minutes and give you a chance to scroll through that.

So the practice guide—some people missed that link on that page that was there—the slide that was there. So David has given you that link there. Why don’t I encourage you the second you have questions about anything we’ve talked about so far or any questions about early math, why don’t we give some people who just barely got a chance to click on the link just a couple more minutes to do so. But I’d love to answer questions from you. And the only way you can do them is by chatting, so I’d love if you open up a chat and send me your questions about the guide, about anything else we talked about so far, or other questions. Because we’re just going to basically give people a chance to get that guide up in front of them.

Putting your questions any time you like. We got one question here. “I’m curious how the roadblocks were developed. I’m sure that these were the most common, most popular, so I’m wondering why these were chosen to be published.” Ah, that’s interesting. This particular guide I wasn’t an author on. I was one of the co-authors on the National Research Council report that we showed you second, but I wasn’t one of the co-authors on this. So I think if I have another time when you guys are working on things, I’m going to quickly write a letter to one of the co-authors that I know very well, [INAUDIBLE], and ask him how that was decided.

From previous committees I was on, those issues are usually done by people on the author team to both gather and discuss questions they’ve heard and then send out an email to a variety of people who work in the field from teachers to, again, policy people, administrators, and the like. And they gather those up and then just the ones that are most common those are the ones they used. Is it Layla? I can’t see the—

**Crystal Day-Hess:** Yes.

**Douglas H. Clements:** Layla, if you have a follow-up question about one, that you think is particularly interesting—or one or two—just chat and the next time we have a Q&A, I’d love to address it. So let’s go to the next slide. And one of the handouts you have is a short—shorter than these online books—this is a short little piece Julie Sarama, my wife and colleague, put together called “Math in the Early Years,” that emphasizes four basic points that I think are really important and shared by all the references we talked about.

First of all, math predicts later school success. Secondly, children can engage in impressively deep and broad mathematical thinking. Third, equity is a concern, and we know how to address it. And fourth, we know a lot about how kids learn math and how to teach it using learning trajectories. So let me elaborate a couple of those.

Math predicts later school success. You may have heard this before, because the research has been out there pretty commonly. You’d think that early math skills would predict later math understandings. And that’s true. You’d think early literacy skills probably predict how kids do in reading later on. That’s true.

But the fascinating thing is early math predicts later school success, later literacy success, as well as the early literacy skills. So there seems to be something about math. Is it just knowing your numbers, knowing your shapes? No, I think it’s deeper than that. I think math is a foundational way of thinking that is important to kids throughout their school career and throughout every subject and topic.

So let me elaborate a little on a different one, the equity concern and the like. Math and language, it really is an equity issue. Mathematical thinking is not just verbal. Right? There’s a lot of visual and conceptual things that aren’t just words, words, words. But knowing the language of mathematics is critical for young children. Research has shown that children from low resource communities have just as strong intuitive mathematical competencies as their

higher resource peers, but they haven't had the opportunity to learn the language of mathematics. That's why it's an important equity issue.

Consider this research. Some children in homes hear about 1,500 number words a year. Sounds like a lot, till you figure out and you'll hear in that research that other children are hearing as many as 93,000 number words per year. That's 60 times as many number words every year before they come to us. Language in math talk predict number knowledge.

But math talk is missing. It's a missing link for many educators. Let's think about that. In classrooms, we could do more. When children make a math utterance in a regular classroom in the United States—early childhood classroom—teachers ignore it about 60 percent of the time and respond mathematically only about 10 percent of the time.

So there are marked differences associated with the income level in children's mathematical knowledge by four years of age, probably because of a difference of experiences. In a different study, math talk by teachers ranged from one instance in the entire visit to 104 instances. That's just a huge difference. And if we're a little bit intentional, if we're a little bit reflective about that, we can be in the 100 category instead of the one category. Because the amount of teachers' math-related talk is significantly related to the growth of preschooler's mathematical knowledge over the school year.

So this is your turn again. Please enter your example of children's math talk into the chat box. So consider, what's the last math question you can remember a child asking you. How did you answer? And what question did you ask kids that had to do with mathematics and how did kids respond? So let's think about that and help me by giving us a few examples in the chat box there.

A question from Brian, "Do we respond here or in the Q&A chat box?" David, can you help us out here. Where would you like them to respond? Keep it right here in this chat box or go to the Q&A chat box?

**Crystal Day-Hess:** All right, so looks like we needed to use the current chat box that we're already in.

**Douglas H. Clements:** It's a good question, Brian. Thank you so much. We'd hate to lose anybody's responses for that. So please, I'd love to hear you guys answer the last math question the child asked you, how did you answer, or just if you loved the question a kid gave. [LAUGHS] You don't want to tell how you responded. We don't care. We'd love to get some examples out there, if you would.

Patty says, "Which would be more inches or yards to measure a house?" Cool. Patty, can you tell us a little more about the age of that child?

**Crystal Day-Hess:** All right. So Patty— "Which would be more inches or yards to measure a house?" —It was her daughter. She was eight.

**Douglas H. Clements:** Eight-years old. You know measurement is such a neglected field in mathematics right up through the primary schools, Patty. That’s fascinating. And Patty, if you have time, I’d love to hear where that conversation went. A lot of kids around the age of eight are getting solid on their ability to say if you use a bigger unit measure, like yards or feet, you’re going to have fewer of them than if you use a smaller number, like inches, where you’re going to have to have a lot of them to line up. And that kind of inverse understanding—the inverse relationship between the size of the unit and the number of units—is really an important thing that some kids can think about at ages four and five, but it usually becomes a relationship, a rational relationship much later. I love this one, Crystal.

**Crystal Day-Hess:** Yeah, Sherry Lynn shared an example of this. She said, “A kindergarten student recognized a book as a rectangle. Another student said, ‘yes,’ and if we open it up it will be a square.”

