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Making an Impact

Impact of the Developing Mathematical Ideas professional development program on grade 4 students' and teachers' understanding of fractions

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Key findings

This study assessed the impact of the Developing Mathematical Ideas (DMI) professional development program on grade 4 students' and teachers' understanding of fractions in eight school districts in three states—Florida, Georgia, and South Carolina—during the 2014/15 school year. It found that:

- DMI did not demonstrate any impact on student proficiency in fractions. Students of teachers who participated in DMI performed at almost the same level as students of teachers who did not participate; the difference was not statistically significant.
- The impact of DMI on teachers' knowledge of fractions was inconclusive. Teachers who participated in DMI performed better than teachers who did not participate, but the difference was not statistically significant.

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Summary

Contemporary state math standards emphasize that students must demonstrate an understanding of the mathematical ideas underlying the computations that have typically been the core of the elementary school math curriculum. The standards have put an increased emphasis on the study of fractions in upper elementary grades, which are the years during which students build a strong foundation in fractions concepts. At the same time, the National Mathematics Advisory Panel (2008) cites limited understanding of fractions as the key reason for the high failure rate in algebra courses. And longitudinal data from both the United States and the United Kingdom have demonstrated that knowledge of fractions in the elementary grades plays a powerful role in subsequent success in algebra, the gateway to math achievement in high school (Siegler et al., 2012).

Members of the Regional Educational Laboratory (REL) Southeast Improving Mathematics Instruction Research Alliance saw teachers' lack of in-depth knowledge of fractions concepts as a major challenge in their teaching these concepts to their students. Alliance members indicated that teachers would benefit from a professional development program that focused on building a deep understanding of the mathematical ideas underlying fractions and of how to apply those ideas in the classroom. This large-scale study investigates the effectiveness of such a program to help inform future district and state investments in professional development. Members of the REL Southeast Improving Mathematics Instruction Research Alliance formed a work group and selected Developing Mathematical Ideas (DMI) as the professional development program that seemed best suited to develop in-depth teacher knowledge of fractions and that could be scaled up in a large number of districts simultaneously.

Developed by the Education Development Corporation, DMI is designed to help teachers think through major mathematical ideas and examine and reflect on how their students develop and understand the ideas (Schifter, Bastable, & Russell, 2010a, 2010b). Teachers examine vignettes of classroom teaching and examples of student work from their own classes and from the classes of other participating teachers. Teachers also work on fractions problems designed to promote their own understanding of fractions concepts.

The primary goal of the study was to assess the impact of DMI on grade 4 students' and teachers' understanding of fractions. The study was conducted during the 2014/15 school year using data from 84 schools in eight school districts in three states (Florida, Georgia, and South Carolina). Participants included 4,204 grade 4 students and 264 grade 4 teachers. Nine trained facilitators provided the professional development. The study used a randomized controlled trial, randomly assigning schools to either the treatment condition or the control condition. Teachers in schools in the treatment condition received 24 hours of DMI professional development on fractions during fall 2014. They attended eight three-hour sessions conducted over four days (two three-hour sessions per day; one day per month). In most cases substitute teachers filled in for teachers during these days; in some cases teachers preferred occasional Saturday sessions and were compensated for the additional workday. Teachers in the control condition did not receive DMI professional development but were free to participate in any type of school- or district-provided professional development in math, including fractions. About a third of teachers in the control condition indicated that they had participated in some form of professional development in fractions.

The student outcome measure was the Test for Understanding of Fractions (Instructional Research Group, 2015), which was administered at the end of the school year to assess students' understanding of fractions concepts and their ability to perform computations and word problems. The teacher outcome measure was the Mathematical Knowledge for Teaching: Fractions Scale (Learning Mathematics for Teaching, 2008), which was administered to all teachers at the end of the study to assess their understanding of the mathematical ideas involved in teaching fractions and their knowledge of the typical errors and misconceptions that can develop as students learn this material.

Key findings include:

- DMI did not demonstrate any impact on student proficiency in fractions. Students of teachers who participated in DMI performed at almost the same level as students of teachers who did not participate; the difference was not statistically significant (p = .637).
- The impact of DMI on teachers' knowledge of fractions was inconclusive. Teachers who participated in DMI performed 0.19 standard deviation better than teachers who did not participate, but the difference was not statistically significant (p = .051).

Thus, DMI had nonsignificant impacts on students' proficiency in fractions and their teachers' knowledge of fractions. The finding of no impact on students' math proficiency is common in the research literature on professional development in math (for example, Garet et al., 2011; Garet et al., 2016; Gersten, Taylor, Keys, Rolfhus, & Newman-Gonchar, 2014). The results suggest that professional development that attempts to build teachers' knowledge of the mathematical ideas underlying the K–8 curriculum, though theoretically compelling, does not always lead to improvements in student learning.

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Why this study?

In 2013 members of the Regional Educational Laboratory (REL) Southeast Improving Mathematics Instruction Research Alliance noted that improving student proficiency in fractions was a major priority in the elementary grades. They thought that many teachers lacked in-depth knowledge of the often subtle mathematical ideas involved in teaching fractions and therefore were unable to go beyond a somewhat superficial emphasis on procedures. Alliance members feared that the problem might worsen as states moved to the new, more challenging Common Core State Standards in math.[1]

To address this issue, alliance members worked with REL Southeast to design a study to determine the impact of Developing Mathematical Ideas (DMI), a professional development program aimed at increasing teachers' knowledge of fractions, on grade 4 students and their teachers. Grade 4 was targeted because grade 4 math emphasizes fractions concepts and because the new Common Core State Standards increased the amount of instruction time and the complexity of the content to be covered in grade 4. DMI was selected because it stresses understanding of the mathematical content and seemed feasible for large-scale implementation. The study was conducted in three states—Florida, Georgia, and South Carolina—during the 2014/15 school year.

Longitudinal data have demonstrated that how well students do in fractions at age 10 (typically grade 4) predicts how well they will do in algebra and overall high school math

Improving grade 4 students' proficiency in fractions is a regional priority

In 2013 only 41 percent of grade 4 students in the United States were deemed proficient in math on the National Assessment of Educational Progress. The percentage of students who were proficient was below that rate in four of the six states in the REL Southeast Region (Alabama, Georgia, Mississippi, and South Carolina) and equal to or slightly above in the remaining two states (Florida and North Carolina; National Assessment of Educational Progress, 2013). Performance on the latest National Assessment of Educational Progress appears similar (National Assessment of Educational Progress, 2015). Thus, in most states in the region, grade 4 students have been performing at an inadequate level.

From an extensive review of the research literature and input from research mathematicians involved in math education, the National Mathematics Advisory Panel (2008, p. xix) concluded that "difficulty with fractions (including decimals and percent) is pervasive and is a major obstacle to further progress in math, including algebra." The panel argued that success in fractions is critical for success in algebra and that a thorough understanding of fractions requires students to confront an array of abstractions that they have never experienced before, making fractions, like algebra, "a demonstrable gateway to later achievement" (National Mathematics Advisory Panel, 2008, p. xiii).

Longitudinal data from both the United States and the United Kingdom have demonstrated that how well students do in fractions at age 10 (typically grade 4) predicts how well they will do in algebra and overall high school math (Siegler et al., 2012). Thus, members of the REL Southeast Improving Mathematics Instruction Research Alliance viewed success in learning fractions as crucial for increasing students' success in subsequent algebra courses.[2]

Building teacher capacity to teach fractions is a regional priority

Math standards in REL Southeast Region states explicitly say that understanding fractions and having the ability to articulate the mathematical principles and reasoning that underlie fractions computation are as critical as being adept at solving fractions addition, subtraction, multiplication, and division problems. Central to understanding fractions are concepts such as fractions number sense (a fraction is a number like any other number on a number line), fraction magnitude (1/2 is bigger than 1/4), and fractions equivalence (1/3 is the same as 2/6). Many elementary teachers lack this level of understanding themselves (Ball & Forzani, 2009; Ball, Hill, & Bass, 2005). Thus, the REL Southeast Improving Mathematics Instruction Research Alliance perceived a need for a professional development program that builds knowledge of the key concepts that underlie operations involving fractions.

Yet Ball (2015) and other researchers stress that skillful math teachers need more than content knowledge. They also need specialized knowledge necessary for communicating mathematical ideas to their students (Hill et al., 2008). Such understanding enables teachers to explain and model mathematical ideas and practices, lead math discussions, and elicit and interpret students' rationales for their solutions to a problem (Ball, 2015). These teaching practices require teachers to think and analyze student responses on the spot (Ma, 1999).

Skillful math teachers need more than content knowledge; they also need specialized knowledge necessary for communicating mathematical ideas to their students

Selecting a professional development program on fractions to improve students' and teachers' understanding of fractions

Members of the REL Southeast Improving Mathematics Instruction Research Alliance felt that teachers would benefit from a professional development program that focused on building a deep understanding of the mathematical ideas underlying fractions and of how to apply those ideas in the classroom. Based on the recommendation of its work group (which reviewed three potential programs designed to improve teachers' knowledge of math), alliance members selected DMI, a professional development program in fractions for grade 4 teachers developed by the Education Development Corporation, because it emphasizes learning and understanding of mathematical content and could be delivered on a large scale in multiple states.[3]

Evaluating the effectiveness of the selected program could inform future district and state investments in professional development programs. Thus, the primary goal of this study was to assess the impact of DMI on grade 4 students' and teachers' understanding of fractions.

DMI is designed to help teachers think through the major mathematical ideas in elementary and middle school and examine how their students develop those ideas (Schifter et al., 2010a, 2010b). DMI consists of seven modules focused on various math topics (such as whole numbers, fractions, geometry, algebra, and functions).

The current study focused only on the second DMI module, Making Meaning for Operations in the Domains of Whole Numbers and Fractions. This module consists of eight three-hour sessions, which are led by a trained DMI facilitator. The majority of the material addresses fractions. Sessions 1 and 2 cover whole number concepts, and sessions 3–8 focus on several mathematical themes involving fractions concepts and operations (table 1). The math content covered in the module is not specific to grade 4. Instead, the module is designed to build teacher understanding of a range of whole number and fractions concepts that are part of the math content taught in the elementary grades.

Sessions 3–8 emphasize the meaning of various procedures used in fractions arithmetic. Participants often solve problems in nontraditional ways and spend time discussing the validity of the approaches that their peers develop.

