What's Happening

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Who repeats algebra I, and how does initial performance relate to improvement when the course is repeated?

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Key findings

This study in a large high school district in California found that:

- Some 44 percent of students repeated algebra I, and the repetition rates varied with student characteristics. The rates were highest among students in special education (69.6 percent), Hispanic students (61.1 percent), and English language learner students (56.7 percent).
- When repeating algebra I, lower performing students are likely to experience improvements in grades and California Standards Test scores while higher performing students are likely to experience improvements on some measures and declines on others. Overall, student achievement improves on average when students repeat the course.

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Summary

Many high school students repeat algebra I, but few studies have examined students' performance when they repeat the course. This study finds that students' achievement improves on average when they repeat algebra I. However, the level of improvement depends on their initial performance and the academic measure used to assess performance (course grades or standardized test scores). For instance, students with initial average algebra I course grades between a "B" and an "A" (more precisely, an algebra I grade point average greater than 3.0 and less than or equal to 4.0) earned lower course grades but higher standardized test scores when they repeated the course.

The idea for this study was developed in collaboration with the Silicon Valley Research Alliance, a group of school districts and researchers in Northern California focused on improving math achievement. The data were collected from the East Side Union High School District in California and five elementary school districts that feed into this high school district. A cohort of first-time grade 7 students in the 2006/07 school year was followed through 2011/12, at which point the students were expected to be in grade 12. Students were included in the analytic sample if data were available on their math course enrollment in the year before taking algebra I, the year they took algebra I, and the year after they took algebra I.

Of the 3,400 students in the sample, 44.3 percent repeated algebra I. While grades and standardized test scores are the most common reasons to retake the course, other considerations may also factor in, such as parent preferences and teacher or counselor recommendations, depending on the school and district. The rates of repeating varied across student characteristics, with students in special education (69.6 percent), Hispanic students (61.1 percent), and English language learner students (56.7 percent) exhibiting the highest rates. Many, but not all, students who repeated algebra I had performed poorly when they first took the course. For instance, among students whose initial algebra I grades averaged between a "B" and an "A", 8.4 percent repeated the course. And among students who scored "proficient" on the algebra I California Standards Test (CST) the first time they took algebra I, 22.2 percent repeated the course.

Students' performance improved on average by approximately half a letter grade and a little less than a third of a CST performance level when they repeated algebra I. But the data showed variation in improvement levels among higher achieving students. For instance, repeating students who initially received algebra I course grades averaging greater than a "C" (that is, greater than 2.0 on a numeric grading scale) had higher CST scores but lower course grades on average when they repeated the course. And students who initially scored "proficient" on the algebra I CST had higher grades but lower CST scores on average when they repeated the course. Students who initially did well in both course grades and the CST, defined as grades averaging at least a "C" and scoring at least "proficient" on the algebra I CST, had declines in CST performance and no statistically significant change in grades when they repeated the course.

These findings show that lower performing students are likely to see improvements in grades and CST scores when they repeat algebra I, while higher performing students are likely to see improvements on some measures of performance and declines on others when they repeat the course. The information on how students of varying achievement levels

perform when they repeat the course can aid educators who are making math placement decisions. More specifically, by giving a sense of how a student might perform if he or she were to repeat algebra I this information can help educators decide whether a student should repeat the course. Educators might also examine the course options available to students of different achievement levels and the processes by which students are placed in math courses.

Contents

Why this study?

Although many high school students repeat algebra I, a course widely considered to be the "gateway" to advanced high school math and science (U.S. Department of Education, 1997), few studies have examined students' performance when they repeat the course. Knowing how algebra I repeaters perform may help educators determine whether to promote a student to a higher math course (usually geometry) or have the student repeat algebra I when that student initially performs at a level that is less than ideal. This study seeks to help educators make informed math placement decisions.

Algebra I is often the first course in which students engage in the abstract reasoning and symbolism that make math powerful (Kieran, 1992; Vogel, 2008), marking an important step beyond arithmetic (Carraher & Schliemann, 2007; Kieran, 1989). Early success in algebra I leads students to take more advanced math courses (Atanda, 1999; Ma, 2000; Paul, 2005; Smith, 1996; Spielhagen, 2006) and achieve higher math scores (Gamoran & Hannigan, 2000; Kurlaender, Reardon, & Jackson, 2008; Smith, 1996).

Of students who first took algebra I in grade 8 39.2 percent repeated the course

However, many students struggle with algebra I. The National Mathematics Advisory Panel (2008, p. xii) concluded that "[a]lthough our students encounter difficulties with many aspects of mathematics, many observers of educational policy see Algebra as a central concern." Student difficulties with algebra I, documented in national and international assessments (Blume & Heckman, 1997; Schmidt, McKnight, Cogan, Jakwerth, & Houang, 1999), may cause students to repeat the course. Approximately 212,000 California students in grades 8–11 in 2008 repeated the algebra I California Standards Test (CST), implying that these 212,000 students were most likely repeating the algebra I course (EdSource, 2009, fig. 4).[1] EdSource identified as test repeaters 2 percent of grade 8 test takers, 38 percent of grade 9 test takers, 52 percent of grade 10 test takers, and 52 percent of grade 11 test takers. Box 1 summarizes the reasons that students may repeat algebra I.

Similarly, this study found that 39.2 percent of students who first took algebra I in grade 8 repeated the course (most likely in grade 9); in comparison, EdSource found that 38.4 percent of grade 9 algebra I CST test takers were repeating the test (most likely having first taken the test in grade 8). This study also found that 56.1 percent of students who first took algebra I in grade 9 repeated the course (most likely in grade 10); in comparison, EdSource found that 52.2 percent of grade 10 test takers were repeating the test (most likely having first taken the test in grade 9). One possible reason that the second finding is higher than EdSource's is that the high school district this study analyzed required students to take algebra I by grade 9,[2] which could have led to more students needing to repeat the course in grade 10. Many students across the state took algebra I for the first time in grade 10 or even grade 11 in 2008 (EdSource, 2009).

Repeating algebra I may be costly for both the student and the education system. Having to repeat a course may demoralize the student or turn the student off the subject, possibly resulting in the student not performing any better when he or she repeats the course. For instance, in a study of nine school districts in California, approximately half the students who were successful in algebra I in grade 8 and were placed in algebra I again in grade 9 received either the same or a lower grade in their second experience (Waterman, 2010).[3] Repeating students also miss the opportunity to take a new and different course. At the education system level the approximately 212,000 students in grades 8–11 who repeated

Box 1. What determines whether a student repeats algebra I?

The reasons students repeat algebra I likely vary by student, school, and district. Most districts rely on course grades and teacher recommendations to determine math course placement (Bitter & O'Day, 2010), with standardized math test scores, student and parent preferences, and counselor recommendations also factoring into the decision (Hallinan, 2003). Oakes, Muir, and Joseph (2000, p. 16) further note that "increasingly, school systems do not use fixed criteria to assign students to particular course levels. Teacher and counselor track-placement recommendations include, in addition to test scores and grades, highly subjective judgments about students' personalities, behavior, and motivation."

