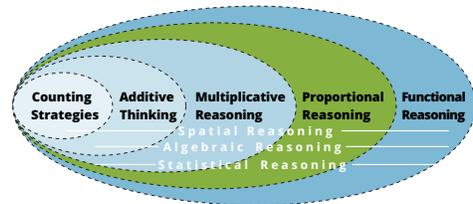## Proportional Reasoning
Part of the Development of Mathematical Reasoning

What is Proportional Reasoning?

Proportional Reasoning builds on Multiplicative Thinking. Students begin to consider the relationship of two linked quantities and how they vary together and scale in tandem or inversely. This reasoning, like multiplicative reasoning, requires students to consider many quantities simultaneously. Ideally students will begin to learn proportional reasoning as they understand and build knowledge of equivalent ratios. This type of reasoning begins to develop in earnest in Grade 6 and continues to grow throughout middle and high school as understanding proportional relationships permeates math and science standards. Proportional Reasoning is important when working with fractions, decimals, percentages, similar figures, trigonometry, transformations and is also critical in the sciences, such as physics and chemistry.
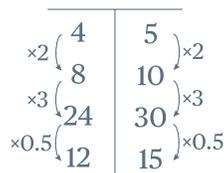

The Development of Mathematical Reasoning

What does Proportional Reasoning look like?

Two quantities varying together	Scaling in tandem	Equivalent ratios

**Two quantities varying together**

4 : 5

8 : 10

12 : 15
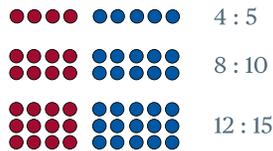
Dealing with more simultaneity
- thinking about two quantities simultaneously changing together,
- it's not about counting the red dots and the blue does one by one, but about perceiving the groups of red dots in relation to the groups of blue dots

Scaling in tandem

$$\times 2 \begin{pmatrix} 4 \\ 8 \end{pmatrix} \times 2$$
$$\times 3 \begin{pmatrix} 24 \end{pmatrix} \times 3$$
$$\times 0.5 \begin{pmatrix} 12 \end{pmatrix} \times 0.5$$

4	5
8	10
24	30
12	15

Scaling in tandem produces equivalent ratios.
- The ratio of y to x is constant, $y{:}x = 5{:}4$
- In this case, $5{:}4 = 10{:}8 = 30{:}24 = 15{:}12$

Equivalent ratios
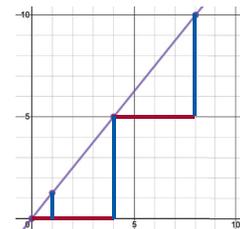
A proportional relation:
- contains the origin
- a constant rate of change $\Leftrightarrow$ linear function
- $y = 1.25\, x$ and $y = 5/4\, x$

What does Proportional Reasoning look like in practice?

Below are four videos that demonstrate students' thinking.

Video 1: Not Proportional Reasoning

Video 2: Not Proportional Reasoning

Video 3: Proportional Reasoning

Video 4: Proportional Reasoning