Table 1. A sampling of themes covered in the Developing Mathematical Ideas module evaluated in the study

Session	Major mathematical themes	Grades that sessions are aligned with based on Common Core State Standards
1: Making meaning for whole-number addition and subtraction	"The same situation can be represented by an addition and a subtraction sentence." (p. 15)	K, 1, 2
2: Making meaning for multiplication and division	"The variety of students' methods for solving story problems involving multiplication and division illustrates relationships among operations." (p. 53)	2, 3
3: When dividing doesn't come out evenly	"The value of a fraction is determined by the relationship between the numerator and the denominator." (p. 87)	3
	"The same quantity can be represented by different fraction names depending on what is taken as 1, the unit or the whole." (p. 87)	
4: Greater than, less than, equal to	"Multiplying the numerator and the denominator by the same constant yields an equivalent fraction." (p. 117)	2, 3, 4
	"To determine which of the two fractions is greater, one may find common denominators and compare numerators, find common numerators and compare denominators, or compare the two fractions to a third number." (p. 117)	
5: Combining shares, or adding fractions	"Students' solutions for sharing situations may result in different additive expressions with fractions. Teachers can help students develop ideas about addition of fractions by challenging them to determine which of their classmates' answers are equivalent and which are incorrect." (p. 149)	4, 5
	"The equivalence of $a \div (b/c)$ and $a \times (c/b)$ can be seen by considering different interpretations of a single diagram." (p. 149)	
6: Taking portions of portions, or multiplying fractions	"It may be necessary to expand ideas about multiplication of whole numbers in order to develop meaning for multiplication involving numbers less than 1." (p. 181)	4, 5, 6
	"Just as multiplication of whole numbers can be represented with a rectangle, so can multiplication involving fractions and mixed numbers." (p. 181)	
7: Expanding ideas about division in the context of fractions	"Diagram solutions for problems involving division of fractions can reveal the relationships among the operations; that is, a division of fractions problem can be solved by calling on addition, subtraction, or multiplication." (p. 207)	5, 6
8: Wrapping up	"The same basic principles that govern operations with whole numbers are called upon to operate with fractions or mixed numbers, but the interpretation of each operation may need to be expanded." (p. 235)	na

na is not applicable.

Note: The Developing Mathematical Ideas module evaluated in the study was Module 2: Making Meaning for Operations in the Domains of Whole Numbers and Fractions. Sessions 1 and 2 cover whole number operations. Sessions 3–8 address fractions.

Source: Authors' compilation based on Schifter et al. (2010b).

In this study most DMI facilitators were elementary math educators who were experienced in providing math professional development in their districts. All had been trained by the developer and had served previously as facilitators of DMI.

DMI uses a set of three recurring instructional activities:

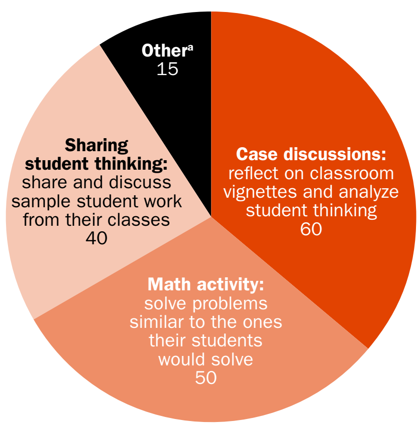
- Sharing student thinking.
- Case discussions.
- Math activity.

During each three-hour session teachers spend 35–45 minutes on the sharing student thinking segment, examining and discussing student work samples from their colleagues' classes. During case discussions teachers spend 55–65 minutes reflecting on case studies they have read as part of their homework. During math activity teachers spend 35–70 minutes solving problems that are designed to promote understanding of underlying fractions concepts (figure 1; see box 1 for further details about the three segments).

Evaluating the effectiveness of the selected professional development program—Developing Mathematical Ideas (DMI)—could inform future district and state investments in professional development programs

Participating teachers complete reading and writing assignments before each session, including collecting and analyzing their students' work samples on math problems relevant to the upcoming sessions. Some sessions include additional activities, such as watching brief videos of fractions instruction, examining curricular material, and planning for student thinking assignments.

Figure 1. Time is allocated for key instructional activities in each Developing Mathematical Ideas session



Note: Total duration of session is 3 hours. On average, Sharing Student Thinking segment is 40 minutes, Case Discussions is 60 minutes, and Math Activity is 50 minutes.

a. Includes such activities as show video, plan student assignment, and plan homework assignment.

Source: Author's construction based on Schifter et al. (2010b).

Box 1. Key instructional activities in each Developing Mathematical Ideas session

Sharing student thinking

In the first segment of each session, teachers examine student work samples shared by other teachers in their group and read their colleagues' analysis of student understanding and misconceptions. By reading their colleagues' analyses of student work samples, they deduce student understandings and misunderstandings about the underlying mathematical ideas. Then, as teachers discuss in their small groups, the facilitator moves from group to group, listening and asking questions about student work. In preparation for this activity, all teachers assign a problem to their students from Developing Mathematical Ideas (DMI). For example, one sample problem given to the students was, "There are 7 brownies to share among 4 friends. How many brownies would each friend get? Solve this problem with a diagram and with an arithmetic sentence. Describe the connections you see between the diagram and the arithmetic" (Schifter et al., 2010b, p. 101).

Case discussions

During this time, teachers reflect on case studies from the DMI *Casebook* (Schifter et al., 2010a), which they have read as part of their homework. The case studies consist of diary entries from different grade levels where teachers describe classroom discussions of mathematical ideas and augment them with pictorial representations of the solutions that students used to illustrate their thinking. For example, in one case a teacher describes a math lesson that asks students to discuss their thinking as they compare the fractions 1/2 and 2/3 (Schifter et al., 2010a, p. 78). To promote small group discussion, the facilitator provides focus questions that draw teachers' attention to the student work that the case study describes. For example, one focus question was, "In the beginning of the case, Harry and Annie use diagrams to explain why 2/3 is greater than 1/2. What ideas about fractions are present in their explanations?" (Schifter et al., 2010b, p. 130). As teachers discuss in their small groups, the facilitator listens, documents issues, and poses them when the groups convene.

Math activity

Teachers solve problems in small groups during this activity. For example, during one session teachers must solve the following problem: "Using diagrams, determine which fraction in each pair is greater: 4/5 or 4/7; 5/6 or 7/8; 3/8 or 2/9." To show understanding, teachers solve the problems without using standard procedures such as converting the fractions to decimals or finding common denominators. The facilitator circulates and interacts with the teachers as they complete the problems and attends to those who do not seem to understand. However, facilitators do not show participants how to solve a problem. Instead, they ask a series of questions that are designed to lead teachers toward understanding the concepts that will help them solve the problems successfully by themselves.

What the study examined

The study addressed two research questions related to the impact of DMI:

- What is the impact of teacher participation in DMI on students' proficiency in fractions?
- What is the impact of teacher participation in DMI on teachers' knowledge of fractions?

To answer the questions related to impact, the study team used a randomized controlled trial in eight school districts in three states—Florida, Georgia, and South Carolina—during the 2014/15 school year. The study sample included 84 schools, 264 teachers, and 4,204 students. Forty-two schools were assigned to the treatment condition, and 42 schools were assigned to the control condition. The 129 teachers from the schools in the treatment condition received the DMI professional development from September to December 2014. The 135 teachers from the schools in the control condition did not receive DMI professional development and instead participated in their school's or district's typical math professional development activities.

The study team also examined three research questions to help in interpreting the impact findings:

- What was the nature of math professional development attended by teachers who did not participate in DMI?
- How satisfied were teachers who participated in DMI with the professional development they received?
- Was DMI implemented as intended?

To answer the questions related to impact, the study used a randomized controlled trial in eight school districts in three states—Florida, Georgia, and South Carolina—during the 2014/15 school year

See box 2 for a summary of the study design, data, and methods and appendix A for further details.

Box 2. Study design, data, and methods

Study design

Schools were randomly assigned to treatment and control conditions. For this cluster-randomized controlled trial 84 interested schools were matched within each district on five variables (grade 4 enrollment, percentage of students who exceeded grade 4 math standards, percentage of students who were Black, percentage of students who were Hispanic, and percentage of students eligible for the federal school lunch program). One school from each matched pair was randomly assigned to the treatment condition (42 schools), and the other was assigned to the control condition (42 schools). All grade 4 teachers of math from the randomly assigned schools who consented to participate in the study formed the teacher sample (129 from schools in the treatment condition and 135 from schools in the control condition). All their students with parent consent formed the student sample for the study (2,091 from schools in the treatment condition and 2,113 from schools in the control condition).

Measures

The study team used total math scaled scores on state math achievement tests (the Florida Comprehensive Assessment Test, Georgia's Criterion-Referenced Competency Tests, and South Carolina's Palmetto Assessment of State Standards) from the spring of grade 3 to measure students' entry-level math skills and knowledge. These test scores were used because grade 4 students do not know much about fractions at the beginning of the school year and might score low on a fractions test, which would hinder the differentiation of students needed for a test to be used as a covariate, and because a test of general math could serve as a good covariate (since general math skills such as whole number arithmetic predict fractions performance; Bailey, Siegler, & Geary, 2014; Hansen et al., 2015). The study team used

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the Test for Understanding of Fractions (Instructional Research Group, 2015) to measure the student outcome (proficiency in fractions). All students were tested 4–6 weeks before the end of the school year, several weeks after the end of the professional development. The test was developed by Instructional Research Group, in collaboration with the Institute of Education Sciences Center for Improving Learning of Fractions. It includes 26 multiple-choice questions, primarily from measures used in the center's research. Items address foundational fractions concepts such as equivalence, magnitude comparison, word problems, and fractions computation for addition and subtraction. Two research math educators examined the measure to ensure that the wording of the items was clear, the mathematical language was precise and accurate, and items covered the full array of grade 4 Common Core State Standards material. See appendix B for information on the measure's reliability.

The study team used the Mathematical Knowledge for Teaching: Number Concepts and Operations scale (Learning Mathematics for Teaching, 2006) to measure entry-level teacher knowledge. It includes 28 multiple-choice questions that assess both the content knowledge and specialized knowledge needed to teach arithmetic to students in the elementary grades. All participating teachers took the pretest prior to the random assignment of schools to the treatment or control conditions. The study team used the Mathematical Knowledge for Teaching: Fractions scale (Learning Mathematics for Teaching, 2008) to measure the teacher outcome (knowledge of fractions). It includes 30 multiple-choice questions involving knowledge of fractions and how to teach fractions to elementary school students. Teachers took the posttest 4–6 weeks before the end of the school year. See appendix B for information on the reliability of the two teacher measures. Sample Mathematical Knowledge for Teaching released items are available at http://lmt.mspnet.org/index.cfm/17924.

Surveys

Information on the professional development activities of treatment and control teachers was gathered using monthly online surveys. The study team gathered information on the professional development activities of treatment and control teachers to facilitate a contrast between the fractions professional development that the control teachers received and Developing Mathematical Ideas (DMI) and any other professional development on fractions that the treatment teachers received. The survey items focus on the content and duration of the fractions professional development activities attended by the teachers. See appendix C for a listing of these survey items.

Teacher satisfaction with DMI was also assessed using online surveys. The study team surveyed the treatment teachers to determine teacher satisfaction with DMI at three points in time: midway through DMI, at the end of DMI, and at the end of the school year. See appendix C for a list of the survey items.

Teachers and facilitators were surveyed about their education and work experience when they consented to participate in the study. The survey included questions about their education, teaching, and mentoring experiences. This information was included in data analysis as potential covariates.

Implementation

Professional development was provided between September and December, 2014. Participating teachers attended the eight three-hour sessions conducted over four days (two three-hour sessions per day; one day per month). In most cases substitute teachers filled in for teachers

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during these days; in some cases teachers preferred occasional Saturday sessions and were compensated for the additional workday.

Nine trained DMI staff facilitated all professional development sessions in the study. In each district, grade 4 teachers from all schools in the treatment condition met as a group in a central location for the DMI sessions. Group sizes ranged from 6 teachers to 33 teachers in a group, with larger groups being facilitated by two DMI facilitators. The same facilitators facilitated all eight sessions in each district (except one session where last minute scheduling conflicts resulted in one of the two facilitators varying across sessions).