The district analyzed in this study (East Side Union High School District) uses various placement criteria, including course grades, CST scores, teacher recommendations, and participation in summer intervention programs (Flamm et al., 2011). Although the exact reason each particular student in this study's dataset did or did not repeat algebra I cannot be ascertained, the study's results show that low student performance (measured by course grades and CST scores) correlates with repeating.

algebra I in 2008 caused California to devote the equivalent of approximately 1,695 full-time algebra I teachers to reteaching the course (EdSource, 2009).[4]

This report shows how students perform when they repeat algebra I and how the level of improvement varies depending on initial course performance and the academic measure

The information provided in this report shows how students perform when they repeat algebra I and how the level of improvement varies depending on initial course performance and the academic measure (course grades or CST scores). This information can help educators determine whether a student should repeat the course.

What the study examined

The study examined three research questions:

- How many students repeat algebra I after taking it for the first time?
- How do student characteristics (such as race/ethnicity, gender, grade 7 math performance, and initial algebra I performance) relate to the likelihood of repeating algebra I?
- How well do students perform when they repeat algebra I compared with the first time they took the course? How does that difference in performance vary based on student characteristics?

The data for this study are from the East Side Union High School District in California and five elementary school districts that feed into the East Side Union High School District: Alum Rock Union Elementary School District, Evergreen School District, Franklin-McKinley Elementary School District, Mt. Pleasant Elementary School District, and Oak Grove Elementary School District. Data from the elementary feeder districts, which span from kindergarten through grade 8, were collected for the 2005/06–2008/09 school years, and data from the high school district, which spans from grade 9 through grade 12, were collected for the 2007/08–2011/12 school years. Student-level longitudinal data, collected from each district, include variables such as identification number, race/ethnicity, gender, math course name, final course letter grade received, math CST taken, CST scale score, and CST performance level (see appendix A for more detailed information about the dataset).

Construction of the sample began with a cohort of first-time grade 7 students in 2006/07 who attended one of the five feeder districts included in the analysis.[5] Because this study

seeks to identify students who repeat algebra I, the sample included only students for whom data were available indicating the student's math course enrollment in the years before and after taking algebra I. For instance, if a student took algebra I in 2008/09, but data were not available on the student's math course enrollments in 2009/10, it could not be determined whether the student repeated algebra I in 2009/10. Similarly, if a student took algebra I in 2008/09 (and, for instance, pre-algebra in 2006/07 and geometry in 2009/10), but the student's math course enrollments in 2007/08 could not be observed, it could not be determined whether the student was repeating algebra I in 2008/09 or taking it for the first time. Additional information is provided in appendix A. Box 2 summarizes how students who were repeating algebra I were identified.

Box 2. Identifying students who repeat algebra I

To identify students who repeated algebra I, the study team first needed to identify instances of algebra I enrollment. A student who took a one-year algebra I course was defined as having taken algebra I. A student enrolled in the first year of a two-year algebra I sequence was not considered to have taken algebra I in that year, but a student enrolled in the second year of a two-year algebra I sequence was considered to have taken algebra I in that year. The two-year algebra I courses were confirmed with each of the school districts participating in the study. More specifically, the study team confirmed with each district whether successful students in each course with "algebra" in the title should have been promoted to geometry. Algebra I courses that did not promote students to geometry the following year even when the students performed very well were not defined as algebra I and were assumed to be two-year algebra I sequences.

Because students in the dataset sometimes enrolled in a math course for only a portion of the school year, a student was defined as having taken algebra I when he or she received a letter grade for the course for some portion of the second half of the school year (for example, the third or fourth quarter in a quarter system or the second semester in a semester system). For instance, a student who received a letter grade in algebra I for the third quarter (out of a possible four quarters) was defined as having taken algebra I in that school year. However, a student who received a letter grade in algebra I for the second quarter before dropping out (and then, for instance, enrolled in a different math course for the rest of that school year) was defined as not having taken algebra I in that school year. These students were likely misplaced in the first half of the school year and so should not be considered as having taken algebra I (or as having repeated it if they took algebra I in the following year). Because algebra I is a cumulative course, a student who took only the first half of the course did not experience the full year of algebra I. The study results would not have been very different if a student who received a grade in the first half of the school year was defined as taking algebra I—the analytic sample would have been 9 percent smaller, and the rate of repeating algebra I would have increased from 44.26 percent to 45.77 percent.

A student who enrolled in algebra I for two consecutive school years (where enrollment in a given year was defined as above) was considered to have repeated algebra I. A student could be considered an algebra I repeater in any of the grades from 8 through 12. A student who enrolled in the second year of a two-year algebra I sequence in one year and then enrolled in a one-year algebra I course the following year was considered to have repeated algebra I. A student who enrolled in algebra I in one year and then enrolled in both algebra I and geometry concurrently the following year was also considered to have repeated algebra I. A student who enrolled in algebra I in one school year then enrolled in algebra I the following summer was also considered to have repeated algebra I. There were 175 summer repeaters in the dataset (representing 11.6 percent of all repeaters). Appendix B provides results for the summer repeaters only, which are similar to the results for all algebra I repeaters.

What the study found

Rates of repeating algebra I varied based on student characteristics. Not all repeaters were low-performing students. Although achievement in algebra I improved on average when students repeated the course, students who initially performed well in the course improved on some measures but performed worse on others the second time around.

Nearly 45 percent of students repeated algebra I—and percentages were nearly 70 percent for some subgroups

Some 44.3 percent of students in the study sample repeated algebra I. The rates of repeating algebra I varied by student characteristic (table 1) and were highest among students in special education (69.6 percent), students with more than 18 absences in the year they

Not all algebra I repeaters were low-performing students

Table 1. Algebra I repeating rates by student characteristic

Student characteristic	Number of repeaters	Percent repeating
Total	1,505	44.3
Race/ethnicity		
American Indian/Alaska Native	[a]	[a]
Asian	307	24.0**
Black	[a]	[a]
Hispanic	1,005	61.1**
White	126	35.7**
Gender		
Female	691	41.2**
Male	814	47.3**
English language learner status		
English language learner student	548	56.7**
Not an English language learner student	957	39.3**
Eligibility for free or reduced-price lunch		
Eligible	1,106	51.3**
Not eligible	399	32.1**
Special education status		
In special education	151	69.6**
Not in special education	1,354	42.5**
Average course grade in grade 7 math[b,c,d]		
Between “F” and “D”	337	72.8**
Between “D” and “C”	351	60.3**
Between “C” and “B”	375	46.1**
Between “B” and “A”	192	22.2**
Performance level on grade 7 math CST[b,d]		
Far below basic	132	74.2**
Below basic	408	67.0**
Basic	550	58.3**
Proficient	317	37.3**
Advanced	76	13.4**

(continued)

