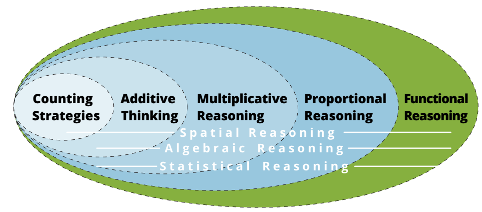
IES Institute of Education Sciences

REL SOUTHEAST
Regional Educational Laboratory
At Florida State University

Functional Reasoning

Part of the Development of Mathematical Reasoning

What is Functional Reasoning?



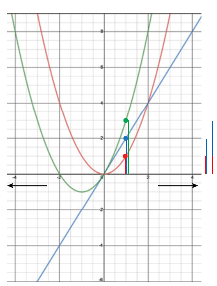
The Development of Mathematical Reasoning

Functional Reasoning builds on Counting Strategies, Additive Thinking, Multiplicative Reasoning and Proportional Reasoning as students are developing Mathematical Reasoning. Functional Reasoning, also called Relational Reasoning or Bivariate Covariation, means that students consider the effect of the rate (which is a ratio - proportional reasoning) on the parent function, which is (often) a set of infinite points that follow a rule (which often contains additive and multiplicative relationships). Because they are considering 2 dimensional points, this means that the student is also considering the effect the domain has on the range as it interacts with the rule. Many things are happening simultaneously and the learner has to focus on parts in the short run while also considering the whole in the long run. Functional Reasoning is important in the study of changing rates of change in calculus, statistical modeling, and scientific applications of formulas.

What does Functional Reasoning look like?

Simultaneity

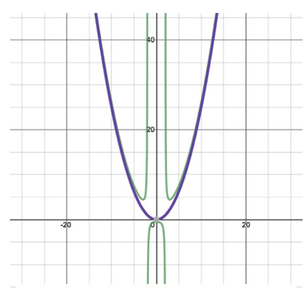
x	x^2	$2x$	$x^2 + 2x$
-	+	-	+
-2	4	-4	0
-1	1	-2	-1
0	0	0	0
1	1	2	3
2	4	4	8
+	+	+	+



Dealing with more simultaneity, considering

- the representation as both a process and an object
- both individual points and the overall pattern
- connections between multiple representations
- the range depends on the domain

Short & Long Run Behavior

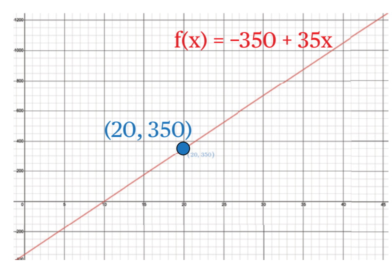


$$r(x) = \frac{0.25x^4 + 2}{x^2 - 4} \approx 0.25x^2$$

Considering both short and long run behavior

- What's happening near x = 0 (short run), zoomed-in look
- How is the function behaving as x gets really large (to the right) or as x gets really small (to the left), zoomed-out look?

Function versus Equation



A "function" approach to teaching algebra includes considering simultaneously

- the funtion (or relation) represents the whole scenario
- Invest $350 into a lawn mower, make $35 per lawn mowed
- An equation, 350 = -350 + 35 (20), represents one point on the graph (20, 350), when you've mowed 20 lawns, you have made $350.

What does Functional Reasoning look like in practice?

Functional Reasoning is multi-faceted, with many things to consider simultaneously, like the interaction between object and process, between short and long run behavior and between function and equation. When we look at the videos of students solving complex problems about functions, we can find evidence of aspects of Functional Reasoning, but one problem is rarely enough to gain a full view of a student's reasoning. Below are four videos that demonstrate students' thinking.

Video 1
Example of Functional Reasoning: Abby

Video 2
Non-example of Functional Reasoning: Noah

Video 3
Example of Functional Reasoning: Grant

Video 4
Non-example of Functional Reasoning: Lusia