

Preparing All Students for Algebra: Applying Research-Based Strategies

Student Success in Mathematics Partnership, Virginia Partnership Divisions: Charlottesville, Harrisonburg, Staunton, Waynesboro, and Winchester

January 24, 2018



Welcome

Agenda

- 8:00 a.m. Welcome, introductions, workshop goals and objectives
- 8:10 a.m. Importance of algebra for student success
- 8:30 a.m. Teaching strategies to support algebra success for all
- 10:00 a.m. Break
- 10:15 a.m. Visual representations to support proportional reasoning
- 11:15 a.m. Next steps and Closing
- 12:00 p.m. Adjourn

Regional Educational Laboratory (REL) Program



- Serves the needs of 10 designated regions, helping them improve education through evidence-based practice.
- Administered by the U.S. Department of Education, Institute of Education Sciences (IES).

Working with the REL Program

- Sustain **partnerships** that use research to address high-leverage issues.
- Complete coherent and cumulative **research agendas**.
- Use REL AP as **key resource** for credible research and support.
- **Increase capacity** to access, understand, interpret, apply, and conduct research.
- **Increase use** of research findings in education decisionmaking.



Partnership Members

Five small city school divisions in Virginia:

1. Charlottesville
2. Harrisonburg
3. Staunton
4. Waynesboro
5. Winchester



Presenters and Student Success in Mathematics Partnership team



Pam Buffington
Partnership Lead
PBuffington@edc.org



Ryoko Yamaguchi
Research Lead
RYamaguchi@plusalpharesearch.com



Laurie McCullough
VASCD
vascded@gmail.com



Jill DePiper
Partnership Member
JDePiper@edc.org

Introductions

Please share your:

- Name.
- Affiliation.
- Role.





FRAME YOUR CHALLENGE

Use the index card provided to describe the challenge or interest that brought you to this workshop.

What do you hope to **learn**, **accomplish**, or **improve** through your participation?

Goals and Objectives



Workshop objectives

- Increase knowledge of research related to algebra I completion and student success.
- Build awareness of the role of ratio and proportion skills and concepts for algebra readiness.
- Increase understanding of how visual representations and language access and production strategies can support students' algebraic problem solving.

The Importance of Algebra for Student Success

WHAT DOES THE RESEARCH SAY?

Exploring the research

Prepare to work in pairs.

Pairs receive collection of quotations citing research.

- Relationship of algebra I and future student success.
- Role of ratio and proportion in the development of algebra readiness.
- Use of visual representations as a support strategy for solving pre-algebra and algebra word problems.

Review and sort by category.

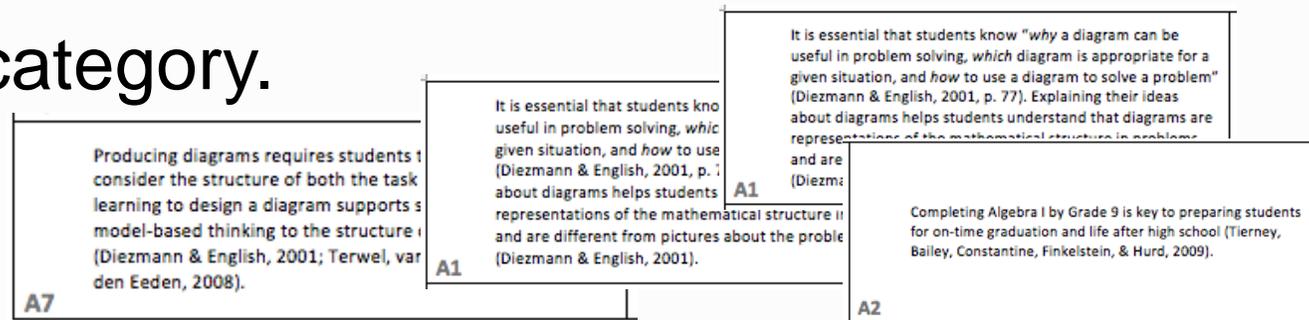


Table discussion

What were the big ideas present in the research quotes?

- Relationship of algebra I with student success.
- Use of visual representations as a strategy in pre-algebra and algebra.
- Role of ratio and proportion in the development of algebra readiness.

Reflect on quotations.