Table 1. Algebra I repeating rates by student characteristic *(continued)*

Student characteristic	Number of repeaters	Percent repeating
Grade level when the student first took algebra I		
7	[a]	[a]
8	627	39.2**
9	866	56.1**
10	[a]	[a]
11	[a]	[a]
Average course grade when the student first took algebra I[c,d]		
Between “F” and “D”	802	89.4**
Between “D” and “C”	409	57.4**
Between “C” and “B”	207	27.1**
Between “B” and “A”	82	8.4**
Performance level on the CST when the student first took algebra I[d]		
Far below basic	227	83.8**
Below basic	553	69.0**
Basic	347	49.3**
Proficient	155	22.2**
Advanced	16	2.9**
Passing status when the student first took algebra I		
Initially passed algebra I with a “D–” or better	930	33.7**
Course grade and CST performance level when the student first took algebra I		
Had an average grade of at least a “C” and scored “proficient” or “advanced” on the algebra I CST when the student first took algebra I	52	5.3**
Number of absences when the student first took algebra I		
0 or 1	192	24.3**
2–4	357	41.3**
5–9	388	57.3**
10–18	267	60.3**
More than 18	158	69.0**
No data	143	36.1**
Algebra I requirement for grade 8 students		
Required for all grade 8 students	95	47.0
Not required for all grade 8 students	1,410	44.1

** Differences within a subgroup are statistically significant at the .01 level.

a. Suppressed to reduce risk of disclosure because there were fewer than 10 students in one of the subgroups.

b. Excludes 229 students who took algebra I in grade 7.

c. See appendix A for details on how letter grades were converted to a numeric scale.

d. Number of repeaters does not sum to 1,505 because some students were missing data on course grade or CST performance level for grade 7 math or when they first took algebra I.

CST is California Standards Test.

Note: *n* = 3,400. Tests of statistical significance were conducted using Pearson’s chi-squared test of independence, which tests the null hypothesis that there is no relationship between repeating algebra I and a given student characteristic.

Source: Authors’ analysis of data obtained from the school districts included in this study.

first took algebra I (69.0 percent), Hispanic students (61.1 percent), and English language learner students (56.7 percent). The lowest rates were among Asian students (24.0 percent), students with no more than one absence in the year they took algebra I (24.3 percent), and students who were not eligible for free or reduced-price lunch (32.1 percent).

Low-performing students were more likely to repeat algebra I, but even students with "B" grades and students with "proficient" scores repeated the course

Students who performed poorly in their grade 7 math class (receiving average grades between an "F" and a "D" or scoring "far below basic" on the CST) had the highest rates of repeating algebra I.[6] Some 72.8 percent of students whose grade 7 math grades averaged between an "F" and a "D" repeated the course, compared with 22.2 percent of students whose grades averaged between a "B" and an "A" (see box 3 for details on how letter grades were converted to a numeric scale). A similar pattern is observed for grade 7 math CST performance levels: 74.2 percent of students scoring "far below basic" repeated algebra I. The rate of repeating algebra I fell with each successive improvement in CST performance level, with the highest performing students (scoring "advanced") repeating at a rate of 13.4 percent.

Students who had low initial algebra I course grades were most likely to repeat the course, and the rate of repeating fell as students did better in the course

Students who had low initial algebra I course grades or low algebra I CST scores were most likely to repeat the course. For instance, 89.4 percent of the students whose initial algebra I grades averaged between an "F" and a "D" repeated the course. The rate of repeating fell as students did better in the course, but even students with initial grades that averaged between a "B" and an "A" repeated at a rate of 8.4 percent.

A similar pattern emerges with the algebra I CST results. Some 83.8 percent of students who scored "far below basic" on the CST when they first took algebra I repeated the course. The rate of repeating fell with each successive improvement in performance level, with students scoring "advanced" repeating at a rate of 2.9 percent. And among high-achieving students who had average course grades of at least a "C" and scored either "proficient" or "advanced" on the CST, the rate of repeating was 5.3 percent.

Box 3. Calculating and defining student grade point averages

To report academic achievement based on course grades, course letter grades were converted to a numeric scale, which were then used to calculate grade point averages. Letter grades were converted to a numeric scale as follows: "A" = 4.0, "A–" = 3.67, "B+" = 3.33, "B" = 3.0, "B–" = 2.67, "C+" = 2.33, "C" = 2.0, "C–" = 1.67, "D+" = 1.33, "D" = 1.0, "D–" = .67, "F+" = .33, "F" = 0.0. The numeric grade equivalents were then averaged for a given course and school year to get a student's grade point average for the course.

Students' grade point averages are categorized into groups throughout the report as follows: "Between 'F' and 'D'," "Between 'D' and 'C'," "Between 'C' and 'B'," and "Between 'B' and 'A'." "Between 'F' and 'D'" is defined as students having average grades greater than or equal to 0.0 and less than or equal to 1.0, "Between 'D' and 'C'" is defined as students having average grades greater than 1.0 and less than or equal to 2.0, "Between 'C' and 'B'" is defined as students having average grades greater than 2.0 and less than or equal to 3.0, and "Between 'B' and 'A'" is defined as students having average grades greater than 3.0 and less than or equal to 4.0.

The rate of repeating among students who attended a school that required algebra I for all grade 8 students (August Boeger Middle School in Mt. Pleasant Elementary School District) was 47.0 percent, compared with 44.1 percent among students who did not attend such a school.[7] However, the difference between these two groups was not statistically significant.

The study team also conducted a mixed-effects logistic regression to isolate the relationship between various student characteristics and the probability of repeating algebra I, while holding other student characteristics constant. See appendix B for a detailed description of this multivariate analysis.

Improvement when students repeated algebra I varied based on initial course performance and academic measure

Overall, 74.1 percent of the students passed algebra I with an average grade of "D–" or better when they attempted algebra I for the second time

Overall, 74.1 percent of the students passed algebra I with an average grade of "D–" or better when they attempted algebra I for the second time (table 2).[8] More than a third of repeaters received grades that averaged between an "F" and a "D," and more than half scored either "below basic" or "far below basic" on the algebra I CST in their second attempt.[9]

The average amount of improvement when repeating algebra I was approximately half a letter grade and 0.28 CST performance level (table 3). Students earned grades that averaged "C–" when they repeated algebra I, compared with an average of "D+" when they first took the course. On the algebra I CST, students improved in performance level on average from 2.3 when they first took the course to 2.6 when they repeated the course.

Although achievement in algebra I improved on average when students repeated the course, the amount of improvement varied depending on the students' initial course performance (table 4). Students who initially received a low grade in the course received a

Table 2. Algebra I performance when students took the course for the second time

Academic performance	Number	Percent
Passed the course with a "D–" or better	1,115	74.1
Average course grade in algebra I		
Between "F" and "D"	541	36.0
Between "D" and "C"	411	27.3
Between "C" and "B"	328	21.8
Between "B" and "A"	225	15.0
Algebra I CST performance level [a]		
Far below basic	163	14.0
Below basic	428	36.6
Basic	361	30.9
Proficient	193	16.5
Advanced	23	2.0

a. Number of repeaters does not sum to 1,505 because some students were missing data on CST performance level when they first repeated algebra I.