**Douglas H. Clements:** Oh, that is so nice. That is so nice because it goes way beyond just shape naming, but a dynamic recognition of the widening of the width of the rectangle, the smaller dimension of the rectangle so it becomes a square. So it’s more transformational, which is really nice visual thinking. Excellent idea. Give us another one, Crystal.

**Crystal Day-Hess:** Yeah, and Leslie’s 3-year-old said, “Two and two make four. And she asked him or her, “How do you know that? Show me.”

**Douglas H. Clements:** Oh, nice. Nice teaching technique. We didn’t even get to that yet. And you’re giving us a good harbinger for a discussion we’ll have later, which is, “How do you know” and, “Show me.” And those kinds of questions—you gave the two probably most powerful one—are wonderful ways to develop mathematical thinking and language. So we really want kids not to just give correct answers. If you’re like me, your whole early school career in math was filled with did you get it right, did you get it right?

But the reason math predicts later literacy in school success beyond other early areas, early measures, is probably because of the kind of reasoning. And that reasoning is part of answering this, “How do you know?,” “Show me.,” “Can you prove it to me?” Because people, this is the genesis of reasoning and proof. Two hundred years ago, there were books that came out that basically said this is human psychology. And they were math books, because they thought all thinking, of course, came from logic.

And logic is a field within mathematics. I think that’s going way too far. We’ve learned a lot more about psychology, but there’s a kernel of truth to the fact that the kind of reasoning we do in mathematics and the kind of reasoning that kids need to be able to be coherent, interested, curious people in STEAM, literacy, language, and the like, are all contained in this kind of how do you know, show me, prove it, reason it through. Give us another one, Crystal.

**Crystal Day-Hess:** Yeah. So Lilly was sharing with us that they were looking at a decimals place value chart in her class and one of the students asked, how come there are not oneths? They used tenth sticks to tinker with the idea to show what those look like.

**Douglas H. Clements:** That—I'm going to borrow some of these stories.

[LAUGHTER]

I'm telling you, because these are really good stories. So oneths—that's really cool, because all the single number are oneths, they're singletons. And we don't group them until 10. But that's because a lot of people think that the decimal point is a symmetric point when you have a number with decimals and stuff. But the oneths, the singles are really the center of the whole number system.

And then you go to one side and you get 10 and then a 100. And you go to the other side and you get tenths and hundredths and the like. That's a wonderful question. Obviously, you guys are already doing wonderful things with your kids talking mathematics. I love it. Are there more, Crystal?

**Crystal Day-Hess:** Yeah. I'm going to go back up just a little bit. Sherry Lynn had a question for us. She said, "Teaching patterns is not in the kindergarten Common Core. How and when is that best addressed?"

**Douglas H. Clements:** Ooh. That's such a great question. Thank you for that one. Why is patterning not in the Common Core? Well, there's a kind of good reason and a kind of reason that's probably a little unfortunate. The little unfortunate reason is patterning as a content area has honestly gotten a little bit of a bad name in early childhood.

That is some understandably slightly critical mathematicians have observed in textbooks and materials for teachers, articles for teachers in a teaching practice, patterning often goes like this, "Okay kids, ready? We're going to do some patterns. Red, blue, what comes next?" And the mathematicians would answer, "Anything you want. You haven't defined a unit. You haven't established a pattern yet." So we often do patterning not so well in early childhood. We can do it better. What's the good side? That's one of the reasons it's not a topic.

What's the good side of patterning not being a topic? In the NRC report, "Mathematics in Early Childhood," that I talked to you about before, that I was co-author on, we decided actually not to make patterning a unit—like a red, red, blue—a unit on red, red, blue, and linear sequential patterns like that—because we thought patterning was so very important. And honestly, the Common Core did the same thing. It said, patterning is a specific mathematical process that sure can be done with sequential patterns—like red, red, blue, and the like, red, red, blue, red, red, blue—but it can also be something that's a part of teaching number, teaching shape.

So when you add one side to a triangle, you get a four-sided shape. You add another side, you get a five-sided shape. Then you get a six-sided shape. And if they're regular shapes, they start

to look more and more like a circle. It's an amazing kind of pattern in that. Or you can look at numbers. And if you have a one connecting cube, and then you have two, and then you have three, this step pattern that looks like stairs is a growing pattern. So it's not just red, red, blue. There's patterns everywhere.

A mathematician, Lynn Steen, half a century ago, defined mathematics as being the patterns in number and space. He didn't say math is number, math is shapes. He said, "patterning is—sorry—mathematics is the study of patterns in number and space." So the good news about patterning is it's all over the place. Where is it in the Common Core? It's in the mathematical practices. Read the last two mathematical practices of the Common Core where we look for structure and we look for the usefulness of repeated reasoning. Repeated reasoning is basically where much patterning comes in. Anything else, Crystal, from our—

**Crystal Day-Hess:** Let's see, we have a lot of great examples. I'll share one more here. Brian shared a couple with us. And he said his daughter asked me, "How many Starbursts do I have at home?" And he said, "I don't know. I think there are some left." And then she said, "I had four and four, which is eight. I ate four, so I have four left."

**Douglas H. Clements:** Oh, okay.

[LAUGHTER]

Brian, no trouble with mathematics for this one. This is great. This is great. Listen, another thing, Brian not only shows us how much mathematics is not an imposition of elementary curriculum down into the early years. It's not done right. Worksheets—I gotcha, I'm with you. The content—it's natural to kids. Kids find it interesting, motivating—like I said before—even joyful. And I think many, many of these examples show the interest kids show in mathematics. Any more, Crystal, or we'll move on?

**Crystal Day-Hess:** Yeah, I think we keep moving.

**Douglas H. Clements:** Let's keep moving. But you can keep filling the chat box, people, all you want, at any time. You don't have to wait for the periods. Put your comments, questions, or the like in there. So let's move on to an important part of language, of course, is our dual language learners. This is really important because children who are members of linguistic minority groups often also deserve very special attention. A defining characteristic of these children in the recent report by the National Academy of Sciences and Medicine is the diversity of these children.