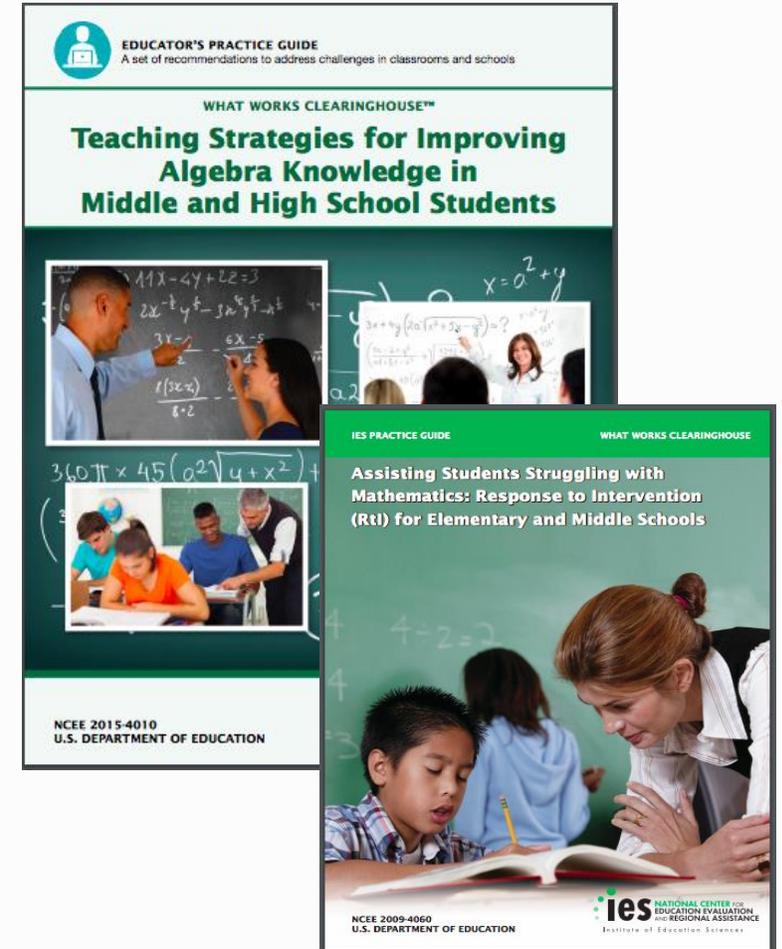
- Choose a quote. Why did you choose that one? Which would you want to explore more fully?

Report out.

Strategies to support algebra readiness and success

IES Practice Guides

- *Teaching strategies for improving algebra knowledge in middle and high school students* (Star et al., 2015).
Focus on worked examples, structure of algebra, alternative algebraic strategies.
- *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools* Practice Guide (Gertsen et al., 2009).
Focus on rational numbers, visual representations, worked examples.
- *Teaching academic content and literacy to English learners in elementary and middle school* Practice Guide (Baker et al., 2014).
Integrate oral and written English language instruction into content-area teaching, pairs and small group talk.
- *Improving mathematical problem-solving in grades 4 through 8.* Practice Guide (Woodward et al., 2012).
Teach students to use visual representations, understand the problem context or language.

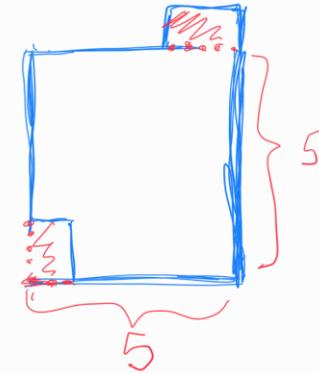


Teaching Strategies to Support Algebra Readiness and Success

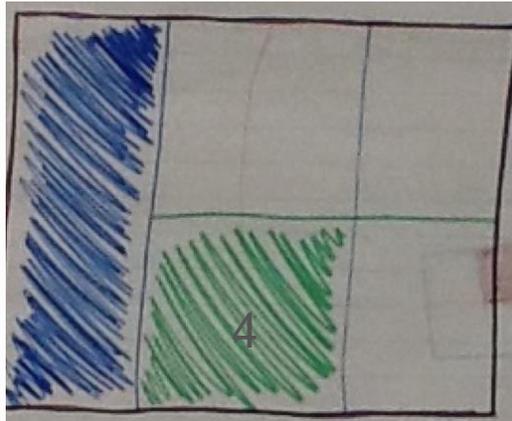
A PRIMER ON MATHEMATICAL VISUAL REPRESENTATIONS

What are mathematical visual representations?

