TRANSCRIPT

Teaching Mathematics Through Student Worked Examples

November 1, 2016

[**Slide:** *Teaching Mathematics Through Student Worked Examples*]

LAURA BUCKNER

Welcome to our webinar today, Teaching Mathematics Through Student Worked Examples.

[**Slide:** *Teaching Mathematics Through Student Worked Examples, November 1, 2016*]

This webinar is hosted by the Regional Educational Laboratory West, or REL West, and also by the National Research and Development Center for Cognition and Mathematics Instruction, better known as the Math Center, And with that, I'm going to hand the ball off to Jodi Davenport, who will introduce you to our presenters and share a little bit about the Math Center. Welcome, Jodi.

[**Slide:** *Presenters*]

JODI DAVENPORT

Thank you, and welcome everybody. So our presenters today bring backgrounds in both research and practice, and we have a really great team. I want to introduce Cathy Carroll here at WestEd; she is a Senior Research Associate and Project Director. Cathy's background is quite extensive; she has over 20 years' experience of teaching middle school math, and since then has become a nationally recognized expert in math leadership and professional development, leading federally funded projects related to middle school math. She is also the President Elect of the California Math Council and has been Chair and Program Chair for the NCSM and NCTM conferences.

And Dr. Julie Booth is our researcher. She's at Temple University and for the past fifteen years, Julie has been conducting research, applying cognitive developmental psychology to education, and her area of work and her passion focuses on identifying ways to help students develop both conceptual and procedural knowledge in mathematics.

And I'm Jodi Davenport. I'm at WestEd. I'm the director for the National Center for Cognition and Learning.

[**Slide:** *About the Math Center*]

I just wanted to give you a little background about the Math Center that Laura just mentioned. This is…our work comes together from a five-year grant from the Institute of Education

Sciences that really aims to apply findings and research on how people learn to try to improve classroom materials and instruction. So I come from a background in cognitive science and there's a lot of findings that the, the Department of Education has identified that are not necessarily being applied in educational materials, educational practice.

[**Slide:** *Organizing Instruction and Study to Improve Student Learning*]

So a few years ago, the Department put together a practice guide, and you'll see on the screen—some of these are going to be kind of small so you could either zoom using the magnifying glass on the bottom or just listen—but they put together a practice guide called *Organizing Instruction and Study to Improve Student Learning*.

And what the book is, is it's a basic…a series of recommendations that come out of basic research that has been carried out in a number of laboratories and settings, university settings across the country. And they created a number of recommendations for these practices—really seem like they actually influence how students pay attention, how students learn, how students remember information—in other words, how cognitive processing of students could be benefited by particular kinds of educational practice.

And in our webinar today, we're going to be talking about two of these recommendations. The first one is to interleave worked examples—this is having students alternate between reading already worked solutions and actually trying to solve problems on their own, and the second one is asking deep explanatory questions. So in addition to just having them solve problems, actually prompting students to answer more deep-level questions that let them respond with explanations and create deeper understanding of course materials.

[**Slide:** *Learning Goals*]

That segues into the learning goals for today's webinar. The first one is, we want everybody to learn about what worked examples are and the research that supports their use in classroom settings, to also identify when worked examples may be particularly effective in your classroom, and finally, to understand how to use them in your math instruction—where do you put them and in what cases might they be a good use of your instructional practice. So Cathy's going to go a little bit more into the background on the research,

CATHY CARROLL

[**Slide:** *Worked Examples*]

Okay this is Cathy Carroll and I'm glad to be with you today. So I'm going to give you a little bit of information, a little bit of background about worked examples and particularly related to the two recommendations for the practice guide that Jodi just mentioned. We'll look at the worked example principle and the self-explanation principle.

[**Slide:** *Worked Example Principle (e.g., Sweller, 1999)*]

So the worked example principle states that replacing some problems in a practice session with an example of how to solve the problem can be helpful to learners in a couple of ways. One is that it reduces the working memory load, so compared with a long string of practice problems—those wonderful pages that many of us recall from our time as students—that this allows learners to focus on learning the steps in problem solving rather than just continually rotely practicing the same thing over again. And, it allows students to process information more deeply when they're not just routinely applying procedures. And, finally, that it might naturally generate more self-explanations, and I'll talk about what that means in a moment. And, you know, just a side benefit that when we were conducting the PD in the Math Center, a number of teachers commented that parents appreciated the worked examples because it gave them a little avenue into helping their students with homework a little bit more.