In other words, some of them come to us with two strong languages, some of them come and they're English language learners. Some of them come and they haven't started really—learning English very much at all. We need to serve all their needs. Most of them lag behind their mono-linguistic peers' educational achievement. And, of course, it's because—most

curriculum, most cultural kind of things are done in English, so limited proficiency in English certainly poses a high barrier.

But—although there’s many challenges, there’s also many developmental benefits. Bilingual children can often see a general mathematical idea more clearly than mono-linguistic children can. That seems like, what? But listen, kids who know that eight means as many—Starburst as Brian’s daughter had, and also know that ocho means that same number, have abstracted the notion that it’s not the word, it’s the idea that’s important. They have two different words for the same idea. That’s powerful.

And that’s the kind of—uh—benefits that bilingualism can give kids. So we want to support their first language. And mono-linguistic English kids—we want to help them learn a second language if we want them to be the most powerful. And listen, children can learn two languages as easily as one. There’s plenty of challenges when they don’t have the opportunity, but the limitation is not in kids learning the language. The research is very clear on that. Two languages as easy as one, as long as we give them the experience.

All teachers can use strategies to support both languages, even if you’re not bilingual yourself. When possible, bilingual approaches in school tend to be best—the research shows. But the following characteristics of instructional programs support oral language learners development of dual language learner kids, regardless of whether you have a bilingual kind of instructional strategy at your school or not. Specialized instruction focused on components of oral language.

Opportunities to interact with speakers, whether at home or school, or in other cultural settings proficient in second language. Real important for kids to keep that second language—growing and the first language strong. And feedback to students during conversational interaction, so they understand, they get feedback. And finally, dedicated time for instruction focused on the oral language, the oral English that they need to learn.

Because it’s a myth that language is based on numbers and symbols—I’m sorry—so math is based on numbers and symbols, so language is less of a concern. So many people think, “Oh, it’s just like I’m used to math. I got worksheets. I added up the digits. That’s what math is.” Understanding math, using math, problem solving with math really demands that you understand the language of mathematics. So the fact is that children need the language of mathematics and they learn math from interacting in oral language. So teachers need to understand the linguistic characteristics of classroom language and also master ways to connect everyday language with the language of math for kids.

So what are some math strategies for DLLs? Well, math talk is more than just using math vocabulary. Sure we have to teach specific vocabulary terms. And if we do it ahead of time, emphasizing cognates—in other words, um—here’s a good example. Um—I’m going to step back a little bit, too, and expand on this example a little bit. If we’re doing subtraction—and we’re going to talk about subtraction a little later, —but I think I’ll use that example now, too.

A lot of people want to help kids understand subtraction by using the word, “takeaway.” And that’s a phrasal adverbial form. In other words, there’s two different words there and you’ve got to put that together. Even the one word altogether still honestly has these two kind of things. And they don’t relate for Spanish-speaking kids directly. That—that can be a little bit of a barrier. If you simply substitute, as you tell the story—and you’ve got—um—let’s use Starburst. You’ve got eight Starbursts and you ate four.

Instead of saying, let’s put out eight and then let’s takeaway four—if you simply use the word, “remove,” which relates to the Spanish remover. Right? So there’s this cognate there that has the same roots. They sound the same. They have the same linguistic root. It’s a lot easier for Spanish people to understand that kind of construct. So that’s a way to emphasize a cognate that both is more understandable and relates the Spanish to the English so closely for those kids that they have an immediate access to the language then.

We have to provide visual and verbal support. So again, having the four and the eight counters as you say these words—ocho or whatever—is certainly helpful. But vocabulary alone is insufficient. Teachers need to help students see multiple meanings and possible conflicts of meanings of different languages, and address the language of mathematics, not just the terms of mathematics. There’s a lot of mathematics that is about how you know, that goes beyond learning words like subtract.

And finally, we have to build on the resources that bilingual children bring to mathematics. Every culture has tons of knowledge that can be used to develop mathematical concepts and understandings. Every culture has mathematics of the home that we could bring into the classroom and help kids make sense of math.

**Crystal Day-Hess:** So Doug, I want to interrupt for just a moment. We had a question come up. And the question is, how do you remediate with students who do not know foundational concepts in their native language? For example, kindergartners who do not know numbers or shapes in their native language.

**Douglas H. Clements:** Yeah, that’s a tough one then. That’s a good one. There’s probably still native language kind of thing. So sometimes the parents can help us with that, because the kids may not have had opportunities to hear those things at home. Some cultures and some cultures within the US and the like, there tends to be a belief that mathematics teaching is for the school. So there’s actually a resistance to talk about numbers, shapes, and other mathematical ideas at home. Working with parents on that can be one thing we can do.

The other thing you could do really for DLLs—Dual Language Learners—who don’t have that resource coming from home is just to use a lot of acting things out and giving the English words to that. So that might be the way you have to go. When we say provide visual and verbal supports, that’s the kind of thing I mean. Use manipulatives. Be very clear about eight, and then count one, two, three, four, five. And then circle them again and say, eight. And then remove four. And then pull that four away. Four—and the like. So a very few limited, but still illustrative

English words tied to visual manipulative kind of experiences and specific actions to build that up.

It is true that because so much of instruction tends to be English oriented, it's very quick even in preschool and also in kindergarten, for kids to be just more competent in math talk in English, even if they're strong and not quite so strong yet in English in the whole and they're very good in their first language. But they learn the math in those kinds of English contexts. So trying to keep it bilingual is great, but if they don't have the first language strengths, doing those actions and visual supports certainly helps.

**Crystal Day-Hess:** Great, thank you.

**Douglas H. Clements:** Anything more you want to add, Crystal, just jump in. Math and language then is a two-way street. I hope I made this clear. Mathematics predicts later reading, as well as early reading does. On the other hand, vocabulary and knowledge of print and ability to tell stories predicts later math scores. It's not either or. We're not trying to say, do one before the other. It's both and. Here I'm trying to emphasize that indeed the math is an important contribution to literacy, but I'm certainly not saying that literacy and language in all spheres are not also contributing to mathematics.