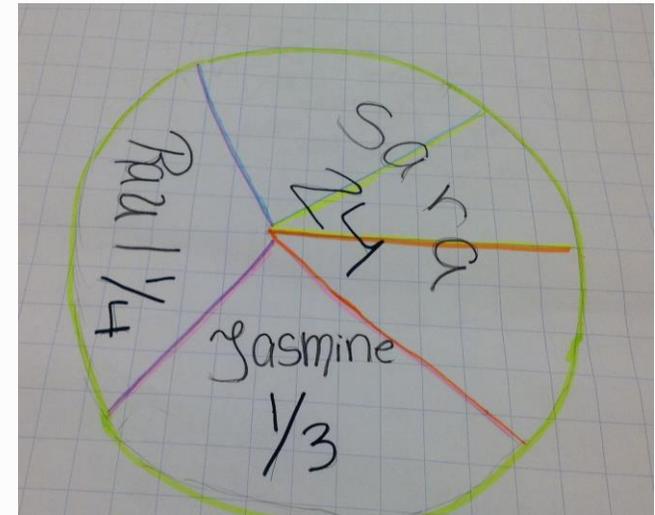
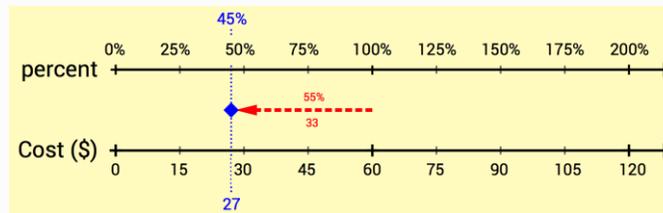
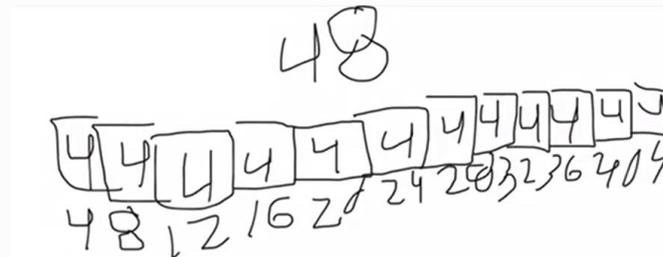
- **Drawing** (e.g., enhancing figures in geometry tasks)
- **Diagramming** (e.g., tape diagrams, number lines, double number lines, area models related to word problems and other quantitative tasks)



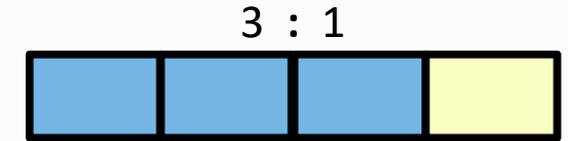
$$\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$



8



Why use mathematical visual representations?



Competent mathematical thinkers use mathematical visual representations flexibly in problem solving. (*Stylianou, 2002; Stylianou & Silver, 2004*)

Mathematical visual representations can reinforce students' conceptual understanding of rational number. (*Gersten et al., 2009; Siegler et al., 2010*)

Understanding how to select the representations most appropriate for solving a task from a variety of visual representations provides more access to solve the task. *IES Practice Guide on Problem Solving (Woodward et al., 2012)*

<p>Producing diagrams requires students to consider the structure of both the task and the learning to design a diagram supports student model-based thinking to the structure of (Diezmann & English, 2001; Terwel, van den Eeden, 2008).</p> <p>A7</p>	<p>It is essential that students know "why a diagram can be useful in problem solving, which diagram is appropriate for a given situation, and how to use a diagram to solve a problem" (Diezmann & English, 2001, p. 77). Explaining their ideas about diagrams helps students understand that diagrams are representations of the mathematical structure in problems, and are different from pictures about the problem context (Diezmann & English, 2001).</p> <p>A1</p>
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Visual representations support English learners, low-literacy learners, & struggling learners in particular

- **Provide an intermediate step between the text and symbolic** — revealing the mathematical structure of a problem. (*Baker et al., 2014; Ng & Lee, 2009*)
- **Support task exploration** and engagement with the mathematics *at the same time* as language development and communication.
- **Serve as artifacts** that can facilitate strategy sharing and negotiation of new ideas.