[**Slide:** *Self-Explanation Principle (e.g., Chi et al., 1994)*]

But the self-exam…self-explanation principle, then, goes hand in hand, as Jodi mentioned, with those recommendations from the practice guide. The, the opportunities to ask students some exploratory questions and really have them digging into their thinking is the sort of the…goes hand in hand with the worked examples. And so this principle prompts learners to explain information as they read or study, and forces them to really try to articulate what it is they're understanding, what their questions are, what they're seeing going on, and it's particularly applicable to math. You know, it gives us a chance to dig into how students get their answers and, and in the case of worked examples, they're trying to explain what they see in the work that is shown.

[**Slide:** *Three Kinds of Worked Examples*]

So let me show you a little example…there's actually three kinds of worked examples that, that we're going to talk about and one is a correct worked example; so where the student work is complete and correct. The second is partially completed so the work is started, it's correct as it goes along, but it's not finished; and then the third type is, is incorrect work. And so this is where we're going to actually look at some work where the, the work is in error, and we'll talk about that in a little more detail in a minute.

[**Slide:** *Implementing Correct Worked Examples*]

So I want to start with implementing correct worked examples. So, if you look at the slide here you'll see that, you know, with the correct worked example, what students will see is a problem correctly solved and then they might be asked to explain the work that's shown for that task. They might be asked to extend the work to another part. They might be asked to offer an alternative approach. Any of the kinds of things that are here. So the explanation, the self-explanation piece that we're looking for here is, it could be in any one of these formats.

[**Slide:** *Examining Correct Worked Examples*]

There's two examples here and I just want to talk about them a little bit and how they're slightly different from one another, but they're both examples of correct worked examples.

So in the case of the first one on the left, the, the textbook assignment was, "Draw a diagram to show how to find the solution to two and six-tenths [2.6] divided by four-tenths [0.4] and explain what the quotient means." So, then, what we see in the, in the shaded blue box is the worked example. So it tells us that Gary drew a correct diagram and provided a good explanation, and so the students are asked to look at his work and then answer the question below. Okay? And the question at the bottom is, "How does the number line show Gary's answer of six and five-tenths [6.5]? Where's the six, where's the five-tenths?" So, what this does is, invites students to look back and make sense of the way that this, that Gary has approached the problem and where his work is appropriately done. And so we would want students to be highlighting that…we see that there are six of the four-tenth values, okay, so we can see those here as we go across—and then that there's another half of one, so five-tenth [0.5] of a four-tenth [0.4]. So that's one kind of thing that they're asked to explain that.

Then the example on the right-hand side is a different kind of explanation. So, again, the work is done correctly; in this case, they were supposed to use the distributor property, tell if the sentence is, is true or not. And so, Nala has explained why it's true that the 432 groups of 50 is the same as 400 groups of 50 and 32 groups of 50. So we know that that's correct, students have that to work with, and then they have the…they're asked to actually now do a similar example. So, could we, instead of saying 400 plus 32, could we rename the 432 as 400 plus 30 plus 2 and extend it that way. And so we asked students to explain the why or why not there.

[**Slide:** *Implementing Partially Completed Worked Examples*]

So those are examples of correct worked examples. Then the second kind of worked example is the partially completed. So in this case, we might ask students to either complete the work where the student was stuck in the example, or we might ask them to complete the work and then answer a question about that work. So, again, I have a couple of examples for you.

[**Slide:** *Examining Partially Completed Worked Examples*]

In the example at the left, this is a problem about a pizza party and people are seated at tables, three different sizes of tables, and have different numbers of pizzas at those tables. And so, the information tells us that at the small table there are 2 pizzas and 5 seats, at the medium-sized table 3 pizzas and 7 seats, and at the large table 5 pizzas and 12 seats. And so, what we ask the students to do here, it says, "Denise started to solve the problem but she got stuck. Complete the problem and then explain your reasoning to Denise." So what would want to see here is for the students to recognize that, you know, that the ratios that are set up are fine, and that there needs to be something done with them mathematically so, you know, find a decimal value or a percent value to be able to determine, then, which table is the best one to sit at to get the most pizza. And so they're asked to, to complete the problem there. Okay? So the…what the worked example does there is give the students sort of a head start on the setting up the problem, and then giving them the opportunity to continue to work on it themselves.

And then, again, the example on the right is a, is a slightly different type of worked example. So, the students are asked to give the values on the number line where the, each of the hash marks is there. Okay? And so Marco has started; he put one and five-hundredths [1.05] on the middle, and so now students are asked, using that information, so what Marco has already added, then they need to fill in the rest of the labels to complete that. Okay? That's the second kind of worked example.