CST is California Standards Test.

Note: *n* = 1,505.

Source: Authors' analysis of data obtained from the school districts included in this study.

Table 3. Improvement in performance between the first and second time taking algebra I

Improvement in algebra I	Mean	Median	Standard deviation
Course grade (*n* = 1,500)	0.49**	0.5	1.25
CST scale score (*n* = 1,002)	14.0**	15.0	38.99
CST performance level (*n* = 1,002)	0.28**	0.0	0.86

** Significant at the .01 level.

CST is California Standards Test.

Note: Paired *t*-tests were used to determine whether the level of improvement between the first and second time taking algebra I was statistically significant. Algebra I CST performance levels were converted to a numeric scale as follows: "far below basic" = 1, "below basic" = 2, "basic" = 3, "proficient" = 4, and "advanced" = 5. Average improvements in the CST performance levels are reported on this numeric scale. Number of repeaters for each area of improvement is not 1,505 because some students were missing data on course grade, CST scale score, or CST performance level for the first or second time they took algebra I.

Source: Authors' analysis of data obtained from the school districts included in this study.

higher grade on average when they repeated, but students who initially received a high grade in the course received a lower grade on average when they repeated.

Students who earned grades that averaged between an "F" and a "D" when they first took algebra I improved by almost a full letter grade on average when they repeated the course

Among students who earned grades that averaged between an "F" and a "D" when they first took algebra I, there was a statistically significant average improvement of 0.94 in course grades (students improved by almost a full letter grade on average when they repeated the course). Students who initially earned letter grades averaging between a "D" and a "C" had a statistically significant increase of about a third of a letter grade when repeating the course (0.36). But students who earned letter grades averaging between a "C" and a "B" on their initial algebra I attempt had a statistically significant decrease of 0.41 in their letter grades on average when repeating the course. And students who earned letter grades averaging between a "B" and an "A" when they first took algebra I earned almost a full letter grade lower on average (–0.93) the second time, a statistically significant decline.

A similar picture emerges when examining CST improvement based on initial CST performance. Students who initially scored "far below basic" or "below basic" had statistically significant improvements in CST performance on average when retaking the course, but students who had already demonstrated proficiency had statistically significant declines on average when retaking the course.

Students who scored "far below basic" improved their performance on average by almost a full performance level (0.92), and students who scored "below basic" improved their performance by almost a third (0.29) of a performance level. But students who scored "proficient" when they first took algebra I on average had a decline of almost a quarter of a performance level (–0.23). Students who initially scored "advanced" had a decline of almost three-quarters of a performance level (–0.73) when they repeated the course (this result should be interpreted with caution due to the low sample size [11 students], but the decline was statistically significant).

One possible explanation for the finding that students who earn high grades (or high CST scores) on their first attempt earn lower grades (or lower CST scores) on their second attempt is that these students experience a regression to the mean—the tendency of extreme values to move closer to the mean when measured a second time. (Another

Table 4. Average improvement in performance between the first and second time taking algebra I, by student characteristic

Student characteristic	Improvement in course grade	Improvement in CST performance level
Grade level when the student first took algebra I		
7	[a]	[a]
8	0.10 (*n* = 627)	0.18** (*n* = 457)
9	0.77** (*n* = 866)	0.36** (*n* = 540)
10	[a]	[a]
Average course grade when the student first took algebra I[b]		
Between "F" and "D"	0.94** (*n* = 802)	0.32** (*n* = 581)
Between "D" and "C"	0.36** (*n* = 409)	0.18** (*n* = 277)
Between "C" and "B"	–0.41** (*n* = 207)	0.27** (*n* = 106)
Between "B" and "A"	–0.93** (*n* = 82)	0.34* (*n* = 38)
CST performance level when the student first took algebra I[b]		
Far below basic	0.51** (*n* = 227)	0.92* (*n* = 193)
Below basic	0.43** (*n* = 553)	0.29** (*n* = 437)
Basic	0.73** (*n* = 347)	0.02 (*n* = 249)
Proficient	0.79** (*n* = 155)	–0.23** (*n* = 112)
Advanced	0.27 (*n* = 15)	–0.73* (*n* = 11)
Course grade and CST performance level when the student first took algebra I		
Had an average grade of at least a "C" and scored "proficient" or "advanced" on the algebra I CST when the student first took algebra I	–0.21 (*n* = 52)	–0.49** (*n* = 43)
Algebra I requirement for grade 8 students[b]		
Required for all grade 8 students	–0.50** (*n* = 95)	0.10 (*n* = 68)
Not required for all grade 8 students	0.56** (*n* = 1,405)	0.29** (*n* = 934)

* Significant at the .05 level; ** significant at the .01 level.

a. Suppressed to reduce risk of disclosure because there were fewer than 10 students in this subgroup.

b. Number of repeaters does not sum to 1,505 because some students were missing data on algebra I course grade or CST performance level when they took algebra I the first or second time.

CST is California Standards Test.

Note: Paired *t*-tests were used to determine whether the level of improvement between the first and second time taking algebra I was statistically significant. CST performance levels were converted to a numeric scale as follows: "far below basic" = 1, "below basic" = 2, "basic" = 3, "proficient" = 4, and "advanced" = 5. The number of students differs for the counts of course grades versus performance levels because not all students took the algebra I CST both the first and second time they took algebra I.

Source: Authors' analysis of data obtained from the school districts included in this study.

reason could be that student performance may decline as students get older, regardless of the course taken.) Regression to the mean would result in both students with lower grades and students with higher grades earning grades that are closer to the average when they repeat the course.[10] But to be sure that regression to the mean is the cause of high-performing students declining in achievement when they repeat the course, a more rigorous study, such as a randomized controlled trial incorporating a comparison group in the design, would need to be conducted.

To provide more information about the possibility of regression to the mean in these findings, the study team examined improvement levels in algebra I course grades based on initial CST scores and improvement levels on the algebra I CST based on initial course grades (see table 4). Analyzing improvement in course grades based on initial CST performance or analyzing improvement in CST performance based on initial course grades is less likely to be affected by regression to the mean because an alternative measure of achievement is being used. The results suggest that regression to the mean could be taking place. All student groups improved on some measure when repeating the course. Students who initially scored "far below basic" on the algebra I CST had an increase of about half a letter grade on average (0.51) when they repeated the course. Students who initially scored "proficient" had an increase of over three-quarters of a letter grade on average (0.79) when they repeated the course, even though their CST scores fell. Students who scored "advanced" also had increases in average letter grades, but the increase was not statistically significant.

The results of this study can guide educators who are deciding whether a student should repeat algebra I

CST scores rose for all student groups when disaggregated by initial course grade. For instance, students who initially had average algebra I course grades between "F" and "D" had a 0.32 average increase in algebra I CST performance levels when they repeated the course. Similarly, students with initial average grades between "B" and "A" had a 0.34 average increase.