DMI facilitators at each site audio-recorded all eight sessions. The DMI staff developed a checklist identifying all key activities for each session, and the study team randomly picked two of the eight sessions to check for implementation: session 3 and session 6. Three members of the study team listened to the audio-recordings and completed the checklists for sessions 3 and 6 at each site. To determine how reliable each of the study team members was in completing the checklists, a second member assessed completion of the activities for four randomly selected sessions (two of session 3 and two of session 6). The average agreement between the study team members for these four sessions was 91 percent.

DMI facilitators recorded teacher attendance for each session. Overall, 80 percent of teachers attended all eight DMI sessions and 98 percent of teachers attended six or more sessions. The smallest number of sessions attended was three (less than 1 percent of teachers). Attendance rates varied by district, from 60 percent of teachers attending all eight sessions in one district to 100 percent attendance in another district (median = 80 percent).

DMI facilitators also tracked homework completion rates. The average completion rate across all assignments was 78 percent (range = 73–81 percent). Completion rates varied by district, from 42 percent in one district to 94 percent in another district (median = 83 percent).

Data analyses

The study team used multilevel models with analysis of covariance to estimate impacts. Multilevel models were used to account for the nested nature of the data (that is, teachers nested in schools and students nested in schools). The study team also used analysis of covariance to adjust for pretest differences between treatment and control samples. See appendix D for more details regarding the analytic models used to determine impacts.

What the study found

This study examined the impact of the DMI professional development program on grade 4 students' proficiency in fractions and on grade 4 teachers' knowledge of mathematical concepts essential for understanding fractions and teaching students about them. Students of teachers who participated in DMI performed at almost the same level as students of teachers who did not participate (impact effect size estimate of –0.03 standard deviation units; $p = .63$). The study also found that the impact of DMI on teachers' knowledge of fractions was inconclusive. Teachers who participated in DMI performed 0.19 standard deviation better than teachers who did not participate, but the difference was not statistically significant ($p = .051$). Thus, DMI resulted in nonsignificant impacts on both teachers' knowledge of fractions and their students' proficiency with fractions.

Students of teachers who participated in Developing Mathematical Ideas did not demonstrate greater knowledge of fractions than did students of teachers who did not participate

DMI did not have a statistically significant impact on the proficiency in fractions of students of teachers who participated in DMI (table 2). The difference in fractions achievement between students of teachers who participated and students of teachers who did not participate was miniscule (–0.03 standard deviation).

Students of teachers who participated in DMI scored slightly higher (16.12) on the posttest, Test for Understanding of Fractions, than did students of teachers who did not participate (15.84). However, students of teachers who participated in DMI scored slightly higher on the pretest (standardized state math tests administered in the spring of grade 3) than did students of teachers who did not participate. After this initial difference was statistically controlled for, the adjusted posttest scores were similar (16.16 for students of teachers who participated in DMI and 16.31 for students of teachers who did not participate), indicating that DMI had no impact on student posttest scores (figure 2).

DMI did not have a statistically significant impact on the proficiency in fractions of students of teachers who participated in DMI; its effect on fractions knowledge of teachers who participated in the professional development was inconclusive

The effect of DMI on fractions knowledge of teachers who participated in the professional development was inconclusive.

Teachers who participated in DMI scored higher on the posttest, Mathematical Knowledge for Teaching: Fractions scale, than did teachers who did not participate (0.19 standard deviation units higher; table 3). An effect size of 0.19 corresponds to a difference of 8 percentile points. This means that, on average, knowledge scores of teachers who participated in DMI differed from teachers who did not participate by 8 percentile points. However, the difference in knowledge of fractions between teachers who participated in DMI and teachers who did not participate was not statistically significant (p = .051).

Teachers who participated in DMI scored slightly higher on the posttest (20.00) than did teachers who did not participate (19.64). However, teachers who participated in DMI scored lower on the pretest, Mathematical Knowledge for Teaching: Number Concepts

Table 2. Pretest and posttest scores of Florida, Georgia, and South Carolina students of teachers who participated in the Developing Mathematical Ideas professional development program and students of teachers who did not participate, 2014/15

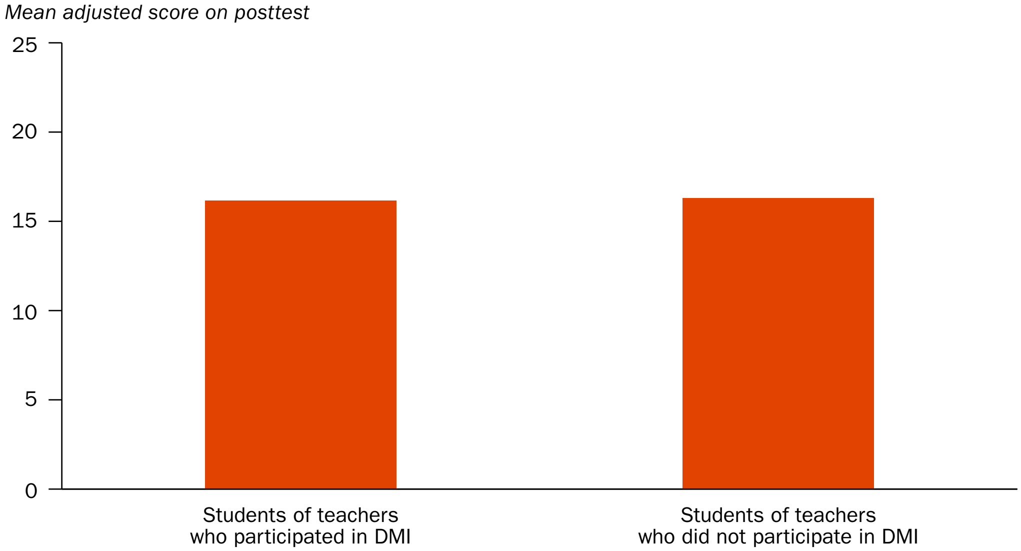
	Pretest (standardized state math Assessment)		Posttest (Test for Understanding of Fractions)				
Group	Mean score (z-score)	Standard deviation	Unadjusted score	Adjusted score	Standard deviation	Hedges' g^a	p value
Students of teachers who participated in DMI (n = 2,091)	0.05	1.01	16.12	16.16	5.43	–0.03	.637
Students of teachers who did not participate in DMI (n = 2,113)	–0.05	0.99	15.84	16.31	5.22		

DMI is Developing Mathematical Ideas.

a. Hedges' g is an effect size index that shows the magnitude of the difference between students of teachers who participated in DMI and students of teachers who did not participate.

Source: Authors' analysis of primary data collected for the study; see appendix A.

Figure 2. Students of Developing Mathematical Ideas teachers and control teachers had similar posttest scores after initial differences were controlled for, 2014/15



DMI is Developing Mathematical Ideas.

Note: The posttest is the Test for Understanding of Fractions.

Source: Authors' analysis of primary data collected for the study; see appendix A.

Table 3. Pretest and posttest scores of Florida, Georgia, and South Carolina teachers who participated in Developing Mathematical Ideas and teachers who did not participate, 2014/15

	Pretest (Mathematical Knowledge for Teaching: Number Concepts and Operations scale)		Posttest (Mathematical Knowledge for Teaching: Fractions scale)				
Group	Mean score (z-score)	Standard deviation	Unadjusted score	Adjusted score	Standard deviation	Hedges' g^a	*p* value
Teachers who participated in DMI (*n* = 129)	13.60	4.06	20.00	20.23	4.19	0.19	.051
Teachers who did not participate in DMI (*n* = 135)	14.10	4.37	19.64	19.42	4.22		

DMI is Developing Mathematical Ideas.

a. Hedges' *g* is an effect size index that shows the magnitude of the difference between students of teachers who participated in DMI and students of teachers who did not participate.

Source: Authors' analysis of primary data collected for the study; see appendix A.

and Operations, than did teachers who did not participate in DMI. After these initial differences were controlled for, adjusted posttest scores were 20.23 for teachers who participated in DMI and 19.42 for teachers who did not participate (figure 3); this difference is not statistically significant.

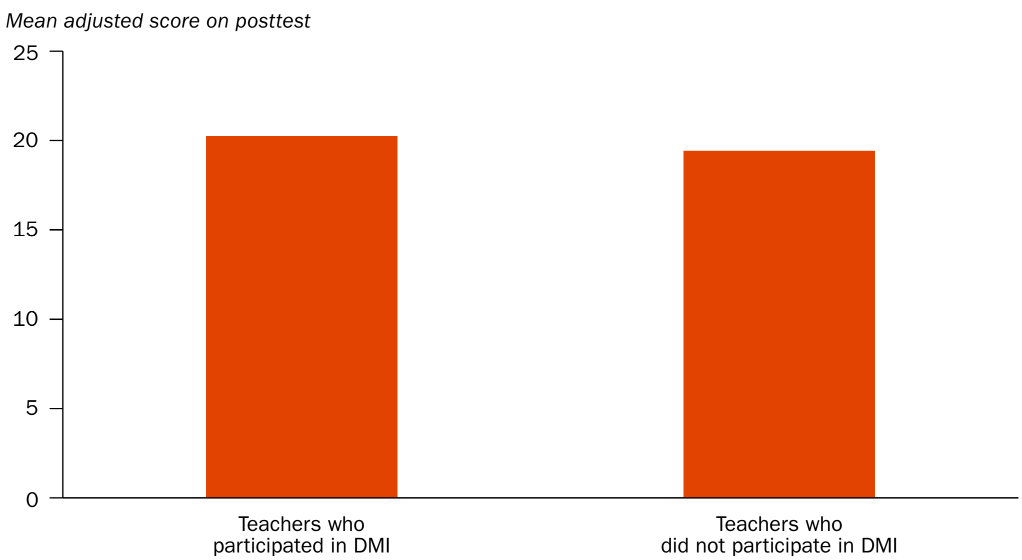
A third of teachers who did not participate in Developing Mathematical Ideas received professional development from another source

Of the 135 teachers who did not participate in DMI, 45 received an average of 5 hours of professional development specifically in fractions, compared with 20 hours for teachers who participated in DMI. Of these 45 teachers, some reported doing activities similar to those in DMI, such as observing videos of math teaching (33 teachers), working with colleagues to plan lessons (35 teachers), discussing student work (25 teachers), and working on math problems (35 teachers).

Most professional development reported by teachers who did not participate in DMI was offered during the same time period that DMI was offered (September–December 2014). Teachers who did not participate in DMI and some teachers who did participate reported receiving other professional development in fractions in the months preceding and following DMI (August 2014 and January, February, and March 2015). See appendix D for additional details on the contrast in professional development received by treatment and control teachers.

Teachers who did not participate in DMI and some teachers who did participate reported receiving other professional development in fractions in the months preceding and following DMI

Figure 3. Teachers who participated in the Developing Mathematical Ideas professional development program scored slightly higher on the posttest than did teachers who did not participate, 2014/15



DMI is Developing Mathematical Ideas.

Note: The posttest is the Mathematical Knowledge for Teaching: Fractions scale.

Source: Authors' analysis of primary data collected for the study; see appendix A.

Most teachers who participated in Developing Mathematical Ideas were satisfied with the professional development they received

Among teachers who participated in DMI, the mean rating for each of four questions on a survey about satisfaction with the program was approximately 3 (agree) on a scale of 1 (strongly disagree) to 4 (strongly agree). This suggests that teachers saw some benefit to participating in DMI. The mean ratings did not change significantly over time, except for one item about improvement in their knowledge of how to teach fractions, which did improve at a statistically significant level over time (table 4).