Sharing jelly beans

- Explore visual representations (VRs) together while engaging with a mathematical task.
- Experience language strategies in the context of problem-solving.



Sharing jelly beans: Three reads

1. What is the problem about?
2. What do you need to find out?
3. What important information is given?



Sharing jelly beans: Three reads



Hector had a bag of jelly beans.

He gave $\frac{1}{4}$ of the jelly beans to Susan.

Then Hector gave $\frac{1}{6}$ of the jelly beans he had left to Pepita.

After giving jelly beans to Susan and Pepita, Hector had 20 jelly beans left in his bag.

How many jelly beans did Hector have at the beginning?

1. What is the problem about?

Sharing jelly beans: Three reads



Hector had a bag of jelly beans.

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After giving jelly beans to Susan and Pepita, Hector had 20 jelly beans left in his bag.

How many jelly beans did Hector have at the beginning?

2. What do you need to find out?

Sharing jelly beans: Three reads



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3. What important information is given?

Sharing jelly beans: Three reads



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After giving jelly beans to Susan and Pepita, Hector had 20 jelly beans left in his bag.

How many jelly beans did Hector have at the beginning?

Work individually to create a diagram that represents the important information.

Sharing jelly beans: Diagram



In pairs:

- Share the diagrams you started.
- If you finish discussing your diagrams:
 - Create additional diagrams that could help solve the problem.
 - Discuss what other questions you could answer using your diagrams.

- *I represented the candies Pepita had by...*
- *I represented the candies Hector had left by...*
- *I see a relationship between... and ... in the diagram.*

Sharing jelly beans: Diagram



Hector had a bag of jelly beans.

He gave $\frac{1}{4}$ of the jelly beans to Susan.

Then Hector gave $\frac{1}{6}$ of the jelly beans he had left to Pepita.

After giving jelly beans to Susan and Pepita, Hector had 20 jelly beans left in his bag.

How many jelly beans did Hector have at the beginning?

This diagram represents the candies [Hector/Susan/Pepita] had by...

I see a relationship between... and ... in this diagram.

Full group sharing – **Document Camera**

Sharing jelly beans experience: Debriefing



- In what ways did or could the sharing jelly beans experience support students **to understand and to use mathematical language**?
- How can visual representations **support students' problem-solving** and *mathematical structure* in the mathematics classroom?
 - What do you notice about how visual representations can support students who are **English learners specifically**?

Language access and production strategies

Strategy name	Language access	Language production
<u>3 Reads</u>	✓	
<u>Acting Out and Realia</u>	✓	
<u>Clarifying Vocabulary I</u>	✓	
<u>Differentiated Teacher Questions</u>		✓
<u>Framer Model</u>	✓	
<u>Teacher Revoicing</u>		✓
<u>Sentence Starters and Frames</u>		✓
<u>Co-Constructed Word Bank</u>		✓
Pairs Work		✓
<u>Clarifying Vocabulary II:</u> <u>Gesturing, Action Words, and Color</u>	✓	

(Driscoll, Nikula, DePiper, 2016)

Break



Teaching Strategies to Support Algebra Readiness and Success

GOING DEEPER INTO MATHEMATICAL VISUAL REPRESENTATIONS:
PROPORTIONAL REASONING AND RATIOS

Comparing driving



Tara and Sam's combined driving distance this week was 60 miles.

Sam drove 4 times as far as Tara.

How many miles did Tara drive?

- Use **2 different methods** to find the answer and show your thinking.
- Use a diagram for at least 1 method.

Sharing comparing driving: Diagram

In pairs:

- Share the diagrams you started.
- If you finish discussing your diagrams:
 - Create additional diagrams that could help solve the problem.
 - Discuss what other questions you could answer using your diagrams.