So, we finally get to Recommendation 4: Help children link formal math vocabulary, symbols, and procedures to their informal knowledge or experiences. This is why the math guide said that everything we've been talking about so far comes home with this recommendation.

Let's take a look at Joan. Joan was a preschool teacher—who used—in Buffalo, who used our building blocks math program. And when she was interviewed, here's what she said. "I love the program," she said. "The children got good at math tasks, being able to verbalize, talk about them, not just doing them. I think this is why my kids do so well on the Brigance. Their verbal skills got better. They transported it to language. I worried about it. I never taught so much math in a day. I thought reading comprehension was being sacrificed. But when I stepped back and looked, I realized doing math was doing language."

Preschool teachers in another study loved math learning, but they were worried that their coordinators and principals cared more about their language and literacy scores. This was a study Julie and I were conducting. So in our next study, the following study, we said, aha, how can we convince them that we've done no harm to literacy and language scores? I know. Why don't we test those things? So that's what we did. And we found that literacy scores did not differ between the two groups. In other words, they were just as good at their ABCs as the kids who didn't spend as much time on mathematics. But the cool thing is the math group we found was significantly higher on our oral language assessment.

Let me tell you about this just real quickly. See those cartoons? The pictures are just three of a long story of a bus who, as you can see in the middle one, was a naughty bus, didn't want to take the kids to school anymore, and just takes off. Oh, that bus learns its lesson and finally

comes back. So what this assessment does is you sit individually with a child—3- or 4-year-old, 5-year-old. In this case, it was fours. And you tell the story and you point to the pictures. And then you say, now here’s the pictures, you tell me the story. And the child is supposed to tell you the story back.

And so you record everything the kid says. It’s a fun 4-minute, 5-minute exercise, and then it’s a painful three hours of coding every word that gets said. And here’s what we found. The math group, the group that had done building blocks, with significantly higher on information. That is the number of vocabulary words the kids used. These were not math vocabulary words. These were the words from the story that the authors of the story said were key conceptual terms.

So, think about that. Learning math in preschool helped them when this test was given, early in their kindergarten year, learn more vocabulary and use more vocabulary words. It’s shocking, but a confirmation of what we said before. Early math teaches you about learning words and ideas, not just math words and ideas. The second one—grammatical complexity—the kids who learn to answer Brian’s question—no, it wasn’t Brian—who was asking the questions, how do you know and show me? It was one of our participants.

**Crystal Day-Hess:** That was Leslie.

**Douglas H. Clements:** That was Leslie. Leslie’s wonderful questions—how do you know and show me and the like. Kids who could—when you have to tell people how do you know something about math, you’ve got to dig down pretty deep to figure out, well, how was I thinking? And when you do that, you learn how to express yourself with greater grammatical complexity. So even in this bus story—this non-math story—they used more grammatically complex sentences than did the kids who hadn’t had the math experiences.

They were more independent. They didn’t need to be encouraged to tell me some more, tell me the next picture. They just went right through the story. So autonomy—Connie Kamii, one of the famous researcher in early mathematics, says that autonomy, thinking for yourself, is the key goal of early mathematics for her. And I think that shows up here. And then, finally, inferential questions. They could reason through some of these inferential questions better than the kids who hadn’t experienced math. So just a little bit—one little study—but I think pretty telling about how important it is to learn mathematics for more than just learning mathematics.

So let’s return to this informal and formal math. There’s a good bit of research on this and it says, children have intuitive strengths in mathematics. That even adults invent strategies to solve math problems. Most of us don’t realize it, but we’re pretty inventive when we solve problems ourselves. But school mathematics is out of balance people. Too often it emphasized the same limited content and rote facts and skills, instead of concepts, problem solving, communication, relational thinking, and reasoning. So too many children lose the connection between that wonderful, creative informal thinking they come to us with, and formal school mathematics later in the primary school, and do not make sense of school mathematics.

Let's give an example. What is informal math anyway? Well, 4-year-old Carmen had almost finished her pretend pizzas—had almost filled her pretend pizzas with toppings. She was rolling a little number cube. And as she got ready to roll the number cube, she said, "I'm going to get a high number and win." And her friend said, "You can't. You have four spaces and the number cube has only ones, twos, and threes on it."

Hey, the numbers might be small, but that reasoning is impressive. Kids can reason mathematically. Carmen's friend probably intuitively used logic. So we could describe that as adults. To win, Carmen must get at least a four. The number cube has only one, two, and three. These numbers are less than four. So Carmen can't win on her next roll. So that's really impressive reasoning. It might seem abstract to put it in that form, but kids are capable of it.

The main question for teachers is some version of, how do you know? So this comes from Bob Davis, one of my heroes in mathematics education—now deceased. His daughter, Alex, was five- years old and her brother Paul was three. And Alex bounds into the kitchen and says, "When Paul is six, I'll be eight. When Paul is nine, I'll be 11. When Paul is 12, I'll be 14." Her father says, "My word. How on earth did you figure all that out?" And Alex said, "It's easy. You just go three, four, five. Five, six, seven. Seven, eight, nine." And so on, up and up and up. She used her informal knowledge of rhythms and counting to solve a pretty tough arithmetic problem up to 18 plus two equals 20.

How do we get them there? Well, children as young as preschoolers can learn to talk about the many strategies they invent. We've got to give them time to think. We have to revoice when necessary. "So you used a diamond," you say. "Yeah, mathematicians call that a rhombus." And we have to model. I'm going to move the boxes as I count to keep track of them. These kind of things really help kids.

We need to ask kids to share, clarify, and justify their ideas. So one teacher was working with a group of children doing geometric puzzles. And she would tell them, "How do you solve that? What if you didn't have any hexagons, could you have solved it at all?" And some of the kids if you know your pattern blocks will say, "I'll use these red trapezoids to do that." Let's proceed.