Overall, the majority of teachers who participated in DMI responded "agree" or "strongly agree" to all four items. For example, 83 percent of teachers agreed or strongly agreed that attending DMI sessions was a good professional learning experience and that their knowledge of fractions had improved. A somewhat smaller percentage of teachers (78 percent) agreed or strongly agreed that their knowledge of how to teach fractions had improved.

Among teachers who participated in DMI, the mean rating for each of four questions on a survey about satisfaction with the program was approximately 3 (agree) on a scale of 1 (strongly disagree) to 4 (strongly agree)

Of the eight districts that participated in the study, one district was a strong outlier. In contrast to the overall positive attitudes of teachers in other districts, teachers from the outlier district were more negative toward DMI (figure 4). Most teachers from this district did not see any positive benefits to their participation. These opinions persisted and did not change over time. A district administrator indicated that teachers wanted professional development that focused more on strategies and techniques that they could take back and implement in their classrooms. Despite the dissatisfaction with the overall focus of the professional development, attendance was at 100 percent, and homework completion at 94 percent in this district.

Table 4. Teacher satisfaction with Developing Mathematical Ideas at three points in time, 2014/15

	Mean rating (1, strongly disagree, to 4, strongly agree) (standard deviation)				
Survey item	Midway through professional development	End of professional development	End of the school year	*F* statistic (degrees of freedom)	*p* value
My knowledge of fractions has improved.	2.94 (0.69)	3.12 (0.74)	3.05 (0.69)	2.15 (1, 7)	.186
My knowledge of how to teach fractions has improved.	2.85 (0.65)	3.02 (0.79)	3.02 (0.74)	11.29* (1, 7)	.012
Attending the DMI professional development sessions was a good professional learning experience.	2.99 (0.72)	3.09 (0.79)	3.07 (0.72)	1.33 (1, 7)	.287
I was able to put into practice what I learned.	3.01 (0.72)	3.07 (0.73)	3.02 (0.74)	0.02 (1, 7)	.895

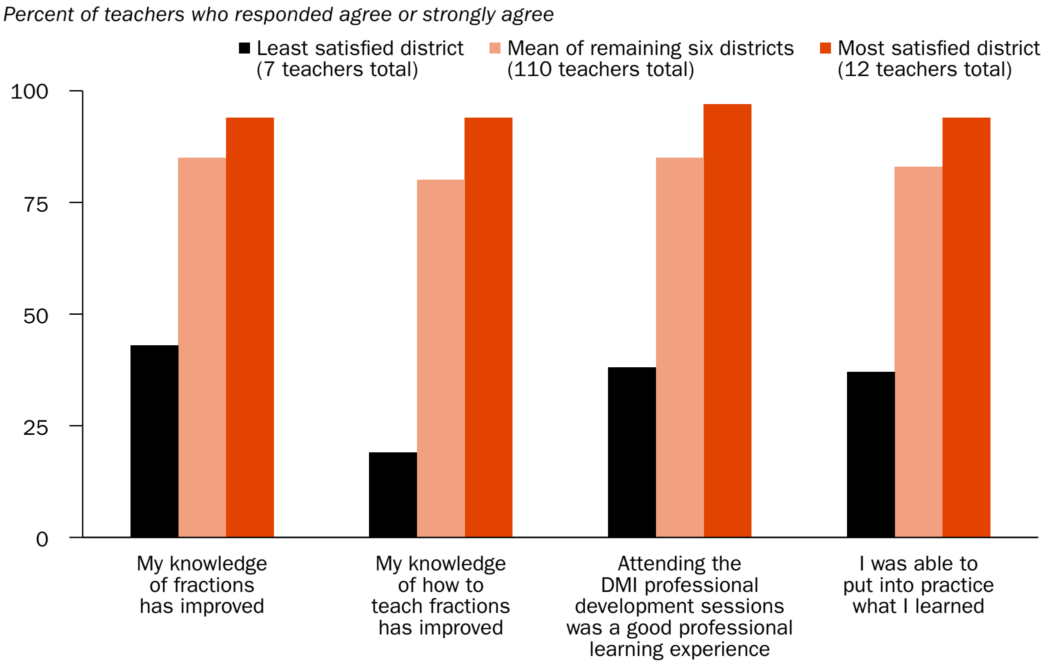
* Significant at $p < .05$.

DMI is Developing Mathematical Ideas.

Note: Of 129 teachers who participated in DMI, 121 responded to all three surveys.

Source: Authors' analysis of primary data collected for the study; see appendix A.

Figure 4. Teachers from one school district were more negative toward the Developing Mathematical Ideas professional development program, compared with teachers from the other seven districts, 2014/15



DMI is Developing Mathematical Ideas.

Note: Values are average satisfaction across surveys at three points in time.

Source: Authors' analysis of primary data collected for the study.

Implementation of the Developing Mathematical Ideas professional development varied across the eight districts and across sessions

To examine implementation of session 3 and session 6 at each of the eight districts, the study team used a fidelity checklist developed by senior DMI staff.

The checklist for session 3 included 24 activities necessary for implementing the program as designed, and the checklist for session 6 included 17 such activities. According to the senior DMI staff, facilitators had to implement at least 80 percent of the activities in each session, a requirement that allowed facilitators to implement the session as intended while providing room to adjust the session according to participants' content knowledge and understanding of major ideas.

For session 3, facilitators implemented at least 80 percent of the activities in six of the eight districts (the median was 90 percent, and the range was 50–96 percent). In contrast, for session 6 facilitators implemented at least 80 percent of the activities in three of the eight districts (the median was 76 percent, and the range was 65–100 percent). The two districts that had below 80 percent implementation for session 3 also had below 80 percent implementation for session 6 (table 5).

Discussion and follow-up activities toward the end of the sessions tended to be omitted in both sessions, though more often in session 6. Because multiplying and dividing fractions

Table 5. Percentage of Developing Mathematical Ideas facilitators who completed 80 percent of activities necessary for implementing the program as designed, by district, 2014/15

District	Session 3	Session 6
1	96	65
2	83	88
3	50	65
4	58	71
5	96	76
6	88	82
7	92	76
8	96	100

Source: Authors' analysis of primary data collected for the study; see appendix A.

without relying on standard algorithms are far more difficult than comparing fractions, the pace of session 6 may have been slower, not allowing for completion of the full lesson.

When combined with previous studies, the findings of the current study raise concerns about the effectiveness of math professional development in bringing about changes in student learning

Implications of the study findings

DMI resulted in nonsignificant impacts on students' proficiency in and teachers' understanding of fractions. The lack of findings at the student level fits the pattern emerging from other large-scale studies of math professional development, where impacts on student achievement have been elusive (Garet et al., 2011; Garet et al., 2016; Gersten et al., 2014). The inconclusive impacts on teacher knowledge also seem to fit the pattern emerging from this research, where changes at the teacher level have been inconsistent.

When combined with previous studies, the findings of the current study raise concerns about the effectiveness of math professional development in bringing about changes in student learning. However, several issues arising from this study may provide guidance for policy and practice and for future research.

Time and support to bring about change

The theory of change in this study was that professional development would bring about a change in teacher knowledge, which in turn would affect student learning because teachers who truly understand the mathematical ideas that underlie the computational procedures are more likely to be able to explain and clarify possible student misconceptions.

Teachers may need more time than was available in the study to learn, understand, and internalize the fractions content and then transfer it from the professional development sessions into the classroom by way of meaningful, high-quality instructional activities. For example, knowledge of fractions content may not translate immediately to providing clear explanations or identifying areas of student understanding or misunderstanding. School district leaders may want to examine the feasibility of providing professional development in the summer just prior to the start of the school year to provide more time for participants to internalize and apply the knowledge in their classrooms.

Teachers may also need additional support beyond participating in the professional development to facilitate the transfer to their classrooms of what they learned during the sessions. For example, knowing about the mathematical ideas underlying fractions equivalence problems may not easily translate to setting up mathematical tasks designed specifically to highlight those concepts for their students. School district leaders may also want to examine the need for additional professional development in the form of coaching or guidance through professional learning communities to help teachers translate the knowledge into effective pedagogical activities (see for example, Gersten et al., 2014).

Professional development implemented as intended

The study team assessed two of the audiotaped sessions (sessions 3 and 6) for procedural fidelity. For session 3, six of the eight districts implemented at least 80 percent of the activities, which was considered acceptable. However, for session 6, which covered a more difficult topic, fewer than half the districts implemented at least 80 percent of the activities. The activities omitted tended to be among the most challenging ones. It is unclear whether the low fidelity of implementation in session 6 and to a lesser extent in session 3 was a result of problems associated with the developers' ability to scale up, such as ability to provide a large number of well-trained facilitators, or a result of other issues encountered by facilitators, such as not having sufficient time to complete all the activities.

Teachers may need additional support beyond participating in the professional development to facilitate the transfer to their classrooms of what they learned during the sessions

During the DMI sessions, teachers explored the content on their own by solving challenging problems and reflecting on case studies and student work samples. The underlying philosophy, as one facilitator noted, was that "when you figure it out for yourself, you learn so much more." To accomplish this goal, facilitators were required to ask probing questions that guided the teachers toward this understanding. They also had to lead discussions so that they, along with the teachers, could co-construct the big ideas in each session (for example, a fractional part can be named by identifying it as a sum or product of fractions; why $a \times (b/c)$ produces the same answer as $a \div (c/b)$).

However, in listening to the audiotapes for sessions 3 and 6, the study team noted that the facilitators did not always pose these questions or have sufficient time available for participants to formulate generalizations on the mathematical ideas they were learning. Perhaps, as seen from the posttest scores, teachers' understanding of fractions concepts improved to some extent but did not reach a threshold necessary for seriously improving the quality and precision in their math instruction. For future implementations, issues relating to scaling up and content coverage need to be examined to ensure that there is sufficient time for completing key activities.

Length of professional development sessions

The developers of DMI asserted that for optimal learning, the preferred option was to conduct one session per day. However, most school districts (and frequently the grade 4 teachers at each school) consistently indicated a preference for two three-hour sessions per day. The reasons for these scheduling arrangements were mainly logistical. Audio recordings of the sessions indicated that the long workdays dedicated solely to math were fatiguing for teachers and perhaps also for facilitators. The fidelity of implementation, as seen, was higher for the morning session 3 than for the afternoon session 6. District leaders may want to consider whether it is possible to implement one session per day as the developers recommended in future implementations.

Relative merits of the professional development activities

While the study team did not survey teachers on the activities that were most useful or instructive, field notes for informal observations of DMI sessions indicated that teachers were most engaged during the math activity segment and least engaged during case discussions and sharing student thinking segments. Future research could survey or interview teachers on the relative merits of DMI activities and other math professional development programs.

Differentiated professional development

The wide distribution of scores on the teacher math content knowledge pretest indicates that a "one size fits all" professional development program may not have been the most suitable approach for the teachers who participated in the study. Informal observations of the DMI sessions by the study team revealed that some teachers seemed to truly grasp the mathematical ideas, while others seemed to struggle. The findings suggest that district leaders may want to consider differentiated material and instruction for teachers with strong math knowledge versus those who have much less initial understanding of the content when planning math professional development activities.