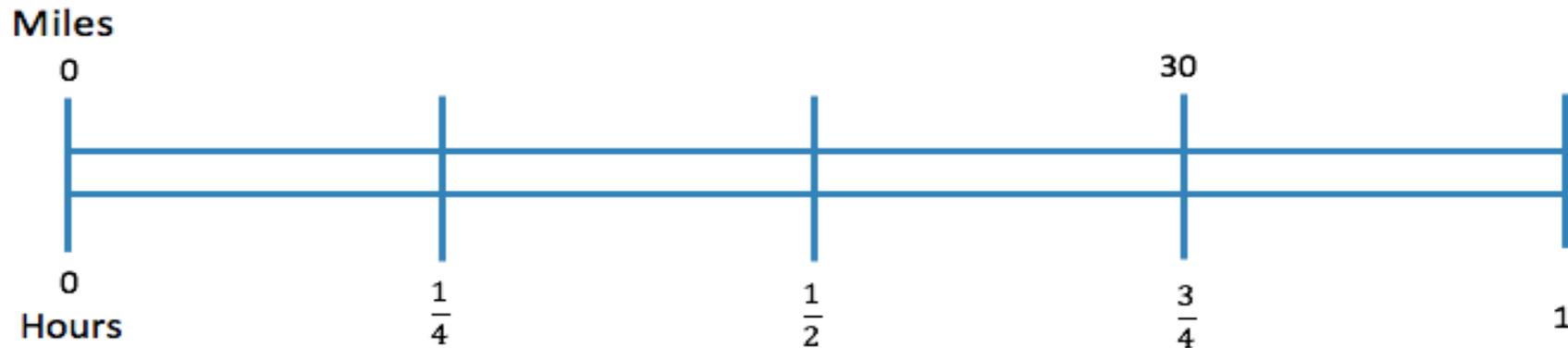
Exploring double number lines

Sam's Motorcycle



Sam bought a used motorcycle. It was on sale because it could not go very fast. Sam was able to go 30 miles in $\frac{3}{4}$ of an hour.

- a) How far can he go in 1 hour? Use a double number line to help solve this problem. Explain your solution.



Exploring double number lines (cont.)

Sam's Motorcycle



b) How far can he go in $3\frac{1}{2}$ hours?

Use a double number line to help solve this problem.
Explain your solution.

Miles



Hours

Task: Covering tables

Paula has fabric that is 4 yards long. She needs a length $\frac{3}{4}$ of a yard of cloth to cover 1 table.

She needs to find out how many tables of the same size she can cover with 4 yards of cloth.

Show 2 different visual representations that can be used to help answer the problem.



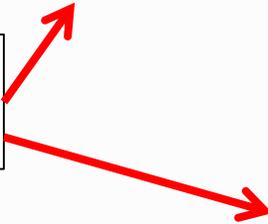
Word Wall, Realia, or Co-Constructed Word Bank

Cover - to spread over



Furniture with flat top and 1 or more legs

Table

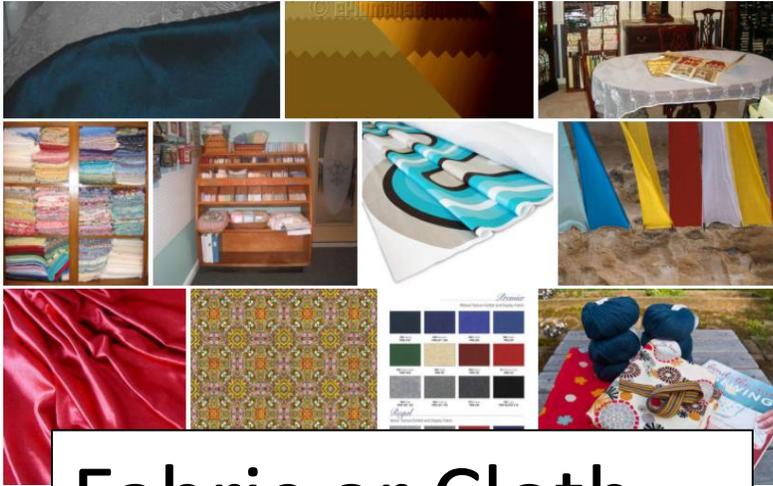


Fact or figures displayed in columns

x	y
1	2
2	4
3	6

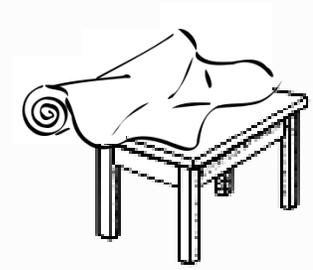
Yard

- Standard unit for measuring length
- Equal to 3 feet or 36"