Kids might "Think-Pair-Share." The teacher asked one kid to figure out how many one more than three is. And when the child has difficulty, the teacher might say, "Can you show me three to get started?" And then the child all of a sudden says, "four." And the teacher says, "Can you teach us how you did that?" That's another way of saying, "Show me.," or "How do you know?," "Did anybody do it a different way?" So all these kind of questions help kids think creatively and understand that there's different ways to solve it and that you can articulate your kind of reasoning in there.

In circle time, two children explained to the group what they did. And the teacher asked another child, "What did Dominic do?" And the teacher asked, "Did anyone do it like Juanita? Would that work for you?" So again, listening to and evaluating each other's ideas and strategies is essential.

Also essential, open-ended questions. So the practice guide has these. I don't think I'll go through them all. But these kind of open-ended questions really help. Before calling on a child, you might allow enough time for more than just a few kids to think of an answer. And when in groups, one child can help another child come up with an answer. Also, rather than saying just yes or no quickly, allow multiple possibilities to be discussed.

For example, a teacher can show the entire class a picture of a mother and daughter holding hands waiting for a school bus. And the teacher can ask, "How are these two people different?" One child might say, "The mother is bigger than the daughter." Another might say, "The mom's wearing stripes and the daughter is wearing dots." Although, the teacher could ultimately focus on the answer that best fits the math context of that particular situation, he or she should acknowledge that there are multiple correct answers to that kind of open-ended question. Do you have something, Crystal?

**Crystal Day-Hess:** Yeah, it's moving back a little bit to when you were talking about the connection between math and language.

**Douglas H. Clements:** Great.

**Crystal Day-Hess:** But we had a question, "How does language develop along with math experiences? How do these help to develop key foundational understandings, such as conversation of number and cardinality? Is there a connection between language development and being able to extract these ideas?"

**Douglas H. Clements:** Yeah, that's a great question. Yeah, conservation of number and cardinality—let me give you a—I'm trying to think how the best maybe example or the connection between language development there. We know that for very young children—right after they turn one, right to the point when they're three or so—they don't develop cardinality as well if they don't have number words first. What we call the intuitive numbers—those are numbers up to about five. And the cardinality notion—cardinality is just a fancy word—good for you using it. Cardinality is just a fancy word that means how many there is.

So we know when we count four objects—one, two, three, four—that that means there's four objects in that group. Kids don't know that right away from the beginning. They'll count one, two, three, four. And if you say, "Give me those four," they'll give you that last one because it had the name four when they counted. But if they understand, if they've got lots of number words for this, and especially, if they can recognize those small sets—so the first learning of cardinality does not come through counting. It comes through a process where you just recognize the sets. If you can do it quickly, it's called subitizing, from the Latin word that means suddenly.

So kids actually learn the word two before they learn the word one, because two is different. All of a sudden you got two. There's one of everything you point to. But now when you have two, that's interesting. So then they learn the word two and one. And if they can learn to recognize a

set of three—not counting, but just seeing that bonk, bonk, bonk—that many objects and recognizing it as three. Then when they count, when they start learning counting one, two, three, they could say, whoa, yeah, one, two, three. I recognize that. That’s three. That’s the same set of three I know how to name.

And so that kind of language that’s attached to those things is essential to developing this kind of idea of how many is in a group and that every group of three actually—if it’s three dogs or three little candies—it’s the same number, even though they’re very different objects or sizes. Is that good?

**Crystal Day-Hess:** Yeah. Thanks for that question, Christopher. Appreciate it.

**Douglas H. Clements:** So discussing strategies with kids, we’ve emphasized already, helps build rich representations of mathematical ideas. In Table 6, in the guide there, there’s a really useful table on using informal representations. So there’s a concept. We’re highlighting subtraction down at the bottom there. There’s the informal representation, like take away or fewer. And there’s teaching the concept, like start with a collection and take away some items to make it smaller. For example, start with three crayons and take one away. And then ask, “How many?”

So building these kind of informal representations and informal knowledge really helps kids. Using the informal phrase “take away” or “remove”—remember I said before—and then later explaining that subtract has the same meaning is far more child-friendly, far more developmentally appropriate, and far more effective, than, for instance, telling kids what subtract means is the mathematical definition—you blather on and on and it doesn’t help kids connect to their informal understanding.

The practice guide also says, “Encourage children to recognize and talk about math in everyday situations.” So a teacher might say, “I have to figure out how many cups we need for the birthday party. Can you help me? How should we do that?” Now, the birthday party may have just the kids in the class or it may have parents coming in or something. So this can be a really tough and interesting question.

Encouraging families to talk about mathematics, number, arithmetic, but also spatial relationships and patterns like we talked about before. The broad idea of patterns. Patterns and number patterns in space, as well as the red, red, blue, red, red, blue patterns and the like, are really helpful. To connect informal and just formal school mathematics help kids learn meaningful problems. Ask kids to share, to justify, to listen to their friends and the like.

As we talked about, revoice, modeling the mathematical vocabulary. So a lot of kids will say—think of a word like straight—and they’ll think of the word straight only when something is perfectly horizontal or vertical. And I wish you could see me, so I could gesture this out. But straight in mathematics doesn’t mean that in most contexts. In most contexts a straight side of a shape means that it is a line segment with no turns or bends in it or the like. So be careful of

that. Model careful mathematical vocabulary. Use their words to revoice and then add the description in mathematics.

Be aware of potential ambiguous words. And carefully introduce new words and meanings after the concepts are understood. That's why we gave that example of taking away and relating it to the word subtract. And you don't have to use the word minus and you don't have to use—you don't have to put the numeral nine minus sign five equals to ask the question to kids. It's better if they come to it via oral language and physical manipulatives.

So if you had nine candies and you ate five, how many do you have left, is a better way to talk about it because it gets conceptual instead of this nine minus five equals kind of thing written on a board, which tends to just bring out, OK, so we have to fill in the right answer instead of thinking about it.