[**Slide:** *Incorrect Worked Examples*]

And then the third kind is an incorrect worked example. So we'll talk a little bit about those, and then Julie will dive in with you into actually creating something and getting to, to use them with your own students and/or teachers with whom you work.

So, incorrect worked examples—in this model what we're doing is actually showing students some common incorrect ways to solve problems and then having them explain why the procedure is inappropriate. And one of the things that was really important is that everywhere where we used an incorrect example, that there was a big X there. It's really clear on the page—and when I show you the pages from the book you'll see—so that it, you know, it was very obvious that the example was incorrect. And the value of this, there's a couple of things; one is that it provides some feedback that reduces the relative strength of the incorrect strategies. So, you know, when we specifically say this is incorrect, you know that is actually helping students to see that the procedure is wrong and also that it can force students to see the differences between the presented problem and others where the procedure does work. So, it has the capacity to expose them to misconceptions.

[**Slide:** *Implementing Incorrect Worked Examples*]

So, in this case, the self-explanation—that, that writing piece or the explanatory piece of the work with the worked examples—was the students might be asked to do a number of things. If they could find where the error was, fix it and explain how they fixed it. They might fix the incorrect response and then complete the work, or if they might simply be asked to explain why, to explain why the response is incorrect or to explain why it's incorrect and then complete it. So a variety things that they, again, that they could be asked to do. Let me show you a couple of examples.

[**Slide:** *Examining Incorrect Worked Examples*]

In the example on the left here, what Syneef has done is that he didn't really…oh, let's take a look at the, the problem first; sorry, I jumped ahead. So this is the case of looking at the temperature in Browning, Montana, and one day the temperature fell from 44 degrees to negative 56 degrees. What was the temperature change that day? So we see Syneef's work here; he says that 44 to negative 56 is 44 minus 56, which is negative 12, so that the temperature only dropped 12 degrees. And so students are asked, "Well, what's his error and why is it a mistake?" So we would want them, you know, to notice that, that the change is actually a difference of 100 degrees, you know, and that the 44 represents above zero, the 56 represents below zero, so to have some explanation in that regard is what we would want students to do there.

And then in the example on the right-hand side, here we have a, a problem situation of making buttons for a conference and the amount that's involved there and so it…we see that Anthony has set up a proportion incorrectly. So he doesn't match the values correctly. He could have set 18 over 12 equals 27 over d if he was matching his things correctly, or he could have done it the opposite way, but he needs to make sure that his ratios are written with the corresponding parts—dollars to dollars, to dozens, and dollars to dozens, so thinking about it that way. And so that's what we are asking students to do in that example.

So that's kind of a summary of what the worked examples are all about. So now I'm going to turn it over to Julie and she'll take you through creating some worked examples.

[**Slide:** *From the Lab to the Classroom*]

JULIE BOOTH

So, all of the work that Cathy was telling you about in terms of evidence that these practices work, most of that was done in laboratory settings. So a researcher at a university sitting down with students and trying to see if a difference could be made in their learning. And so many teachers have questioned about whether these kinds of approaches can make a meaningful difference in a real-world classroom. I wanted to tell you about some of that work, including some of the work that, that shows what happens with struggling students or special ed students with all of this, as one of our participants asked.

[**Slide:** *Example-based condition outscored control on all measures*]

So, this is a study that was done with a very large sample, a randomized controlled trial, so, you know, different classrooms were put into different conditions and some of them worked with correct and incorrect examples over the course of the year and some of them didn't. And what we found in this study was that the students in the example-based conditions, condition, outscored the control condition, which is your business as usual, like what your normal…what you would normally do in a math class on all of the measures. They were better at their conceptual understanding of math by the end of the year. They were better at solving problems by the end of the year. And they were also better on, excuse me, items that were taken from standardized tests.

[**Slide:** *Gains in conceptual knowledge even greater for struggling students (up to 10 percentage points)!*]

Interestingly, even though it seemed to benefit all of the students across the year, we find that when it comes to improving students' conceptual knowledge, the benefit is the greatest for those struggling students. So the students who are maybe at the lowest end of the, of the spectrum at the beginning of the year, who have the, the least amount of prior knowledge, you would expect to end the year with about the least amount of prior knowledge. Hopefully everybody's knowledge has increased but their placement in the classroom is probably still about the same. And we found that the students that were in our experimental conditions and got these correct and incorrect examples greatly shifted their trajectory and were doing so much better than would be predicted for, for other students that, that came into the year with their level of knowledge, but then had a traditional math course.

So this, these results are about the combination of the correct and incorrect examples and the approach in general.