The wide distribution of scores on the teacher math content knowledge pretest indicates that a "one size fits all" professional development program may not have been the most suitable approach for the teachers who participated in the study

Breadth versus depth

DMI covered a wide range of fractions content, from foundational aspects such as unit fraction and equivalence to multiplication and division of fractions. Thus, the program included fractions content from grades 3–6. District leaders may want to consider focusing more deeply on fewer concepts, so that teachers develop a deeper understanding of the mathematical ideas relevant for teaching their grade 4 students.

Limitations of the study

Although the study sites were both urban and rural, they were not a nationally representative sample. Thus, the findings are generalizable only to schools, teachers, and students similar to those who participated in the study.

The results are generalizable to only the DMI professional development program and not necessarily to other programs that focus on building teachers' content knowledge of fractions or techniques in using student errors and misconceptions to enhance students' understanding.

While the self-report data show that some teachers who did not participate in DMI had professional development on topics and activities similar to those in DMI, their professional development experiences may have been different from those of teachers who participated in DMI. The self-report data also do not provide information on whether the professional development received by teachers who did not participate in DMI consisted of isolated short duration sessions or was cumulative, building on knowledge from one session to the next.

Appendix A. Study design and study sample

This appendix explains the study's design (matching and randomization) and sample characteristics (demographic characteristics, baseline equivalence, and attrition).

Study design

The research team used a multisite cluster randomized controlled trial and randomly assigned schools within districts (Donner & Klar, 2000; Shadish, Cook, & Campbell, 2002). Schools within each district were first matched and then randomly assigned to treatment and control conditions.

The study team used an optimal greedy matching method (King et al., 2007) to match schools because it performs similarly to optimal matching but is less computationally intensive (Bruhn & McKenzie, 2009; Gu & Rosenbaum, 1993). Pairs of schools were formed by calculating the Mahalanobis distance between all schools in each district based on five baseline observables: grade 4 enrollment, percentage of students who exceeded grade 4 math standards, percentage of students who were Black, percentage of students who were Hispanic, and percentage of students eligible for the federal school lunch program. Then pairs were identified sequentially by first choosing the pair with the smallest Mahalanobis distance, then choosing the pair with the second smallest, and so on, until the sample was completely matched. After the pairs were formed, one school from each pair was randomly assigned to the treatment condition. This was done by generating random numbers for each school, and then assigning the school with the smaller random number to the treatment condition.

Study sample

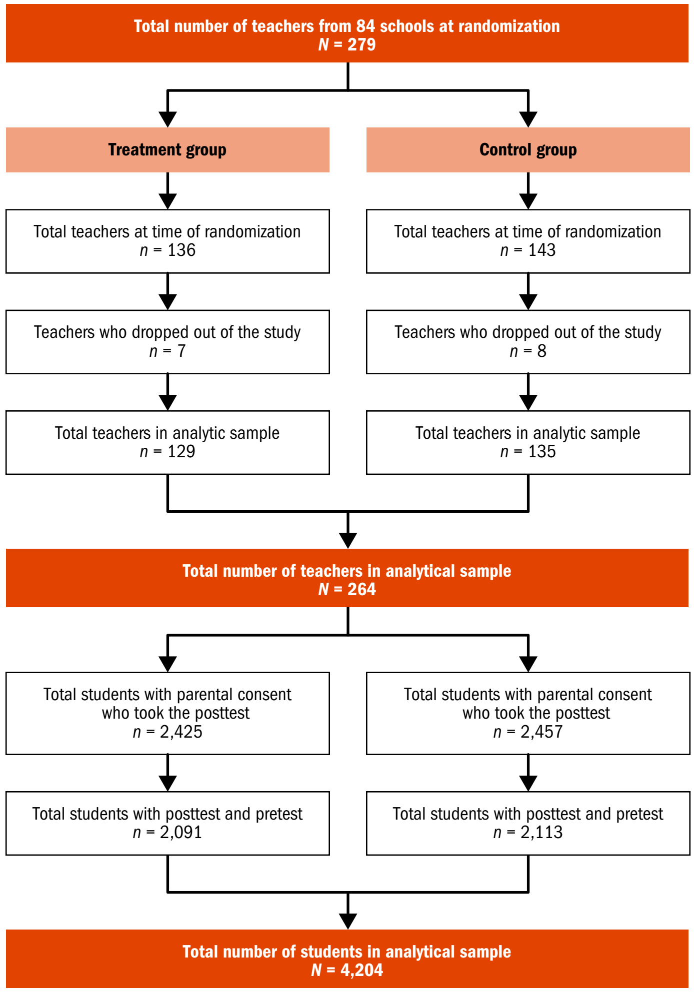
The study took place in eight districts across three states (one in Florida, four in Georgia, and three in South Carolina). A total of 84 schools agreed to participate in the study (42 were assigned to the treatment condition and 42 were assigned to the control condition). The analytic sample includes 264 teachers and 4,204 students (figure A1).

No schools dropped out of the study. Overall, 5.4 percent of the teachers left the study. Differential attrition between treatment and control teachers was 0.5 percent. Reasons for leaving the study across both treatment and control teachers included health and family situation, leaving the district, or taking a different teaching position within the district. Student attrition could not be determined as the study team was unable to verify which students with parent consent were present at many of the schools at the time of randomization.[4] However, in each class all students with parent consent took the posttest, and their pretest scores were retrieved from school databases.

School sample. Of the 84 schools, 14 were rural schools, and 70 were urban schools (table A1). Treatment and control schools did not differ significantly on the percentage of grade 4 students who passed the state math test or on any of the demographic characteristics (table A2).

Teacher sample. Most teachers were female, had more than 11 years of teaching experience, and had 2 to 5 years of experience teaching grade 4 students (table A3). The number

Figure A1. Formation of teacher and student samples



Source: Authors' analysis of primary data collected for the study.

Table A1. Geographic location of schools in the study (number of schools)

Location of school district	Florida		Georgia		South Carolina	
	Treatment group	Control group	Treatment group	Control group	Treatment group	Control group
Rural	na	na	2	3	5	4
Urban						
City[a]	na	na	9	10	9	9
Suburb[b]	6	6	5	3	6	6
Town, fringe[c]	na	na	na	1	na	na

na is not applicable.

a. Territory inside an urbanized area and inside a principal city.

b. Territory outside a principal city and inside an urbanized area.

c. Territory inside an urban cluster that is less than or equal to 10 miles from an urbanized area.

Source: Authors' analysis based on 2014–2015 data from U.S. Department of Education (n.d. a, n.d. b).

Table A2. Baseline equivalence of the 84 schools in the study, 2014/15

Characteristic	Mean (standard deviation) Treatment condition ($n = 42$)	Mean (standard deviation) Control condition ($n = 42$)	Hedges' g[a]	p value
Number of students in grade 4	97.83 (32.05)	98.69 (40.80)	−0.02	.755
Percentage of grade 4 students who passed the state math test	74.37 (11.21)	74.60 (10.55)	−0.02	.925
Percentage of students who were Black	36.86 (28.80)	36.21 (28.57)	0.02	.918
Percentage of students who were Hispanic	12.41 (15.89)	15.85 (16.45)	−0.21	.332
Percentage of students eligible for the federal school lunch program	61.72 (26.97)	66.84 (23.27)	−0.20	.354
Percentage of students who were English learner students	8.10 (14.74)	9.95 (14.14)	−0.13	.558

Hedges' g is an effect size index that shows the magnitude of the difference between treatment group and control group.

Source: Authors' analysis based on data from the Florida Department of Education, the Georgia Department of Education, and the South Carolina Department of Education.

of teachers with a bachelor's degree or master's degree was comparable for the two samples. More treatment teachers than control teachers had worked as math coaches, and this difference was statistically significant.

Teachers in the treatment condition scored 0.12 standard deviation below teachers in the control condition on the math pretest (Mathematical Knowledge for Teaching: Number Concepts and Operations). However, the difference was not statistically significant (table A4). The groups showed other differences (such as the number of years of teaching grade 4 and the number of courses taken in a teaching math type of certification) that were also nonsignificant.

Teachers in the study used a variety of math curricula (table A5). Curriculum variations are by district, and therefore the curriculum is the same for teachers in the treatment condition and teachers in the control condition.

Student sample. Student state math achievement scores from the spring of grade 3 and student demographic data were collected from district databases. Treatment students scored 0.10 standard deviation higher on the math pretest than did control students, but the difference was not significant (table A6). However, students in the treatment and control conditions differed significantly on the percentage of Hispanic students and the percentage of talented and gifted students.

Facilitator sample. Nine facilitators, provided by the developers of Developing Mathematical Ideas (DMI), facilitated the DMI sessions in the study. All were experienced DMI facilitators, with an average 11 years of experience (table A7). Each facilitator was responsible for facilitating all DMI sessions at each school district site. Five of the nine facilitators facilitated sessions in one school district each. Each of the remaining four facilitators facilitated sessions at two school district sites.

Table A3. Demographic characteristics of the analytic sample of 264 teachers, 2014/15

Characteristic	Percent: Total sample (N = 264)	Percent: Treatment condition (n = 129)	Percent: Control condition (n = 135)	Chi-square test (degrees of freedom)	p value
Gender					
Female	93.94	94.57	93.33	0.178 (1)	.673
Degree type					
Bachelor's	47.35	46.51	48.15	0.111 (2)	.946
Master's	44.70	45.74	43.70		
Coursework beyond master's	7.95	7.75	8.15		
Total years of classroom teaching					
0–1	11.74	10.85	12.59	2.102 (3)	.552
2–5	13.64	11.63	15.56		
6–10	22.35	25.58	19.26		
11 or more	52.27	51.94	52.59		
Total years of teaching grade 4					
0–1	30.30	27.13	33.33	1.608 (3)	.658
2–5	40.15	43.41	37.04		
6–10	19.70	20.16	19.26		
11 or more	9.85	9.30	10.37		
Has experience teaching math in grades 6–12[a]					
Yes	5.70	5.43	5.97	0.036 (1)	.850
No	94.30	94.57	94.03		
Certifications[b]					
Elementary	98.11	97.67	98.52	0.980 (3)	.806
Math	6.82	6.20	7.41		
Middle school	16.67	14.73	18.52		
Special education	6.06	6.98	5.19		
Number of undergraduate or graduate classes in math[c]					
0	6.11	6.20	6.02	0.141 (3)	.987
1–2	41.98	42.64	41.35		
3–5	42.75	42.64	42.86		
6 or more	9.16	8.53	9.77		
Number of undergraduate or graduate classes in methods for teaching math[d]					
0	11.49	10.08	12.88	2.129 (3)	.546
1–2	62.45	61.24	63.64		
3–5	21.46	24.81	18.18		
6 or more	4.60	3.88	5.30		
Worked as a math coach or mentor in the past three years[e]					
Yes	8.08	11.72	4.55	4.504* (1)	.034
No	91.92	88.28	95.45		

* Significant at $p < .05$.

Note: Percentages may not sum to 100 because of rounding.

a. Data for one teacher in the control condition were missing.

b. Percentages do not sum to 100 because some teachers have multiple certifications.

c. Data for two teachers in the control condition were missing.

d. Data for three teachers in the control condition were missing.

e. Data for one teacher in the treatment condition and three teachers in the control condition were missing.

Source: Authors' analysis of primary data collected for the study.