**Crystal Day-Hess:** Sure. So Doug, we have a comment or question from Carrie. She says, "I've heard math educators being more specific with defining equal as same amount as, rather than same as, because for young children four plus four is not necessarily the same as eight.

**Douglas H. Clements:** Yeah.

**Crystal Day-Hess:** So your opinions?

**Douglas H. Clements:** That's a great one, Carrie. Thanks for that. Yeah. The same amount as or the same number as is right. That's what makes two things equal in number. So let's pretend we have a situation like the birthday party. So we want to have the—what was it? Cups? Where we were trying to get cups for the birthday party—we have already forgotten. We wanted to have enough cups for kids, let's say. So we want the number of cups to be equal to the number of kids. But those aren't the same. 18 kids isn't the same as 18 cups. The number is the same.

So I appreciate what Carrie is saying. The same number as is more important concept for equals there, unless you mean the same weight as or the same length as. And then it's different, but it's still not the same. So every good foot ruler might be the same length, but it's not the same ruler. So well said Carrie. I hope I answered that. And Carrie, if you or anybody else says, "He didn't really answer my question. I still have a follow-up question," love to hear it.

Where were we here? Observe children's use of words, negotiate new meanings for practical experiences. I think that is something we covered. Hey, even toddlers—toddlers talk math. Toddlers around 18-months learn one simultaneously with two. Like I said, there are some reports that more kids use two before one, but most of the reports would at least say it's very close when they learn those two words together. And at about two years, they use two spontaneously and consistently. And they learn three at about three years—most kids in our culture.

But, listen, they can do more. Because number words are not emphasized in many homes. Do you remember the research we talked about with the great divergence of the experiences kids

have with number words in both home and school? And not only that, we don't do everything right. For instance, non-examples are rarely used. I'm going to go to the next slide and say, reminder, in one week some parents used only 28 number words compared to other parents using 1,800. You can see that kids just have an experiential difference there when they come to us. And parents using these words, teachers using these words, extends kids' quantitative knowledge and contributes to differences in math competence, like the cardinality idea that we talked about before.

And again, children need examples and non-examples. So that if kids point to the one of the left and say that's three—or no, that's the wrong example for this—sorry about that. That if the kids see these two groups and the parent says, "Which one is two?" and then they point to the one on the left and say that's two. Then the parent can say, "That's not two, that's three. This is two." So once again, it's not just always saying this is two, this is two, this is two. But sometimes saying this is two, this is not two, helps kids separate what you're talking about and distinguish and learn those concepts.

People, it takes a community to do this well. The teacher has to create a nurturing and supportive math talk community, eliciting thinking from students, helping students explain, and help each other explain and solve problems. Getting kids talking to each other. We love "Think-Pair-Share." And actually when I first learned about "Think-Pair-Share," I tended to think that you think first, and then you pair up, and then everybody shares with the whole group. Nope.

The most important sharing of "Think-Pair-Share" is to slow down, to think first yourself, then to pair up with somebody, and then to share each person with the other. So I share with Crystal. Crystal shares with me. And then the teacher can decide, if they're in a small or a whole group, how many of those can I get to share with the whole group. But the most important sharing is between the kids like that. And it really develops kids' mathematical thinking.

I'm going to give you two examples now that came from an article by Karen Fuson, and Julie Sarama, and myself. That actually I will give credit to Karen. She's the one who has found that this is really effective. This is an activity that Karen, and the teacher she works with, of course, has used in hundreds of kindergarten classrooms way at the beginning of the year. So kids see the five group patterns. We call it a five in tens frame. Can you see that up to five fits in the group and then you start the six on the next line with the little dots there.

And they relate them to the five groups on their fingers. They then practice counting from one to 10, as one student leader points to each number. Like picture that one through 10 up on a board or something like that, in a chalk tray or something. And the one child is pointing at it and they call it a number parade. And the kids can focus on the numerals or the dots, or they can even raise their fingers as they count, or they can jump with each word. You decide. Feedback from the teachers and observations in classrooms indicate the kids really liked doing that and enjoy and can do this activity. And then, you do the math talk.

So that's very quick. It's a warm-up. The heart of the activity is in this math talk where you relate the visual quantity three to three fingers. "Show me three fingers." "Let's make three sounds." "Let's jump three times or slap our thighs three times" or the like. "Let's practice visual imaging. Close your eyes everybody, visualize. Think of something that's three. Now 'Think-Pair-Share.' Turn to your partner. What did you see when you closed your eyes? What three did you see?" And then you could describe different arrangements of your fingers to make each number. So with three, there's not many arrangements, but you can make all three on one hand, you can make two and one, and you can make one and two on the other hand.

Here's another one. This is a follow-up. So picture each kid has a mat like the one pictured. The numerals one through five, originally not ordered yet. And red and blue tiles at the bottom. Do you notice that five are red, five are blue, and one side is plain and the other side has a white dot? That comes up later. To begin, the children, their first job is put the number tiles in order at the top. They can look up at that number parade that's still up there in the classroom to help them put them in order.

Then what's next? The teacher or a child points to the number tiles in order and says the number on that tile. So first one, then two, we're up to three now. Children pull down the number tile for that number. Can you see how they pull down the three? And then, children show that number of tiles. So cool, but once again, heart of the activity comes right here. The counting mats then helps kids because now they can describe the different arrangements by color. So I put—me being the picture there— "I put two blue and one red."

Or spatial relationships—actually, I put all three in a row. But you can imagine kids put two at the top and one in the bottom. You can change your arrangement. Discuss why you still have three. You can talk about how many have dots and how many don't have dots. You could say, "Copy the person next to you." If you were "Think-Pair-Sharing," copy your partner's way of doing it and they have to copy yours. And you can see and describe all the partners of three, in other words two and one again, and the like.

**Crystal Day-Hess:** I like this example, Doug, not getting us too far off track. But with the different of dots, different colors, it really helps support cognitive flexibility and put things in functions.