Students who initially performed well in both academic measures (had an average grade of at least a "C" in the course and scored "proficient" or "advanced" on the algebra I CST) tended to perform worse when they repeated the course. These students had a statistically significant decline of almost half a CST performance level (0.49) when they repeated the course. They also had a decline of 0.21 in average course grades when they repeated, but this decline was not statistically significant.

Students who attended a school that required algebra I for all grade 8 students had a decline in course grades on average (–0.50) when they repeated the course, but students who did not attend such a school had an improvement of approximately half a grade (0.56) when repeating.

Study implications and suggested next steps

The results of this study can guide educators who are deciding whether a student should repeat algebra I. The findings show that students who initially performed poorly in algebra I improve on average when they repeat the course. But students who initially performed better in algebra I experienced improvements on some academic measures and declines on others. The results from table 4 can be informative in understanding how students may perform if they repeat algebra I. For instance, a student with initial algebra I grades between "C" and "B" may experience a decline in average grades of approximately 0.4

(almost half of a grade) but an improvement on average in CST performance levels of approximately 0.3. Equipped with this information, educators can then decide whether a student with this particular prior achievement history should repeat the course.

This study replicates a key finding in Waterman (2010) that many students who initially perform well in algebra I earn lower grades when they repeat the course. However, analysis of variation in improvement levels disaggregated by course grades and CST scores suggests that Waterman's finding could be due to regression to the mean. Although these results may weaken Waterman's conclusion that students who initially earn high grades should not repeat algebra I, only a more rigorous study (such as a randomized controlled trial) can provide a definitive answer.

While this study answered some questions, other remain. To better understand why students—especially high-performing students—repeated algebra I, interviews could be conducted with educators at each of the high schools analyzed.[11] For instance, repeating students may not have grasped certain content standards that the educators considered critical for success in future math classes. A further study could analyze student performance on these content standards when students repeated the course.

The socioemotional impacts of repeating algebra I could also be examined. For instance, how did repeating the course affect students' motivation, self-confidence, and attitudes toward math?

Future research could also examine how students who do or do not repeat algebra I perform in future math classes, particularly algebra II. This could involve comparing the outcomes on algebra II end-of-course tests such as the algebra II CST.[12] This focus on future student achievement may indirectly incorporate socioemotional impacts, since student motivation and self-confidence can affect student achievement in the more challenging course.

Appendix A. Detailed information about the dataset and analysis

This appendix provides detailed information about the dataset, including student demographics, rules for including students, procedures for calculating average grades, and data strengths and limitations.

Detailed information about the dataset

Study data were provided by the East Side Union High School District and five of its seven elementary feeder school districts: Alum Rock Union Elementary School District, Evergreen School District, Franklin-McKinley Elementary School District, Mt. Pleasant Elementary School District, and Oak Grove Elementary School District. (Berryessa Union Elementary School District and Orchard Elementary School District also feed into the East Side Union High School District, but the study team was unable to obtain the necessary data from these districts to include them.) The East Side Union High School District covers grades 9–12, while the elementary feeder school districts cover kindergarten–grade 8. The California State Department of Education provides a unique identification number for each student in the state's traditional public, alternative, and charter schools. Students can be linked from district to district with this state identification number.

Most of the feeder school districts provided data covering the school years 2005/06–2008/09, but one feeder school district (Franklin-McKinley Elementary School District) provided data only for 2006/07–2008/09. East Side Union High School District provided a dataset covering 2007/08–2011/12. The feeder school districts provided data for all students in grades 6–8, whereas the high school district provided data for all students in grades 9–12.

The dataset contains demographic information, including race/ethnicity, gender, English language learner status, eligibility for free or reduced-price lunch, special education status, and grade level. It also contains math course information, including course name, California Basic Educational Data System course number (standardized four-digit codes used by all public schools in California to reflect the curriculum covered in each course), course letter grade for each term in the school year, the math California Standards Test (CST) taken, the CST scale score, the CST performance level, and the school year each CST was administered. The dataset also includes behavioral data such as number of days absent in the school year and a suspension indicator.

Some data elements were not provided because the data did not exist during the school years requested or could not be located:

- Alum Rock Union Elementary School District did not provide the California Basic Educational Data System course number for 2005/06–2007/08.
- Evergreen School District did not provide suspension data or California Basic Educational Data System course numbers for any years requested.
- Franklin-McKinley Elementary School District did not provide data for 2005/06 or California Basic Educational Data System course numbers and course grades for 2006/07.
- Mt. Pleasant Elementary School District did not provide suspension or California Basic Educational Data System course numbers for any years requested.

- Oak Grove Elementary School District did not provide data on special education status or suspension data for any years requested, data on eligibility for free or reduced-price lunch for 2005/06 and 2006/07, or CST information for 2005/06.

Because so few districts provided suspension data, the study team could not examine the rate of repeating algebra I based on whether a student had ever been suspended from school.

Although the dataset spans the school years 2005/06–2011/12, the analysis in this report examines only 2006/07–2011/12. The sample follows one cohort of first-time grade 7 students in 2006/07 who were enrolled in one of the five feeder school districts and attended one of the comprehensive high schools in the East Side Union High School District. A student who took algebra I in grade 7 and a non–algebra I math course in grade 8 was included in the sample, regardless of whether the student was observed in any comprehensive high school in grade 9.

The sample started with 5,391 first-time grade 7 students in 2006/07, comprising 1,356 students from Alum Rock Union Elementary School District, 1,400 students from Evergreen School District, 984 students from Franklin-McKinley Elementary School District, 337 students from Mt. Pleasant Elementary School District, and 1,314 students from Oak Grove Elementary School District. (Four grade 7 students in the 2006/07 school year who were repeating the entire grade level were not included in the sample.)

The following students were dropped from the sample:

- 87 students who did not have a state student identification number.
- 65 students with multiple district or state identification numbers.
- 68 students in math courses with no math course grade for any school term or school year.[13]
- 791 students without data showing that they took algebra I in any middle school or comprehensive high school.
- 980 students without data showing math course enrollment in the years before or after taking algebra I.

Among the 980 students with missing or incomplete data, 58 did not have a math course in the year before taking algebra I, 761 students did not have a math course in the year after taking algebra I, and 161 students did not have a math course in the years before or after taking algebra I. (This last category includes some students with no math course in the first year, algebra I in the second year, algebra I in the third year, and no math course in the fourth year.) Furthermore, 43 of the 58 students who did not have a math course in the year before taking algebra I later repeated algebra I; 220 of the 761 students who did not have a math course in the year after taking an algebra I course were observed repeating algebra I (for instance, they could have taken algebra I, then algebra I again the following year, and then no math course in the year after that); and 66 of the 161 students who did not have a math course in the years before or after taking an algebra I course repeated algebra I.

