Handout 2: Algebra and Student Success Card Sort

It is essential that students know "*why* a diagram can be useful in problem solving, *which* diagram is appropriate for a given situation, and *how* to use a diagram to solve a problem" (Diezmann & English, 2001, p. 77). Explaining their ideas about diagrams helps students understand that diagrams are representations of the mathematical structure in problems and are different from pictures about the problem context (Diezmann & English, 2001). **A1**	Completing algebra I by grade 9 is key to preparing students for on-time graduation and life after high school (Tierney, Bailey, Constantine, Finkelstein, & Hurd, 2009). **A2**
Mastering proportional thinking and reasoning in upper elementary and middle grades can lead to students acquiring pre-algebra and algebra concepts later in their schooling (Empson, Levi, & Carpenter, 2011; Siegler et al., 2010). **A3**	Algebra knowledge and skills are important for future postsecondary success and creating a skilled workforce for scientific and technical careers (Katz, 2007). **A4**
"Visual representations help students solve problems by linking the relationships between quantities in the problem with the mathematical operations needed to solve the problem. Students who learn to visually represent the mathematical information in problems prior to writing an equation are more effective at problem-solving" (Woodward et al., 2012). **A5**	Proportionality has been called the "cornerstone of higher mathematics and the capstone of elementary concepts" (Lesh, Post, & Behr, 1988, p. 98.). **A6**

Producing diagrams requires students to reflect on and consider the structure of both the task and the diagram, and learning to design a diagram supports students in bringing model-based thinking to the structure of mathematical tasks (Diezmann & English, 2001; Terwel, van Oers, van Dijk, & van den Eeden, 2008). **A7**	Proportional thinking and reasoning, a topic covered heavily in middle grades mathematics, is foundational to students' algebraic understandings and therefore critical to algebra I course readiness (Siegler et al., 2012). **A8**
Reviews of research, however, have found unintended consequences when students are mandated to take algebra I (Stein, Kaufman, Sherman, & Hillen, 2011). Success is more than simply taking algebra I by grade 9. Students must be *ready* for the content in order to succeed (Allensworth, Nomi, Montgomery, & Lee, 2009; Stein, Kaufman, Sherman, & Hillen, 2011). **A9**	The mixed results of universal algebra I emphasize the importance of examining policies and instruction when seeking to change course-taking pathways. Mathematics learning is much more nuanced than simply taking the algebra I course and assessment before high school (Allensworth, Nomi, Montgomery, & Lee, 2009; Gamoran & Hannigan, 2000; Loveless, 2008; Stein, Kaufman, Sherman, & Hillen, 2011). **A10**
Flexible use of visual representations is part of a competent mathematical thinker's problem-solving repertoire (Stylianou, 2002; Stylianou & Silver, 2004). **A11**	While middle grades mathematics is not always organized around a theme, proportionality is an overarching concept in middle grades mathematics and "one that unites, relates, and clarifies many important middle grades topics" (Lanius & Williams, 2003, p. 392). **A12**

Course-taking pathways may force students to continue in a progression that limits opportunities to learn mathematics in the current course as well as in later higher level mathematics (Oakes, 1990). **A13**	To advance algebraic thinking and problem-solving skills, it is important to study a variety of VRs [visual representations] and understand how to select the representations most appropriate for solving a task. For example, tape diagrams can be useful for solving rational number tasks focused on equipartitioning (Siegler et al., 2010, while number lines "clearly illustrate the magnitude of fractions; the relation between whole numbers and fractions; and the relations among fractions, decimals, and percents," as well as equivalent and nonequivalent fractions (Siegler et al., 2010, p. 20). **A14**
Understanding proportional reasoning is a prerequisite to higher-level mathematics (such as in-depth studies of algebra, probability, geometry, and measurement) because relationships between quantities are key to functions and variation. Given that proportionality is foundational to higher mathematics, having specific mathematics coursework in this area may be key to supporting students' algebraic understanding (Lesh, Post, & Behr, 1988). **A15**	Algebra is a demonstrable gateway to later achievement. Students need it for any form of higher mathematics later in high school; moreover, research shows that completion of algebra II correlates significantly with success in college and earnings from employment. In fact, students who complete algebra II are more than twice as likely to graduate from college than students with less mathematical preparation (National Mathematics Advisory Panel, 2008). **A16**

A17 "Understanding algebra is a key for success in future mathematics courses, including geometry and calculus. Many mathematics experts also consider algebra knowledge and skills important for postsecondary success as well as for producing a skilled workforce for scientific and technical careers. Algebra requires proficiency with multiple representations, including symbols, equations, and graphs, as well as the ability to reason logically, both of which play crucial roles in advanced mathematics courses" (Star et al., 2015, p. 1).	**A18** "Proportional *reasoning* refers to detecting, expressing, analyzing, explaining, and providing evidence in support of assertions about proportional relationships" (Lamon, 2007, p. 647). Reasoning proportionally does not mean just using a cross-product approach for solving problems, but includes understanding *when* problems are proportional or not and *what* the situations mean (Lamon, 2007).
A19 "Well-chosen visual representations help students focus on what is central to many mathematical problems: the relationship between quantities" (Woodward, J., Beckmann, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., et al., 2012, p. 8).	**A20** "Both general education students and students with learning disabilities performed better when taught to use visual representations such as identifying and mapping relevant information onto schematic diagrams" (Woodward, J., Beckmann, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., et al., 2012, p. 8).