**Douglas H. Clements:** Oh, tell us how. Tell us how do you see that?

**Crystal Day-Hess:** Yeah, so we know that cognitive flexibility comes into play where kids are able to see multiple representations of some item or some concept. So here, they could see three as being two blue and one red. Or in this example, three is also two tiles with one dot and one tile with no dot. So it's really supporting that flexible thinking.

**Douglas H. Clements:** Yeah, that's really good. And you've got to step back and look at it a different way to do that. And like I said, and then for another arrangement, the kid might have

one on top and two at the bottom, and then that would be different too. And, of course, as we get up to six, we're going to have a lot more different ways of doing that. Nice example.

So this is your turn. So we'd like to ask you now to reflect on the strategies that we've talked about and develop one concrete step based on the strategies for improving math talk in your classroom. So if you're with colleagues right now, please talk to each other, right about this—and come up with more than one. But if you're alone, then it's fine, still think of one. And please enter the comments or questions you have about this, anything you wish, in the chat.

We'll give you a couple of minutes quietly to work without us blabbing on. But then as you enter them in the chat, even if we're answering some of the things on the chat or verbally, um—please you can keep talking to each other and entering it, if you're not done with your conversation. So a couple of minutes just for you.

So we've got—please put more in the chat box. You guys were great before. In fact, we've got some old entries into the chat box—old—[LAUGHS] a few 30 minutes ago—old only. But we do have one new one. What was that one?

**Crystal Day-Hess:** This one?

**Douglas H. Clements:** No, no, the one that just came in.

**Crystal Day-Hess:** Oh, the one that just—

**Douglas H. Clements:** Doing that first. Tony.

**Crystal Day-Hess:** Oh, sometimes we think we're talking—sorry about that—so it's just emphasizing the importance of being intentional with our math talk. Sometimes we think of it as just being small and insignificant.

**Douglas H. Clements:** Ah, yeah.

**Crystal Day-Hess:** It really does have a big impact.

**Douglas H. Clements:** It's got a big impact. Right. And once again, I think we've said it a lot of times, but I'll say it once more. Its impact far exceeds what we would have thought. That okay, we've prepared for the next grade math. We're giving them executive function like Crystal just said. And Crystal, it's not just cognitive flexibility either, right? There's other executive functions. From your research on this, because you're doing a lot of research on this right now, can you give us an example of another executive function and how a math activity really exercised it?

**Crystal Day-Hess:** Yes, maybe in the math board example that you gave us, Doug, we could think about how working memory comes into play there? Kids—you need to remember, okay, I'm supposed to be representing the number three. So holding that information in mind and

thinking about the other—maybe we’re asking them to find multiple examples of how do you make three. And with the example you gave with showing it on their fingers. Three, as you said, may not be the best example, because there’s so few options. You have to remember, okay, last time, I showed three and zero on my hands. Now, let’s see, I could do something different, because I already did that one. Now I’m going to do two and one.

**Douglas H. Clements:** Yeah, and you’ve got to keep all those in line going forward.

**Crystal Day-Hess:** Exactly. So we have a question from Sherry Lynn. She was wondering if you can clarify the difference between cardinality and subitizing?

**Douglas H. Clements:** Oh, Sherry Lynn, that’s a great question. And I’m sorry I was not distinct about those two. So subitizing, from the Latin word that means suddenly—so it’s quick recognition of small sets. And so if you can see two—and two dogs walk in the room and the kid says two doggies. Or three bananas—how many do you want? Two cookies. Two cookies. That’s a quick recognition of the set without counting.

Subitizing is an alternative to counting. Both are important. Both are powerful. In which you don’t name one, two, three, four, like you do in counting, where you do one to one correspondence between each counting word in turn, but you just see a group and say it. So I hold up all five fingers and you all, if you could see me, would see fives. You wouldn’t go one, two, three, four, five. You’d just see the five. And you’d say that’s five, I recognize that right away.

How is that different from cardinality? The cool thing about subitizing is cardinality is built into it. When you see three, you know there’s three there. But when kids count, for years kids will verbally count and then they’ll learn to count with one to one correspondence, keeping one number counting word name for each object. But still not know when they get to the last object that that says how many in the group. Subitizing—you always know how many in the group, because that’s what you’re doing. You’re naming the number in the group.

So subitizing has cardinality built in. Counting you have to learn the cardinality principle. So what does that mean? It means the last counting word tells how many in the group. If kids can subitize sets up to five and then they learn to count up to five, it’s easier for them to understand that cardinality principle, because when they get to the end they step back and say, “Yeah, I just said five and I know that. That’s five. I know how to recognize that.”

So now when they’re counting 10, or 20, or 25 objects, they can apply the same principle. Where you can’t subitize 25 very easily on ordered collections. Most people can’t at all quickly recognize that. But you can count it and you can know exactly how many. And you know that cardinality idea. So the cardinality is separate. That’s just the idea of how manyness. Subitizing has cardinality built in. Counting they have to learn the cardinality principle of counting, which is the last number means how many in that set.

I bet I beat that one to death, but I hope that answered your question. Other questions? Or Crystal, are there other remarks you want to feature? Well, let's finish. We only have a little bit to go. Let's turn to the next slide and do that. The next slide just returns to Recommendation 5. And I think we only have a few minutes left.

Recommendation 5 says, "Dedicate time each day to teaching math, and integrate math instruction throughout the school day." So we talked about this early. I want to emphasize everything we talked about with math talk can be a part of every one of these contexts. Language is essential to every one of these math topics, whether you have a quick planned daily instruction.

And listen, people for you—people in kindergarten or preschool, threes and fours, for instance—this daily instruction does not—when I picture it, I'm not picturing kids in rows with worksheets. I'm picturing kids marching around the classroom, stamping, saying one, two, three, four, five, six; doing patterns in how they say the numbers; doing verbal counting; and doing one to one correspondence, because every time I stamp the ground I'm saying one more thing. So it can be as simple as that. But that's still daily intentional instruction.