Table A4. Baseline equivalence on math pretest and demographic characteristics of the 264 teachers in the analytic sample, 2014/15

Characteristic	Mean (standard deviation) Treatment condition (n = 129)	Mean (standard deviation) Control condition (n = 135)	Hedges' g	Cox's d	Chi-square test	p value
Score on math pretest—Mathematical Knowledge for Teaching: Number Concepts and Operations	13.60 (4.06)	14.10 (4.37)	–0.12	na	na	.352
Years of teaching grade 4	4.14 (3.28)	3.85 (3.35)	0.09	na	na	.582
Number of courses taken in math	1.53 (0.74)	1.54 (0.77)	–0.01	na	na	.865
Number of courses taken in teaching math	1.22 (0.68)	1.13 (0.72)	0.13	na	na	.290
Percentage with a master's degree[a]	53.49 (50.07)	51.85 (50.15)	na	0.04	0.07	.790
Percentage who taught math in grades 6–12[a]	5.43 (22.74)	5.93 (23.70)	na	–0.06	0.03	.861
Percentage with math certification[a]	6.20 (24.21)	7.41 (26.29)	na	–0.12	0.15	.698
Percentage with middle school certification[a]	14.73 (35.58)	18.58 (38.99)	na	–0.17	0.68	.409
Percentage with special education certification[a]	6.98 (25.57)	5.19 (22.26)	na	0.19	0.37	.542

na is not applicable.

Note: Both Hedges' g and Cox's d are effect size indexes that show the magnitude of the difference between treatment group and control group. Hedges' g is used for continuous measures, and Cox's d is used for dichotomous measures.

a. Dichotomous measures, for which both Cox's d and chi-square tests are reported.

Source: Authors' analysis of primary data collected for the study.

Table A5. Math curriculum use by district, 2014/15

District	Math curriculum used
1	Envision
2	Go Math
3	Singapore Math
4	Math Expressions
5	Envision/V-Math
6	Investigations
7	Go Math
8	My Math

Source: Authors' analysis of primary data collected for the study.

Table A6. Baseline equivalence of the 4,204 students in the analytic sample, 2014/15 (percent unless otherwise indicated)

Characteristic	Mean (standard deviation) Treatment condition (n = 2,091)	Mean (standard deviation) Control condition (n = 2,113)	Hedges' g	Cox's d	Chi-square test	p value
Math pretest (z-score)	0.05 (1.01)	-0.05 (0.99)	0.09	na	na	.334
Male[a]	51.32 (49.99)	50.83 (50.00)	na	0.01	0.10	.752
Black[a]	31.88 (46.61)	29.15 (45.46)	na	0.08	3.69	.055
Hispanic[a]	13.21 (33.87)	16.23 (36.88)	na	–0.15	7.36**	.006
White[a]	48.36 (49.98)	48.75 (50.00)	na	–0.01	0.07	.797
In special education[a]	8.94 (28.54)	8.85 (28.41)	na	0.01	0.01	.916
English learner students[a]	10.23 (30.32)	11.97 (32.47)	na	–0.11	3.22	.073
Talented and gifted[a]	8.56 (27.98)	11.36 (31.47)	na	–0.19	9.17**	.003

** Significant at $p < .01$.

na is not applicable.

Note: Both Hedges' g and Cox's d are effect size indexes that show the magnitude of the difference between the treatment group and control group. Hedges' g is used for continuous measures, and Cox's d is used for dichotomous measures.

a. Dichotomous measures, for which both Cox's d and chi-square tests are reported.

Source: Authors' analysis of primary data collected for the study.

Table A7. Demographic characteristics of Developing Mathematical Ideas facilitators, 2014/15

Characteristic	Mean	Median	Range	Number of facilitators (*N* = 9)
Total years				
Total years in position	11.6	12	5–22	na
Total years of classroom teaching	26.2	25	7–40	na
Total years of teaching math grades 6–12	5.6	1	0–40	na
Total years of teaching grade 4	5.9	5	0–25	na
Total years of providing math in-service for districts	16.4	16	10–25	na
Total years as DMI facilitator	11.3	15	4–16	na
Certifications				
Elementary	na	na	na	8
Math	na	na	na	4
Middle school	na	na	na	4
Administration	na	na	na	2
National board certified	na	na	na	3
Highest degree earned				
Bachelor's	na	na	na	1
Master's	na	na	na	3
Coursework beyond master's	na	na	na	5
Current position				
Classroom teacher	na	na	na	1
Math coach, specialist, or instructional support	na	na	na	6
Consultant	na	na	na	2

DMI is Developing Mathematical Ideas; na is not applicable.

Source: Authors' analysis of primary data collected for the study.

Appendix B. Reliability of measures

The study team examined the internal consistency reliability of the measures used in the study using two types of analyses:

- Classical test score analysis to examine item to total correlations and compute coefficient alphas for both measures.
- Item response theory analysis using a two-parameter logistic model to examine items along two parameters: discrimination and difficulty.

Internal consistency (coefficient alpha) reliability was .86 for the student measure, .72 for the teacher pretest measure, and .73 for the teacher posttest measures. All are well over the threshold for an outcome measure established by What Works Clearinghouse (2014).

Test for Understanding of Fractions (student posttest)

Based on data from a sample of 5,005 grade 4 students, Cronbach's alpha was .86 for the 26-item Test for Understanding of Fractions, which was used as the student posttest measure (table B1). Item response theory analysis indicated that reliability was greater than .80 from an ability level of –2.4 to 1.6 and greater than .90 from an ability level of –0.8 to 0.4. Maximum information for the 26 items (that is, the peak of the information curve) was at an ability level of 0 (I[θ] = 10.62).

Item-level percentages for a correct response ranged from 30 percent for item 23 to 95 percent for item 2 (table B2). Of the 26 items, 24 had discrimination estimates above 0.60, ranging from 0.75 to 2.13. Items with higher discrimination estimates (that is, steeper slopes) are considered more informative. Item difficulty estimates ranged from –9.94 for item 3 to 0.73 for item 23. Items with negative difficulty estimates are considered easier than items with positive difficulty estimates.

Mathematical Knowledge for Teaching: Number Concepts and Operations scale (teacher pretest)

Based on a sample of 279 grade 4 teachers, the Cronbach's alpha for the 28-item Mathematical Knowledge for Teaching: Number Concepts and Operations scale, which was used as the teacher pretest measure, was .72 (table B3). Based on item response theory analysis, the reliability was greater than .75 from an ability level of roughly –2.0 to 2.4 and greater than .80 from an ability level of –0.4 to 1.2. Maximum information (that is, the peak of the information curve) for the 28 items was at an ability level of 0.4 (I[θ] = 4.43).

Table B1. Reliability of the student posttest—Test for Understanding of Fractions, 2014/15

Measure	Cronbach's alpha from classical test theory analysis	Reliability estimates from item response theory analysis
Test for Understanding of Fractions	.86	> .80

Note: Analysis based on a sample of 5,005 students. Item response theory reliabilities were determined using a two-parameter logistical model based on the following parameters: discrimination and difficulty.

Source: Authors' analysis of primary data collected for the study; see appendix A.

Table B2. Item-level analysis of the student posttest—Test for Understanding of Fractions, 2014/15

		Cronbach's alpha		Item response theory analysis	
Item number	Percent correct	Item-total correlation	Alpha if item deleted	Discrimination	Difficulty
1	44.10	–.24	.86	–0.56	–0.46
2	95.06	.18	.85	1.02	–3.30
3	89.33	.06	.85	0.22	–9.94
4	71.45	.39	.84	1.20	–0.99
5	47.43	.52	.83	1.57	0.06
6	49.71	.29	.84	0.75	0.01
7	42.14	.47	.84	1.35	0.29
8	48.93	.48	.84	1.43	0.01
9	76.20	.33	.84	0.97	–1.43
10	34.44	.57	.83	2.06	0.49
11	73.65	.30	.84	0.87	–1.38
12	87.57	.35	.84	1.69	–1.64
13	57.70	.52	.83	1.75	–0.31
14	52.70	.48	.84	1.38	–0.14
15	87.53	.25	.84	0.93	–2.40
16	64.04	.30	.84	0.79	–0.84
17	85.17	.30	.84	1.14	–1.88
18	61.10	.51	.83	1.68	–0.44
19	77.74	.29	.84	0.81	–1.74
20	47.49	.41	.84	1.11	0.09
21	39.98	.52	.83	1.60	0.34
22	55.18	.54	.83	1.91	–0.22
23	30.33	.50	.84	1.61	0.73
24	70.51	.38	.84	1.08	–1.01
25	37.92	.59	.83	2.13	0.36
26	54.21	.52	.83	1.56	–0.19

Source: Authors' analysis of primary data collected for the study; see appendix A.

Table B3. Reliability of the teacher pretest—Mathematical Knowledge for Teaching: Number Concepts and Operations scale, 2014/15

Measure	Cronbach's alpha from classical test theory analysis	Reliability estimates from Item Response Theory analysis
Mathematical Knowledge for Teaching: Number Concepts and Operations scale	.72	> .75

Note: Psychometric data based on a sample of 279 teachers. Item response theory reliabilities were determined using a two-parameter logistical model based on the following parameters: discrimination and difficulty.

Source: Authors' analysis of primary data collected for the study; see appendix A.

Item discrimination parameters for the 28 items ranged from 0.24 (item 15) to 1.40 (item 16). Items with higher discrimination estimates (that is, steeper slopes) are considered more informative. Item difficulty estimates range from –3.09 (item 1) to 5.53 (item 27). Items with negative difficulty estimates are considered easier than items with positive item difficulty estimates (table B4).

Table B4. Item-level analysis of the teacher pretest—Mathematical Knowledge for Teaching: Number Concepts and Operations scale, 2014/15

Item number	Cronbach's alpha		Item response theory analysis	
	Item total correlation	Alpha if item deleted	Discrimination	Difficulty
1	.12	.72	0.31	–3.09
2	.26	.71	0.72	–1.65
3	.20	.71	0.44	–0.29
4	.19	.71	0.56	–2.92
5	.29	.71	1.06	1.78
6	.15	.72	0.45	1.52
7	.36	.70	1.16	0.20
8	.27	.71	0.97	1.76
9	.20	.71	0.67	–2.01
10	.34	.70	1.04	0.49
11	.33	.70	1.13	–0.27
12	.22	.71	0.39	–0.56
13	.22	.71	0.91	2.18
14	.31	.70	0.96	0.25
15	.18	.71	0.24	–1.60
16	.42	.70	1.40	0.41
17	.30	.71	0.96	–0.86
18	.25	.71	0.86	–1.44
19	.21	.71	0.53	0.80
20	.26	.71	0.73	0.68
21	.15	.72	0.44	3.34
22	.13	.72	0.50	–3.11
23	.33	.70	1.06	–1.62
24	.23	.71	0.58	0.36
25	.21	.71	0.53	0.83
26	.21	.71	0.71	–2.78
27	.11	.72	0.41	5.53
28	.32	.71	1.14	1.73

Source: Authors' analysis of primary data collected for the study; see appendix A.

Mathematical Knowledge for Teaching: Fractions scale (teacher posttest)

Based on a sample of 266 grade 4 teachers, the Cronbach's alpha for the 30-item Mathematical Knowledge for Teaching: Fractions scale, which was used as the teacher posttest measure, was .73 (table B5). Item response theory analysis indicated that reliability was greater than .80 from an ability level of –2.8 to an ability level of 0.4. Maximum information for the 30 items (that is, the peak of the information curve) was at an ability level of –2.0 ($I[\theta] = 6.05$).