If the students with missing data who were observed repeating algebra I (43 plus 220 plus 66 = 329 students) were added to the analytic sample, the algebra I repeating rate would have increased from 44.3 percent to 49.2 percent. However, adding only the students with

incomplete data who were also observed repeating algebra I (but not adding all 980 students with incomplete data) would artificially inflate the repeating rate. If all 980 students with incomplete data were included in the analytic sample, the repeating rate would have decreased from 44.3 percent to 41.9 percent. Because it cannot be ascertained whether many of the students with incomplete data repeated algebra I, and because one of the main goals of this study was to as accurately as possible calculate the rate of repeating algebra I, all 980 students with incomplete data were excluded from the analysis.

There are 229 students in the analytic sample who took algebra I in grade 7. These students are excluded in the results that report on the rate of repeating algebra I based on grade 7 math CST and course performance.

The final analytic sample included 3,400 students, with 632 students from Alum Rock Union Elementary School District, 1,049 students from Evergreen School District, 595 students from Franklin-McKinley Elementary School District, 208 students from Mt. Pleasant Elementary School District, and 916 students from Oak Grove Elementary School District.

A major strength of this dataset is that it includes a statewide student identification number that follows students from elementary school district to high school district. This made it possible to observe course enrollments and course performance over six years.

However, one key limitation of the dataset (and of this study) is that it does not provide the exact reason each student repeated algebra I. Although most students likely repeated the course because of poor initial course performance, some students repeated the course even though they had earned average grades better than "C." Unfortunately, schools and districts do not record administrators' and educators' reasons for having students repeat algebra I.

Data analysis

Calculating algebra I and grade 7 math course grade point averages. Each student's algebra I and grade 7 math grade point average for each school year were calculated based on the student's grades in the course from all terms. Most school districts provided grades for each term.[14] The procedure for calculating an average grade for a group of students is as follows:

1. Each student's letter grades were converted to a numeric scale: "A" = 4.0, "A–" = 3.67, "B+" = 3.33, "B" = 3.0, "B–" = 2.67, "C+" = 2.33, "C" = 2.0, "C–" = 1.67, "D+" = 1.33, "D" = 1.0, "D–" = .67, "F+" = .33, "F" = 0.0.

2. Each student's grades were averaged over multiple terms for each course to come up with a single numeric course grade for the student. A grade averaging between a "B" and an "A" is equivalent to a grade averaging more than 3.0 and less than or equal to 4.0 on the numeric scale, a grade averaging between a "C" and a "B" is equivalent to a grade averaging more than 2.0 and less than or equal to 3.0, and so forth.

3. The students' course grades were averaged to arrive at the group's course grade.

Each individual student's course grade point average is thus the sum of the student's numeric scale grades for all terms divided by the number of terms, and the course grade

point average for a group of students is the sum of all students' numeric course grade point averages divided by the number of students.

Defining an algebra I course when there was no California Basic Educational Data System number. The California Basic Educational Data System number identifies courses. For instance, number 2403 identifies the beginning algebra/algebra I (one-year) course, and number 2428 identifies the beginning algebra part 1 (first year of a two-year) course. At the high school level, 95 percent of the math courses included a California Basic Educational Data System number. Courses without a number were identified with names that included the word "algebra" and defined as algebra I courses except in cases where the course name also included the terms "II," "geometry," "analysis," or "intermediate."

Most math courses in the feeder school districts did not have California Basic Educational Data System numbers. The instructional services division or the educational services department in each district was consulted to identify the algebra I courses based on course names. One- and two-year algebra courses were distinguished.

Within the Evergreen School District there was a class titled "Algebra Basic" that the district had identified as the first year of a two-year algebra I sequence (the district's one-year algebra I course was titled "Algebra Adv"). Students who completed the "Algebra Basic" course in Evergreen were expected to take a one-year algebra I course in grade 9. Among the 570 students that took "Algebra Basic" in Evergreen, 568 took the grade 8 general mathematics CST, and 2 took the algebra I CST.

However, 37 students in Evergreen who took "Algebra Basic" in grade 8 did not take algebra I in grade 9; they instead took geometry. For these 37 students to be included in the analytic sample, they would have needed to be defined as having taken the full algebra I course in the year they took "Algebra Basic." The other option would have been to define "Algebra Basic" as a full-year algebra I course for all Evergreen students.[15] But this option did not seem correct given that the district had identified this course as year one of a two-year algebra I sequence and that almost all the students who took this course took the grade 8 general mathematics CST. If the "Algebra Basic" course in Evergreen had been defined as a full-year algebra I course, the overall rate of repeating algebra I would have risen by approximately 8 percentage points to 52.0 percent.

The final analytic sample included 819 students who took a course with the word "algebra" in its title that the study team did not consider a full-year algebra I course. Among these 819 students, 801 took the grade 8 general mathematics CST, 14 took the algebra I CST, and 4 did not take any math CST. All 819 students took the full-year algebra I course the following year in grade 9, and 215 of them earned average course grades of better than a "B" and scored either "proficient" or "advanced" on the CST. The fact that these 215 students had done well in their course with the word "algebra" in its title but still needed to take a full-year algebra I course the following year provides assurance that courses with the word "algebra" in the title were correctly coded as not being full-year algebra I courses.

Defining demographic characteristics. Additional rules for defining student demographic characteristics were as follows:

- The Asian ethnicity category includes Filipino students.

- English language learner students include all students who were classified as an English language learner in any of the school years under observation.
- Students eligible for free or reduced-price lunch include all students eligible for free or reduced-price lunch in any of the school years under observation.
- Students in special education include all students in special education in any of the school years under observation.

Appendix B. The multivariate analysis, multiple repeaters, and summer repeaters

To isolate the relationship between various student characteristics and the probability of repeating algebra I while holding other characteristics constant, a mixed-effects logistic regression was performed. A logistic regression was used because the dependent variable was binary (repeated algebra I or did not repeat algebra I). Mixed effects were incorporated into the model to account for the fact that each high school may have its own way of determining which students repeat algebra I. The mixed-effects logistic regression used was of the following form:

$$\Pr(Repeater_{is} = 1) = \text{logit}^{-1}(\beta_0 + \beta_1' Characteristics_{is} + \zeta_s + \varepsilon_{is}),$$

where *Repeater* is a dichotomous variable indicating that the student repeated algebra I, β_0 is the intercept, β_1 is a vector of parameters to be estimated from the data, *Characteristics* is a vector of student characteristics,[16] ζ is the school-level random effects (in this case a random intercept for each high school),[17] and ε is the independent and identically distributed error term. Subscript *i* refers to the student and subscript *s* refers to the high school that the student attended. Odds ratios are reported for ease of interpretation.

The results of the mixed-effects logistic regression show how the odds of repeating algebra I vary by student characteristic (table B1). The omitted categories in the model are Hispanic students, students whose average grade 7 math grade was between a "B" and an "A," students who scored "advanced" on the grade 7 CST, students with zero or one absence in grade 7, students whose average algebra I grade when they first took the course was between a "B" and an "A," and students who scored "advanced" on the algebra I CST when they first took the course.