Fabric or Cloth

Task: Covering tables



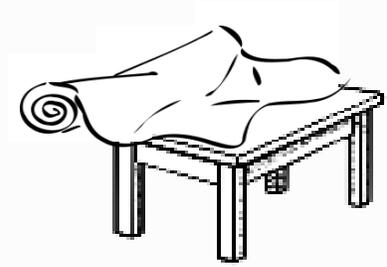
Review task Individually.

- Create 2 representations to assist in solving the problem.
- Pair up.
- Discuss the representations with your partner.

Mutually determine...

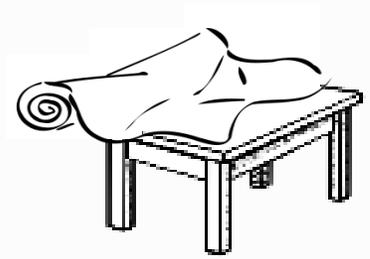
- How many tables can Paula cover with 4 yards of cloth using $\frac{3}{4}$ of a yard of cloth on each?
- How many yards of cloth will be left over after using $\frac{3}{4}$ of a yard of cloth for each table?

Task: Covering Tables



Share representations using the document camera.

Task: Covering Tables



Connecting to symbolic representation

- Review how Paula worked out the problem.
- How does Paula's work relate to your diagrams?

$$\begin{aligned} &4 \text{ divided by } \frac{3}{4} \\ &4 \div \frac{3}{4} \\ &\frac{4}{1} \times \frac{4}{3} = \frac{16}{3} \\ &\begin{array}{r} 5 \\ 3 \overline{)16} \\ \underline{15} \\ 1 \end{array} \\ &5 \frac{1}{3} \end{aligned}$$

Task: Covering tables

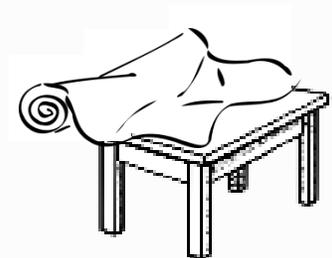
Connecting VR, symbolic, ratio tables

e) Fill in the ratio tables.

Yards of Cloth	Tables
1	
2	
3	
4	

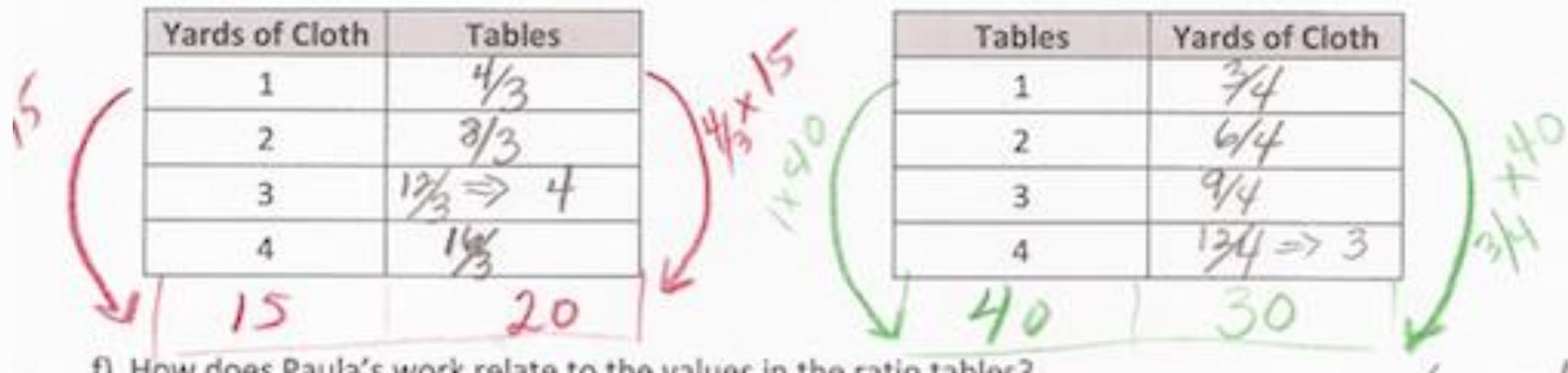
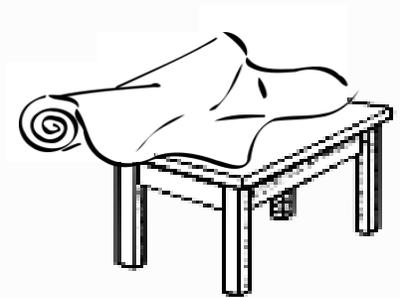
Tables	Yards of Cloth
1	
2	
3	
4	