Item discrimination parameters for the 30 items ranged from 0.06 (item 28) to 1.99 (item 10). Items with higher discrimination estimates (that is, steeper slopes) are considered more informative. Item difficulty estimates for the data ranged from –31.18 (item 28) to 3.63 (item 24). Items with negative difficulty estimates are considered easier than items with positive difficulty estimates (table B6).

Table B5. Reliability of teacher posttest—Mathematical Knowledge for Teaching: Fractions scale, 2014/15

Measure	Internal consistency of items Cronbach's alpha from classical test theory analysis	Reliability estimates from item response theory analysis
Mathematical Knowledge for Teaching: Fractions scale	.73	> .80

Note: Psychometric data based on a sample of 266 teachers. Item response theory reliabilities were determined using a two-parameter logistical model based on the following parameters: discrimination and difficulty.

Source: Authors' analysis of primary data collected for the study; see appendix A.

Table B6. Item-level analysis of teacher posttest—Mathematical Knowledge for Teaching: Fractions scale, 2014/15

Item number	Cronbach's alpha		Item Response Theory analysis	
	Total item correlation	Alpha if item deleted	Discrimination	Difficulty
1	.23	.72	0.62	–1.01
2	.32	.72	0.98	–1.45
3	.25	.72	0.67	–0.87
4	.31	.72	0.89	0.89
5	.22	.72	0.50	0.22
6	.17	.73	0.39	0.59
7	.19	.73	1.66	–2.66
8	.25	.72	1.34	–2.27
9	.20	.73	1.28	–2.58
10	.21	.73	1.99	–2.57
11	.38	.71	1.30	–0.55
12	.36	.71	1.37	–1.19
13	.05	.74	0.18	–7.64
14	.16	.73	0.47	–2.76
15	.38	.71	1.38	–0.74
16	.17	.73	0.49	1.72
17	.38	.71	1.33	–0.87
18	.11	.73	0.41	–4.91
19	.20	.72	1.08	2.71
20	.28	.72	1.08	–1.47
21	.33	.72	1.06	–1.42
22	.25	.72	0.62	–1.49
23	.43	.71	1.28	0.33
24	.18	.73	0.56	3.63
25	.15	.73	0.40	–1.86
26	.24	.72	0.76	–2.13
27	.32	.72	0.83	–0.11
28	.02	.73	0.06	–31.18
29	.29	.72	0.81	–1.32
30	.29	.72	0.85	–1.30

Source: Authors' analysis of primary data collected for the study; see appendix A.

Appendix C. Online surveys

This appendix presents the two online surveys used in this project. One survey gathered information on the professional development activities of teachers who participated in Developing Mathematical Ideas and of teachers who did not participate. The other survey gauged satisfaction with the program among teachers who participated in DMI.

Survey of professional development activities

To facilitate a contrast between the professional development activities of teachers who participated in DMI and of teachers who did not participate, the study team surveyed all teachers in the study to gather information about their professional development activities in math (table C1). The team administered the survey online each month, from September 2014 to April 2015. Teachers received an email reminder each month to complete the survey. On average, the response rate for the eight surveys was 91 percent. The survey items were drawn from previous professional development research that the study team conducted (Gersten, Dimino, Jayanthi, Kim, & Santoro, 2010) and from the teacher survey used in the Middle School Mathematics Professional Development Study by Garet et al. (2011).

Survey of teacher satisfaction with Developing Mathematical Ideas

To gauge treatment teacher satisfaction with DMI, the study team surveyed the teachers three times: midway through the professional development, at the end of the professional development, and at the end of the school year. The average response rate for the three surveys was 97 percent. The study team developed the survey items and administered them to the teachers online (table C2).

Table C1. Monthly fractions professional development survey questions given to treatment and control teachers

Fractions professional development questions	Response	
1. Did you participate in math professional development?	☐ Yes	☐ No
2. Was the topic of fractions covered in the math professional development you participated in?	☐ Yes	☐ No
3. For how many hours did the professional development you attended cover fractions?	________ hours	
4. Which of the following topics were included in the fractions portion of the professional development? Check all that apply.		
Understanding fractions as numbers	☐	
Equivalence of fractions	☐	
Fraction addition/subtraction	☐	
Fraction multiplication/division	☐	
Improper fractions/mixed numbers	☐	
5. Check the activities that were part of the math professional development sessions you attended. Check all that apply.		
Observing videos or live demonstrations of mathematics teaching during the professional development session	☐	
Working with colleagues to plan or enhance mathematics lessons/activities	☐	
Discussion of students' mathematics work samples with colleagues to analyze how students think about and learn mathematics (including common student difficulties and misconceptions)	☐	
Working on mathematics problems (either individually or with a partner)	☐	
Engaging in group discussions about the mathematics concepts we were learning	☐	

(continued)

Table C1. Monthly fractions professional development survey questions given to treatment and control teachers *(continued)*

Fractions professional development questions	Response
6. Check the follow-up activities that were part of the math professional development you attended. Check all activities that apply.	
I was required to observe demonstrations of mathematics teaching in classroom settings	☐
I was required to develop mathematics lessons/activities outside of the professional development session, for use in my classroom	☐
I was required to implement a mathematics lesson or activity in my classroom	☐
I received feedback from the professional development staff about my understanding of mathematical ideas	☐
I was observed in my classroom by a coach and received feedback	☐
7. Which of the following did you learn during the math professional development sessions you attended? Check all activities that apply.	
I learned how to use manipulatives (concrete objects) to teach mathematical concepts	☐
I learned how to use visual representations to convey mathematical concepts (such as number line, strip diagrams, pictorial representations, etc.)	☐
I learned how to ask students questions and provide feedback	☐
I learned how to use a curriculum or how to use supplemental curricula materials	☐
8. Did your professional development use a commercially available mathematics professional development program such as DMI, NUMBERS, INTEL MATHEMATICS?	☐ Yes ☐ No
If Yes, specify the name of the program	________

DMI is Developing Mathematical Ideas.

Source: Authors' creation.

Table C2. Survey questions given to treatment teachers to determine their satisfaction with the Developing Mathematical Ideas program

Survey item	Strongly disagree	Disagree	Agree	Strongly agree
My knowledge of fractions has improved.	1	2	3	4
My knowledge of how to teach fractions has improved.	1	2	3	4
Attending the DMI professional development sessions was a good professional learning experience.	1	2	3	4
I was able to put into practice what I learned during the DMI professional development sessions.	1	2	3	4

DMI is Developing Mathematical Ideas.

Source: Authors' creation.

Appendix D. Data analysis

This appendix presents data analyses procedures and findings from the main impact analyses, sensitivity analyses, and imputation analysis. In addition, the self-report data from teachers in the treatment and control conditions on their professional development activities are also summarized.

Assessing impacts at student level

Impact analyses for student outcomes were based on a multilevel model with students at level 1 and schools at level 2 (table D1). For the student impact, the student-level covariates included the previous year's state math test score standardized within state and five dummy-coded covariates that identify students as in special education, talented and gifted, English learner, Black, or Hispanic. Five school-level covariates were also included: the percentage of grade 4 students who passed the state math test, the percentage of students who were eligible for the federal school lunch program, total number of students in grade 4, the percentage of students who were Black, and the percentage of students who were Hispanic.

Schools were assigned within school pairs and districts (that is, blocked on pairs and districts). The model used in the analysis—students nested within schools—is a two-level model that ignored school pairs and districts as blocking factors. Ignoring blocking, though not faithful to the actual design used, does not appear to cause problems with either the Type I or Type II error rate (Dong & Lipsey, 2010). The decision to use a two-level model is consistent with the recommendation in the literature about how to handle clustering for studies that randomly assign schools to conditions within districts (Raudenbush & Sadoff, 2008; Schochet, 2008; Zhu, Jacob, Bloom, & Xu, 2011).

The models were fitted to the data with SAS PROC MIXED version 9.2 (SAS Institute, 2009) using restricted maximum likelihood, generally recommended for multilevel models (Hox, 2002). Following What Works Clearinghouse 3.0 guidelines (What Works Clearinghouse, 2014), Hedges' *g* (Hedges, 1981) was also computed for each fixed effect.

Sensitivity analysis. A sensitivity analysis was conducted to see whether analytic models that include blocks produce results similar to those without blocks. A third level was added to the models that accounted for either school pairs or districts as either random or fixed blocks. The results from the models that accounted for blocks differed only slightly from the models that excluded blocks for students, which was expected given that the school pair and district were not highly correlated with outcomes (table D2).

Assessing impacts at teacher level

As with students, teacher-level analysis used a two-level fixed effects model, with teachers at level 1 and schools at level 2. The teacher analysis included several teacher-level and school-level covariates (table D3). The 11 teacher-level covariates included teacher's pretest math test score, teacher's education level (master's vs. bachelor's), years of experience teaching in grade 4, years of experience teaching grades 6–12, math certification (yes or no), middle school certification (yes or no), special education certification (yes or no), number of courses in math, number of courses in math instruction, and two variables

Table D1. Impact of Developing Mathematical Ideas on student fractions achievement, 2014/15

Model	Parameter estimates	Standard errors	*p* value
Fixed effects			
Intercept	18.86	1.87***	<.001
Condition	–0.15	0.32	.638
Student			
Math pretest	3.25	0.06***	<.001
Special education status	–0.52	0.18**	.005
Talented and gifted status	1.42	0.20***	<.001
English learner student	–0.58	0.26*	.025
Black	–1.06	0.15***	<.001
Hispanic	–0.23	0.22	.296
School			
Percentage of grade 4 students who passed state math test	0.00	0.02	.862
Percentage of students eligible for the federal school lunch program	–0.04	0.01***	<.001
Total number of students in grade 4	0.00	0.01	.740
Percentage of students who were Black	0.00	0.01	.682
Percentage of students who were Hispanic	0.03	0.01	.089
Variances			
School intercept	1.84	0.35***	<.001
Residual	11.61	0.26***	<.001
Intraclass correlation	0.14	na	na
Hedges' *g*[a] (condition)	–0.03	na	na
p value (condition)	.637	na	na
Adjusted means			
Treatment	16.16	na	na
Control	16.31	na	na

* Significant at $p < .05$; ** Significant at $p < .01$; *** Significant at $p < .001$.

DMI is Developing Mathematical Ideas; na is not applicable.

a. Hedges' *g* is an effect size index that shows the magnitude of the difference between treatment group and control group.

Note: Tests of fixed effects accounted for schools as the unit of analysis within the treatment condition. Analyses conducted with a mixed-model analysis of covariance used 77 degrees of freedom.

Source: Authors' analysis of primary data collected for the study; see appendix A.

related to coaching. At the school level, five covariates were included: percentage of students who passed the state math test in grade 4, enrollment in grade 4, percentage of students eligible for the federal school lunch program, percentage of students who were Black, and percentage of students who were Hispanic. As with student-level analysis, the model used did not include district level.

Sensitivity analysis. Sensitivity analyses were conducted at the teacher level to determine whether models with districts as either random effects or fixed effects produce impacts similar to those with no blocks. The three-level models did not differ notably from two-level models that excluded districts as a blocking variable (table D4).