Among the student characteristics included in the regression model, the variable that correlated most strongly with repeating algebra I was the student's academic performance when he or she first took algebra I. The odds of repeating algebra I for students who scored "far below basic" on the algebra I CST the first time they took the course were 11.34 times the odds for students who scored "advanced." Similarly, the odds for students who scored "below basic" were 10.71 times the odds for students who scored "advanced." A student's average algebra I grade tells a similar story: The odds of repeating algebra I for students whose grades were between an "F" and a "D" were 53.07 times the odds of students whose grades were between a "B" and an "A."

Students who repeated algebra I multiple times

Some 437 students repeated algebra I twice (took the course three times), 51 students repeated algebra I three times, and 4 students repeated algebra I four times. In the results that follow, all repeating outcomes are combined (the results from the first time repeating algebra I are presented in the main text).

When students repeated algebra I multiple times, the modal average course grade was between an "F" and a "D" (38.6 percent; table B2). The modal CST performance level was "below basic" (45.5 percent). These modal patterns are the same as those for when the student repeated algebra I for the first time (see table 2 in the main text).

Table B1. Random intercept logistic regression relating student characteristics to repeating algebra I

Fixed effect	Odds ratio
Race/ethnicity	
American Indian/Alaska Native	1.000
Asian	0.546**
Black	0.501*
White	0.549**
Gender	
Female	0.979
English language learner status	
English language learner student	0.920
Free or reduced-price lunch eligibility	
Eligible	1.028
Special education status	
In special education	2.469**
Average course grade in grade 7 math	
Between “F” and “D”	0.920
Between “D” and “C”	1.005
Between “C” and “B”	0.839
Performance level on the grade 7 math CST	
Far below basic	0.858
Below basic	0.925
Basic	1.339
Proficient	1.158
Number of absences in grade 7	
2–4	0.850
5–9	1.142
10–18	1.087
More than 18	1.017
Average course grade when the student first took algebra I	
Between “F” and “D”	53.070**
Between “D” and “C”	9.760**
Between “C” and “B”	2.165**
Performance level on the CST when the student first took algebra I	
Far below basic	11.340**
Below basic	10.710**
Basic	7.448**
Proficient	3.792**
School random effect	0.09

* Significant at the .05 level; ** significant at the .01 level.

CST is California Standards Test.

Note: *n* = 2,582.

Source: Authors’ analysis of data obtained from the school districts included in this study.

Table B2. Algebra I performance when repeating algebra I for the second, third, or fourth time

Academic performance	Number	Percent
Passed the course with a "D–" or better	363	74.4
Average course grade in algebra I		
Between "F" and "D"	190	38.6
Between "D" and "C"	158	32.1
Between "C" and "B"	93	18.9
Between "B" and "A"	51	10.4
Algebra I CST performance level[a]		
Far below basic	46	15.3
Below basic	137	45.5
Basic	74	24.6
Proficient	[b]	[b]
Advanced	[b]	[b]

a. Number of repeaters does not sum to 492 because some students were missing data on CST performance level for the second, third, or fourth time they took algebra I.

b. Suppressed to reduce risk of disclosure because there were fewer than 10 students in one of the subgroups.

CST is California Standards Test.

Note: *n* = 492.

Source: Authors' analysis of data obtained from the school districts included in this study.

Students who repeated algebra I multiple times usually performed better than when they first took the course, experiencing average increases in algebra I grades (0.5), CST scale scores (14.1), and CST performance levels (0.2; table B3).

As with first-time repeaters, students who initially performed poorly in algebra I were likely to improve when repeating the course multiple times, but the amount of improvement for students who initially performed well depended on the academic measure (table B4). For instance, students who initially scored "proficient" on the algebra I CST had a decline in CST score but an improvement in course grade when they repeated the course multiple times.

Table B3. Improvement in performance between the first time taking algebra I and the second, third, or fourth time repeating algebra I

Area of improvement in algebra I	Mean	Median	Standard deviation
Course grade (*n* = 491)	0.5**	0.5	1.3
CST scale score (*n* = 245)	14.1**	14.0	40.1
CST performance level (*n* = 245)	0.2**	0.0	0.9

** Significant at the .01 level.

CST is California Standards Test.

Note: Paired *t*-tests were used to determine whether the level of improvement between the first and second time taking algebra I was statistically significant. Number of repeaters for each area of improvement is not 492 because some students were missing data on course grade, CST scale score, or CST performance level for the first, second, third, or fourth time they took algebra I.

Source: Authors' analysis of data obtained from the school districts included in this study.

Table B4. Average improvement in performance between the first time taking algebra I and the second, third, or fourth time repeating the course, by student characteristic

Student characteristic	Improvement in course grade	Improvement in CST performance level
Grade level when the student first took algebra I		
7	[a]	[a]
8	0.52** (*n* = 261)	0.25** (*n* = 150)
9	0.57** (*n* = 229)	0.17* (*n* = 95)
10	[a]	[a]
Average course grade when the student first took algebra I[b]		
Between "F" and "D"	1.00** (*n* = 296)	0.15* (*n* = 105)
Between "D" and "C"	0.20 (*n* = 112)	0.40** (*n* = 86)
Between "C" and "B"	–0.47** (*n* = 68)	0.2 (*n* = 38)
Between "B" and "A"	–1.16** (*n* = 15)	[a]
CST performance level when the student first took algebra I[b]		
Far below basic	–0.15 (*n* = 46)	0.74** (*n* = 42)
Below basic	0.22* (*n* = 137)	0.28** (*n* = 127)
Basic	0.77** (*n* = 73)	–0.05 (*n* = 59)
Proficient	0.85** (*n* = 41)	–0.60* (*n* = 15)
Advanced	[a]	[a]
Course grade and CST performance level when the student first took algebra I		
Had an average course grade of at least a "C" and scored "proficient" or "advanced" on the algebra I CST	[a]	[a]
Algebra I requirement for all grade 8 students[b]		
Required for all grade 8 students	0.04 (*n* = 65)	0.18 (*n* = 39)
Not required for all grade 8 students	0.62** (*n* = 426)	0.22** (*n* = 206)

* Significant at the .05 level; ** significant at the .01 level.

a. Suppressed to reduce risk of disclosure because there were fewer than 10 students in this subgroup.

b. Number of repeaters does not sum to 492 because some students were missing data on algebra I course grade or CST performance level when they first took algebra I.

CST is California Standards Test.

Note: *n* = 492. Paired *t*-tests were used to determine whether the amount of improvement between the first and subsequent times taking algebra I was statistically significant. The number of students differs for the counts of course grades versus performance levels because not all students took the algebra I CST both the first and second time they took algebra I.

Source: Authors' analysis of data obtained from the school districts included in this study.