f) How does Paula's work relate to the values in the ratio tables?



Task: Covering tables

Identifying & using unit rates



$$15 \text{ yards} \times \frac{4}{3} \text{ tables / yard}$$

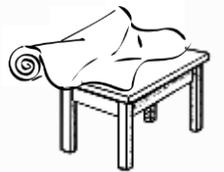
$$\frac{15 \times 4}{3} = 20 \text{ tables}$$

$$40 \text{ tables} \times \frac{3}{4} \text{ yard / table}$$

$$\frac{40 \times 3}{4} = 30 \text{ yards}$$

Covering tables experience: Debriefing

- In what ways did or could the covering tables experience support students **to understand and to use language** in the context of the mathematics classroom?
- How can visual representations **support mathematics content and practices?**
- How can the use of these embedded research-based strategies broaden access to algebra for all students?



Wrap-up and Next Steps

Reflecting back forward ↔ planning

With your team:

- **Revisit** the challenge or goal you wrote on an index card earlier this morning.
- Based on this morning's session, would you **revise** what you wrote earlier?
- What **ideas** or **take-aways** from this session apply to your challenge or goal?

Individual planning (Make notes on the back of your card)

In your own context:

- **What** strategies will you integrate into your own practice?
- **How** will you ensure that you follow through?
- **Who** among your colleagues can you share strategies/information with, and how?

And back to your team.....

- What steps can you take as a team to **share** and **apply evidence-based strategies** with teachers of students in grades 4–6 in partnership schools?
- What **training, resources, or support** would help teachers integrate these strategies into practice?

How did we do?

- What have you learned? What will you do next?
- Did we answer all your questions?
- Do you have other burning questions or concerns?
- Please complete the stakeholder feedback survey.



Thank you for your participation!

Pam Buffington,
Ryoko Yamaguchi, and
Laurie McCullough
REL Appalachia

Regional Educational Laboratory Appalachia

RESOURCES & SUPPORTS FOR TEACHERS

Ask-A-REL

Ask-A-REL is a collaborative reference desk service provided by the 10 regional educational laboratories (REL) that, by design, functions much in the same way as a technical reference library. It provides references, referrals, and brief responses in the form of citations on research based education questions.

<https://ies.ed.gov/ncee/edlabs/askarel/>

Example:

What do we know from research about the impact of online algebra I courses on student achievement?

Ask-A-REL

Ask A REL Instructions

To ask an education-focused question, please complete the question submission form below:

1. Include your name and email address
2. Select your state from the drop-down menu
3. Type your question in the box
4. To receive a copy of your question, check the box "I would like to receive a copy of my question sent to my e-mail."

Note: The questions you submit are sent directly to the REL selected and not stored on this site or by the Institute of Education Sciences. To ask a question or to provide a comment about the Regional Educational Laboratory Program or the Institute of Education Sciences, select the "Contact" button at the top of this page.

* Full Name:

* Email:

* Confirm Email:

* State:

* Question

I would like to receive a copy of my question sent to my email.



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September 2017 - In This Issue:

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- PROJECT UPDATES
- STAFF HIGHLIGHTS
- COMING SOON

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MESSAGE FROM DIRECTOR

As a resident and parent in the REL Appalachia (REL AP) region, I have the honor to serve the amazing and diverse REL AP communities throughout Kentucky, Tennessee, Virginia, and West Virginia. As the Director of REL AP, I am thrilled to be leading an incredibly talented team of researchers, technical assistance providers, and communications experts who work in partnership with equally talented leaders in school districts, state departments of education, and elsewhere to carry out and use research to improve student academic outcomes in the region.