Embedding math in classroom routines and activities. Everybody gets two celery sticks today. Highlighting math within topics of study across the curriculum. I gave you the example of the book—ku-plink, ku-plank, ku-plunk book. But anywhere—you're doing social studies work, you're doing social emotional work or something—there's very often either reasoning topics, patterning topics, measurement topics or number topics that occur. Math-rich environment, games, manipulatives. Hey, building blocks.

One of the things we didn't talk about today is we—we do encounter still some teachers—I don't think probably most of you on this call—but anybody who has this opinion is absolutely welcome to type it in the chat box and disagree with me if you like. That I think there's an unfortunate tendency for some teachers to say, "I just don't want to—I don't think it's right to teach math to really young kids. I think they should be playing." And I often ask them, well, playing with what? And they say, well, like building with blocks. Teachers College and the like has made that as an essential thing for good play for young kids. And I agree 100 percent.

What I would like to add to that, though, is, "Where did building blocks come from?" They came from Friedrich Froebel 100 years ago, and Patty Smith and others at the beginning of the previous century, who designed the building blocks or kindergarten blocks—some people call them—used in our classrooms to do what? Teach math. That's why they're there. They're there to teach math through play. And that's exactly what I like to tell people is this kind of math-rich environment doesn't mean kids stop playing and do math. They learn math while they're playing.

Think of the blocks. What do you call that rectangular block that's as long as two square blocks, two square prisms? You call it a "unit block." That's the unit. Cut it in half, it's two squares. Cut it in half the other way, it's two wraps or two triangular prisms. Double it, it's the larger block.

Double it again, it's the longest block in most of the block sets. Put those two triangles back together, you can make a shape exactly like the unit block. Shape composition—the putting together things. Geometry that's there—doubling—hey, who was doubling? I forgot. Brian, I think. Was Brian the doubling story? I think so.

**Crystal Day-Hess:** Brian had sent some doubling and Christy had some as well.

**Douglas H. Clements:** Oh, Christy. Yeah, yeah, I thought there was another doubling thing up there. Christy, playing 2,048 or 2,046 with her son.

**Crystal Day-Hess:** Playing an app, yeah, with her 6-year-old.

**Douglas H. Clements:** Yeah. And said, “What number happened to numbers when they're combined?” “They doubled.” “What does that mean?” “It means a number plus itself. Like five plus five equals 10.” Well, if kids see that these two blocks are the same as this long block, if they see that these two triangles you put them together and they make this bigger rectangle if you put them this way and a triangle if you put them that way—another cognitive flexibility exercise, right Crystal?

This is doubling. This is adding a shape to itself. It builds a mental spatial geometric model for kids that becomes the foundations of all later mathematical thinking. That's a math-rich environment. It's a playful environment, but it's a math-rich environment filled with the language of mathematics.

So the last couple of minutes we've got—any more questions for you. If you have a story to tell, that's okay too. But we only have four more minutes left and maybe a little less than that, because we've got to say thank you. Right, Crystal?

**Crystal Day-Hess:** Yes, I think David has some things to add as well.

**Douglas H. Clements:** Okay, David, you just chat to us as soon as you want us to stop taking questions. Does anybody else have another last minute question for us? Crystal will warn you, if you don't ask a question pretty soon, I'll just keep talking.

**Crystal Day-Hess:** It's true.

[LAUGHTER]

**Douglas H. Clements:** Thanks a lot, Crystal.

**Crystal Day-Hess:** So Patty is asking, “Are there videos that show what we talked about today?”

**Douglas H. Clements:** Could you type in learning directories to her?

**Crystal Day-Hess:** Absolutely.

**Douglas H. Clements:** We have the best website for you guys, ever, ever, ever. Crystal just typed it in to the chat box. It's LearningTrajectories.org. She's typing it again. There we go. There we go. Just click on it, Crystal. Does that come up for you just fine? Just to make sure we got it right. Yep.

**Crystal Day-Hess:** Yes, it does.

**Douglas H. Clements:** Yeah, we got it right. People, please, please, we invite you to go to this free website. You can sign on to the website or just—just view the videos on the home page of it without signing on, if you want to just get an idea about it. But all you have to do is sign on. Signing on is free. We will never ask you for money. It—it—a lot of the things we talked about today—we have videos about it. The learning trajectories, which is the main point of the site, are nicely laid out with videos that both show you kids thinking—that's the developmental progression—and instruction.

So you can always write Crystal and me for more information about the website and the like. My email is Douglas.Clements@du.edu. And so I'd be your best contact for that. Feel free if you have other questions. Crystal's typing that in right now for you. About LT squared or about anything else. But it's a free website that has a lot of this information and a lot of videos that are of a general use, as well as learning trajectories videos. So please, that's a great resource for you right there. David, I think you'd better take it away or I'm going to fill the last 30 seconds I gave you.

**David Yanoski:** All right, well, thank you everybody for joining us today. We really appreciate Doug and Crystal, and the information that they brought us. And REL Central really has enjoyed working with these two. And actually, just a hint, look forward to a couple of other projects and products from REL Central with Doug and Crystal coming up in the next year or so.

Just two quick things. First, we will be sending out a—what's the word I'm looking for, sorry—a survey about this particular webinar in the next couple of days. We would really appreciate your feedback and any comments or suggestions. So we will send that out to all people who have been registered and we'd really appreciate you getting back to us and completing that survey.

Also, within the next two to three weeks, the slides from this webinar will be available on the REL Central website. Just search for REL Central. And in addition, there will be within the next month or so, we'll be releasing a video of the entire website. We have a lot of things to do to make it perfect. But again, thank you very much. And we look forward to presenting more webinars for you. Thanks, everybody. And have a wonderful day.

**Crystal Day-Hess:** Thank you.

**Douglas H. Clements:** Thanks, everybody. Thanks for sticking in with me.



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