Table D2. Sensitivity analyses for the impact of Developing Mathematical Ideas on the student posttest—Test for Understanding of Fractions, 2014/15

Model description	Hedges' g[a]	p value
Two-level models		
Pretest covariate	–0.02	.806
Pretest and demographic covariates	–0.01	.936
Pretest, school, and demographic covariates	–0.03	.637
Three-level models		
Pretest, school, and demographic covariates nested within classrooms and schools	–0.03	.614
Pretest, school, and demographic covariates nested within classrooms and matched school pairs	–0.03	.640
Pretest, school, and demographic covariates nested within schools and school pairs	–0.03	.638
Pretest, school, and demographic covariates with districts as random blocks	–0.04	.392
Pretest, school, and demographic covariates with districts as fixed blocks	–0.04	.357

Note: Analyses conducted with a mixed-model analysis of covariance used 82 degrees of freedom for the test of treatment condition in models 1 and 2, 77 degrees of freedom for models 3–6, 76 degrees of freedom for model 7, and 70 degrees of freedom for model 8.

a. Hedges' *g* is an effect size index that show the magnitude of the difference between treatment group and control group.

Source: Authors' analysis of primary data collected for the study; see appendix A.

Imputation of missing student pretest data

At the end of school year, 4,882 students from the classes of teachers in the study took the posttest (the Test for Understanding of Fractions). However, the study team was able to obtain pretest scores—grade 3 spring state math achievement scores—from the districts for only 4,204 of these students. The analytic sample included only these 4,204 students with posttest and pretest scores. The study team imputed missing pretest values based on other available pretest data and found that it did not appreciably change the results. In the original analysis of covariance model with pretest as the only covariate, the difference between conditions was –0.38, $t = -0.25$, $p = .806$. With the imputed data, the difference was –0.35, $t = -0.22$, $p = .825$.

Correlations between pretest and posttest scores

The correlation between student state math achievement scores and student fractions posttest scores (on the Test for Understanding of Fractions) was .70. The correlation between the teacher pretest (the Mathematical Knowledge for Teaching: Number Concepts and Operations scale) and the teacher posttest (the Mathematical Knowledge for Teaching: Fractions scale) was correlated at .65. Both correlations were statistically significant at $p < .001$. These correlations indicate that using these pre-measures in the analysis improves the precision of the statistical analyses conducted because they are suitable covariates.

Contrast between the fractions professional development received by teachers in the treatment and control conditions

Teachers in both the treatment and control conditions reported on the fractions professional development they participated in during the school year from August 2014 to March

Table D3. Impact of Developing Mathematical Ideas on teachers' knowledge of fractions, 2014/15

Model	Parameter estimates	Standard errors	*p* value
Fixed effects			
Intercept	9.73	2.48***	<.001
Condition	0.81	0.41	.051
Teacher			
Math pretest	0.64	0.05***	<.001
Master's degree	–0.35	0.42	.406
Years of teaching grade 4	–0.09	0.06	.137
Years of teaching grade 6–12	0.37	0.90	.680
Math certification	0.63	0.95	.509
Middle school certification	0.01	0.65	.993
Special education certification	0.20	0.84	.808
Courses in math	0.19	0.30	.525
Courses in teaching math	–0.31	0.32	.333
Coaching with reduced teaching load	0.25	1.32	.849
Coaching in addition to teaching	–1.55	0.75*	.040
School			
Percentage of grade 4 students who passed state math test	0.03	0.02	.189
Percentage of students eligible for the federal school lunch program	–0.02	0.02	.167
Total number of students in grade 4	..	0.01	.977
Percentage of students who were Black	0.01	0.01	.600
Percentage of students who were Hispanic	0.02	0.02	.415
Variances			
School intercept	0.16	0.63	.795
Residual	9.81	1.05***	<.001
Intraclass correlation	0.02	na	na
Hedges' g^a (condition)	0.19	na	na
p value (condition)	.051	na	na
Adjusted means			
Treatment	20.23	na	na
Control	19.42	na	na

* Significant at $p < .05$; *** Significant at $p < .001$.

DMI is Developing Mathematical Ideas; na is not applicable; .. is a negligible value.

a. Hedges' *g* is an effect size index that shows the magnitude of the difference between treatment group and control group.

Note: Tests of fixed effects accounted for schools as the unit of analysis within the treatment condition. Analyses conducted with a mixed-model analysis of covariance used 77 degrees of freedom.

Source: Authors' analysis of primary data collected for the study; see appendix A.

2015. About a third of the teachers in the control condition (45 out of 135) indicated that they participated in some form of fractions professional development during the school year. Teachers in the control condition reported spending on average 5 hours on fractions professional development, compared with the 20 hours reported by teachers in the treatment condition.

While 129 teachers in the treatment condition participated in the DMI professional development (based on attendance data collected for the study), only 113 (88 percent) reported

Table D4. Sensitivity analyses for the impact of Developing Mathematical Ideas on the teacher posttest—Mathematical Knowledge for Teaching: Fractions scale, 2014/15

Model description	Hedges' g^a	*p* value
Two-level models		
Pretest covariate	0.16	.078
Eleven teacher-level covariates and five school-level covariates with math pretest covariate	0.19	.051
Three-level models with schools and districts		
Eleven teacher-level covariates and five school-level covariates with math pretest covariate and districts as random blocks	0.19	.051
Eleven teacher-level covariates and five school-level covariates with math pretest covariate and districts as fixed blocks	0.19	.060

Note: Analyses conducted with a mixed-model analysis of covariance used 82 degrees of freedom for the test of treatment in model 1, 77 degrees of freedom in model 2, 76 degrees of freedom in model 3, and 70 degrees of freedom in model 4.

Hedges' *g* is an effect size index that shows the magnitude of the difference between treatment group and control group.

Source: Authors' analysis of primary data collected for the study; see appendix A.

participating in a fractions professional development during the school year. Based on this underreporting by teachers in the treatment condition, it is possible that teachers in the control condition also underreported their participation in fractions professional development. Still, the self-report data from the eight monthly surveys show a contrast between teachers in the two conditions (table D5).

A larger percentage of teachers in both the treatment and control conditions reported participating in professional development in fractions during the months of the DMI professional development (September to December 2014) than during the month preceding (August 2014) or the months following (January to March 2015). While there was no statistically significant difference between the percentage of teachers in the two conditions who participated in fractions professional development during the months preceding DMI ($\chi^2(1) = 0.2466$, $p = .620$) or following DMI ($\chi^2(1) = 0.0215$, $p = .883$), there was a significant difference during the months DMI was offered ($\chi^2(1) = 106.4322$, $p < .001$).

Table D5. Topics and activities covered in the fractions professional development sessions attended by treatment and control teachers, 2014/15

Fractions professional development	Percentage of teachers before DMI (August 2014)		Percentage of teachers during DMI (September to December 2014)		Percentage of teachers after DMI (January to March 2015)	
	Treatment condition[a]	Control condition[b]	Treatment condition	Control condition	Treatment condition	Control condition
Had fractions professional development	9.91	11.97	87.60	24.44	13.95	13.33
Topics						
Understanding fractions as numbers[c]	6.31	9.40	82.95	20.00	10.08	11.85
Equivalence of fractions[c]	6.31	11.11	78.29	18.52	10.08	12.59
Adding and subtracting fractions[c]	5.41	7.69	75.19	18.52	10.08	8.15
Multiplying and dividing fractions[c]	3.60	5.13	77.52	10.37	9.30	8.15
Improper fractions/mixed numbers[c]	4.50	7.69	62.02	16.30	8.53	8.15
Activities						
Observed videos or live demonstrations of math teaching during the professional development session[c]	4.50	7.69	86.82	16.30	10.08	6.67
Worked with colleagues to plan or enhance math lessons/activities[c]	3.60	7.69	81.40	17.04	10.08	11.11
Discussed students' math work samples with colleagues to analyze how students think about and learn math[c]	0.90	4.27	84.50	8.89	10.08	5.93
Worked on math problems (either individually or with a partner)[c]	6.31	6.84	87.60	19.26	10.85	6.67
Engaged in group discussions about the math concepts being learned[c]	6.31	5.98	86.82	17.78	10.08	8.89
Implemented a math lesson or activity in my classroom[c]	0.90	1.71	83.72	12.59	6.20	7.41
Received feedback from the professional development staff about my understanding of mathematical ideas[c]	1.80	2.56	75.97	6.67	6.98	2.96
Learned how to use manipulatives (concrete objects) to teach math concepts[c]	5.41	8.55	63.57	16.30	9.30	10.37
Learned how to use visual representations to convey math concepts (such as number line, strip diagrams, pictorial representations, and so forth)[c]	4.50	7.69	84.50	17.04	8.53	7.41
Learned how to ask students questions and provide feedback[c]	2.70	4.27	78.29	11.11	6.20	5.93
Was required to observe demonstrations of math teaching in classroom settings	0.00	2.56	57.36	5.93	3.88	2.96
Developed math lessons/activities outside of the professional development session, for use in my classroom	0.90	2.56	55.81	6.67	6.20	3.70
Was observed in my classroom by a coach and received feedback	0.00	0.00	8.53	1.48	1.55	1.48

DMI is Developing Mathematical Ideas.

Note: Data are based on teacher responses to eight monthly surveys. A total of 129 treatment teachers and 135 control teachers responded to the monthly surveys. Control teacher response rate for the eight monthly surveys is as follows: survey 1 = 85.19 percent, survey 2 = 81.48 percent, survey 3 = 83.70 percent, survey 4 = 94.07 percent, survey 5 = 93.33 percent, survey 6 = 96.30 percent, survey 7 = 95.56 percent, survey 8 = 98.52 percent. Treatment teacher response rate for the eight monthly surveys is as follows: survey 1 = 89.15 percent, survey 2 = 86.05 percent, survey 3 = 86.05 percent, survey 4 = 98.45 percent, survey 5 = 99.22 percent, survey 6 = 97.67 percent, survey 7 = 96.90 percent, survey 8 = 96.12 percent.

a. Based on responses of 111 treatment teachers. Teachers from one school district started DMI in August 2014 and one school district began participation in the study after August 2014.

b. Based on responses of 117 control teachers. Teachers from one school district started DMI in August 2014 and one school district began participation in the study after August 2014.

c. These topics and activities were addressed in the DMI professional development program.

Source: Authors' analysis of primary data collected for the study; see appendix A.

Notes

1. In 2013 all states in the REL Southeast Region used the Common Core State Standards as their state standards. Their current state standards differ from the Common Core standards in some respects but continue to focus on fractions concepts in grade 4.
2. Math educators in the region have been concerned with poor student performance in algebra. For example, in 2014, the pass rate in the algebra 1 end-of-course assessment was 68 percent in Florida (Florida Department of Education, 2016).
3. Members of the REL Southeast Improving Mathematics Instruction Research Alliance from Georgia and South Carolina formed a work group to select a professional development program that stressed content knowledge of fractions and that provided teachers with instructional activities that could help their students develop understanding of key mathematical ideas. The work group included members of the alliance and, in some cases, their designees (who were state education agency math leaders responsible for math education in their states and math specialists). The study team solicited information from vendors that were able to provide professional development to the large number of schools the staff anticipated being involved in the study—about 40 schools with more than 100 grade 4 math teachers in three states. Only three vendors indicated that they had adequate capacity to conduct this type of large-scale professional development, and their programs focused primarily on building content knowledge.
4. The study team believes student attrition to be students missing at random. Teachers participated in the professional development effort, and students would have been unaware of the project. Any shifts in classroom instruction would have been subtle and probably undetectable for grade 4 students because textbooks and other curricular material would have stayed the same.

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