Students who repeated algebra I in the summer

As noted in the main text of the report, 175 of the algebra I repeaters, or 11.6 percent, repeated the course in the summer. Like other algebra I repeaters, summer repeaters tended to improve (table B5).[18] In fact, among the summer repeaters, even students who originally had algebra I grades that averaged between "C" and "B" improved their grades when they repeated the course. However, this finding should be interpreted with caution due to the low sample size (13 students).

Table B5. Average improvement in performance between the first and second time taking algebra I among summer repeaters

Student characteristic	Course grade improvement
Grade level when the student first took algebra I	
7	[a]
8	0.85** (*n* = 16)
9	1.60** (*n* = 158)
10	[a]
Average course grade when the student first took algebra I	
Between "F" and "D"	1.92** (*n* = 96)
Between "D" and "C"	1.26** (*n* = 60)
Between "C" and "B"	0.68** (*n* = 13)
Between "B" and "A"	[a]
Algebra I requirement for all grade 8 students	
Required for all grade 8 students	[a]
Not required for all grade 8 students	1.54** (*n* = 174)

** Significant at the .01 level.

a. Suppressed to reduce risk of disclosure because there were fewer than 10 students in this subgroup.

Note: *n* = 175. Students who repeat algebra I in the summer do not take the CST, so this table does not report improvement in CST performance level.

Source: Authors' analysis of data obtained from the school districts included in this study.

Notes

1. Before the implementation of the Common Core State Standards, students in California took end-of-course CSTs in math in grades 2–11. The end-of-course CST options in math were grades 2–7 math, grade 8 general math, algebra I, geometry, algebra II, summative high school math, integrated math I, integrated math II, and integrated math III. Students in a two-year algebra I sequence took the grade 8 general math CST after completing year one of the sequence and the algebra I CST at the end of the second year (EdSource, 2009, p. 8). Students taking the CST can score in one of five performance levels: far below basic, below basic, basic, proficient, and advanced. For this study the CST performance levels were converted to a numeric scale as follows: "far below basic" = 1, "below basic" = 2, "basic" = 3, "proficient" = 4, and "advanced" = 5. Average improvements in CST performance levels are reported on this numeric scale.
2. Most students in the district analyzed in this study first take algebra I either in grade 8 or grade 9, with grade 9 the more common grade level.
3. Among the 219 students who received at least a "B–" in algebra I in grade 8 and took algebra I again in grade 9, 50.6 percent received a grade of "B" or below in algebra I in grade 9 (Waterman, 2010, tab. 12). Many students who were successful in algebra I in grade 8 ended up retaking algebra I in grade 9 because either "middle schools were calling a class algebra when they weren't teaching algebra, or … high schools were not accepting eighth-grade algebra as a legitimate class" (p. 1). The current study confirmed with each of the participating school districts that all courses defined as algebra I were in fact legitimate full-year algebra I courses that were teaching the algebra I standards and were accepted by the high schools (see box 2 for more on this point).
4. EdSource (2009) assumes that a full-time algebra I teacher instructs 125 students (five course sections of 25 students each).
5. Student grade levels are reported as expected grade levels rather than the actual grade level that the dataset indicates the student is enrolled in. For example, if a student was in grade 7 in 2006/07, he or she would be an expected grade 8 student in 2007/08, an expected grade 9 student in 2008/09, and so on. This is the same strategy used in Clotfelter, Ladd, and Vigdor (2012, fn. 18 and 29) and Kurlaender et al. (2008, p. 9) to include students who repeated grade levels. Using expected grade levels allows for the identification of algebra I repeaters who are also repeating the entire grade. This would be relevant primarily for students repeating grade 8 who may also be forced to repeat algebra I if geometry is not offered at the school. However, in this study's sample, only one student repeated grade 8 (and one student repeated grade 7 between 2006/07 and 2007/08). As a result, the analysis is not affected by students repeating an entire grade. Students who took algebra I before grade 7 and did not take the course in grade 7 or later are excluded from the analytic sample. It is unlikely that there were many of these students.
6. The rationale for reporting findings for grade 7 math performance is to enable educators to predict which students are likely to struggle in algebra I before they have taken the course, with the hope of targeting these students for extra support. The findings for grade 7 math performance exclude students who took algebra I in grade 7.
7. As noted earlier, most students in this district first take algebra I in grade 8 or grade 9, with grade 9 being the most common grade level.
8. Some students repeated algebra I more than once. In the study dataset, 437 students repeated algebra I twice, 51 students repeated algebra I three times, and 4 students

repeated algebra I four times. The main text of this report presents students' outcomes when they first repeated algebra I. Appendix B presents the outcomes of students when they repeated algebra I a second, third, or fourth time.

9. Based on communications with the East Side Union High School District, it was confirmed that repeating algebra I students would take an algebra I course with the same curriculum, content standards, and grading criteria as their first algebra I course.
10. Although various estimators have been proposed to quantify the effect of the regression to the mean when repeated measurements are taken (Chen & Cox, 1992; Lin & Hughes, 1997; Mee & Chua, 1991; Naranjo & McKean, 2001), these estimators are typically applied to determine the effect of an intervention in the absence of a control group. Because this report does not seek to estimate the effect of a particular intervention but simply to provide basic descriptive statistics, it does not employ any of these adjustment estimators.
11. There are 13 comprehensive high schools in the East Side Union High School District, so conducting interviews at each school could entail as few as 13 total interviews. As shown in table 4, there were 52 high-performing repeating students who scored "proficient" or "advanced" on the algebra I CST and had average algebra I course grades of at least a "C"; based on these results, each of the 13 interviews could focus on an average of four high-performing students who repeated the course.
12. As a robustness check, which may be necessary if only a select group completes algebra II, additional outcome measures could be incorporated, such as algebra II course grades (since the CSTs are taken only through grade 11), highest math course completed, and whether the student earned a high school diploma.
13. Any student with no math course grade for the entire year was dropped in that school year. All students from the Franklin-McKinley Elementary School District lacked course letter grades in grade 7, but these students were retained to preserve this district in the analytic sample. These students were excluded from analyses that reported on grade 7 math course grades.
14. Oak Grove Elementary School District provided only second-semester course grades. The grade point average for Oak Grove students is based on these grades.
15. The 37 students were located in all three of the Evergreen middle schools. Of students who took "Algebra Basic," 4 of 193 at school A, 4 of 111 at school B, and 29 of 266 at school C were promoted to geometry.
16. The student characteristics included in the model are race/ethnicity, gender, English language learner status, eligibility for free or reduced-price lunch, special education status, grade 7 CST performance level, average grade 7 math grades, number of absences in grade 7, average algebra I grade the first time the student took the course, and algebra I CST performance level the first time the student took the course. The 229 students who took algebra I in grade 7 are excluded.
17. Random school effects have been included in this model to account for the fact that high schools may vary in their decision whether a given student should repeat algebra I. Modeling for these effects in the analysis allows for each high school to have its own (unobserved) impact on the probability that a student repeats algebra I at the high school.
18. Students who take algebra I over the summer do not take the CST.

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