[**Slide:** *Inclusion of incorrect examples appears especially important*]

But we have evidence as well that this use of incorrect examples is especially important. So especially, again, whenever you are talking about trying to improve students' conceptual understanding in math, using incorrect examples, having the students explain incorrect examples, is particularly important.

[**Slide:** *Incorrect examples are particularly beneficial for struggling students*]

And even though many of the teachers…as Cathy suggested, many of the teachers we've discussed this with usually start by being very hesitant when they think about incorrect examples. It is very, very common for a teacher to, to be like, you know, "I can see how this might be useful, but I'm really, really worried about giving this to my struggling students. I think they're going to be confused. I really don't want to show them the wrong thing and then they, they, you know, they associate that with, 'Well, the teacher showed me that so that must be right.'" Well, recent evidence that, that has come from my lab shows that these incorrect examples are particularly important for the students that are struggling. So the greatest benefit from the incorrect examples comes from the…for the students that those teachers are, might typically want to shield from them.

So we have a growing amount of evidence that the approach, in general, and the use of incorrect examples, in particular, can be really useful for students learning math, across the, the spectrum of, of prior knowledge but maybe particularly for the students that struggle the most with learning math.

[**Slide:** *Hands-on: How to Create Incorrect Examples*]

So, how do you take this information and then implement it in your classroom? It's not like your textbooks have a lot of incorrect examples incorporated throughout them, and when they do they might not be structured in a way that has the student engaging with the example and really trying to understand why it's wrong. So what we want to do today is talk about how you can create incorrect examples that will be effective for your students to engage with in your classroom.

So, just like any lesson, the first step is to determine what your objective is, so this might be, depending on your particular state or district, it might be a standard, it might be a particular lesson objective that's been laid out for you by your department, etc. But there's, there's something that you want the students to learn how to do in this particular exercise.

When you think about what that objective is, what it is you want your students to do, you can brainstorm a number of different ways that students might go wrong with trying to accomplish this objective. So, I encourage you to think through, you know, a few different kinds of mistakes or misconceptions that you've seen students have when trying to solve problems that fit with that particular objective.

You're going to choose only one misconception for each example that you use in the classroom, and this can be something that's a little…that's, that's challenging. Once you identify all those misconceptions, you may think, "Oh, let's, let's dive right in and make sure we can fix all of these things at once." Well, yes, there are a number of ways that students can go wrong but each particular example should only focus on one misconception and work really hard to eradicate that misconception. You can use another example, then, for a different misconception in a different lesson.

You're going to create the incorrect worked example using that misconception, essentially showing the student how to do it that wrong way that would be consistent with the misconception. But you're going to make sure that it is clear to them that the problem is incorrect. There should be no opportunities for them to have to try to determine whether it's correct or incorrect, especially if it's a common misconception or mistake that they might make. They might think it's correct because it's the way that they do it, and that's not really going to help them. So, the problem needs to be clearly marked as incorrect. And you also want to give them a way to relate to the problem by helping them to think of it as being completed by a fictitious student other than themselves, so it's not them that's incorrect—somebody else has tried this and has done it wrong.

But when you do use a student name to try to draw the, draw your student in, you want to remember to be diverse within these; that you choose across a number of examples. You don't want it to always be traditionally white male names that are getting problems correct and traditionally Latino females or etc. who are getting it incorrect. You want to use this as an opportunity to show the, your students that anybody can be correct, anybody can be incorrect, and getting something wrong doesn't mean that you're not a math person; it's just part of the learning process.

And then, finally, you're going to write the self-explanation prompt that focuses on the misconception. And this can be the most challenging, but also the most exciting part. So we're going to practice going through these steps together to think through how we might create an incorrect worked example that would be useful in the classroom.

[**Slide:** *Self-Explanation Prompt Tips*]

So, when you're writing these self-explanation prompts, we have a couple of different tips for you to keep in mind. Some things for you to remember to do are to ask the "why" questions—so not just *what* did so and so do, *why* is this wrong? Why did he think this was okay to do it this way? And have students explain their reasoning, not just necessarily solve the problem in the correct way. You really want them engaging with the alternate forms of reasoning for the problem. And you want to call students' attention to the features of the problem that you think are important. If you picked out a misconception for a particular objective, you have an idea of what they are misunderstanding and so there are parts of that problem that if you call the students' attention to them and you are able to fix the way they're thinking about those parts, you should be able to make good headway towards fixing the misconceptions.

Things not to do: to just ask questions that instruct students to state the procedure—what did that person do? And don't just ask what's wrong with this example, what mistake was made, or what is the correct answer? You really want them to engage in the *why*, and really dive into the problem in a deeper way than they would with just these sort of more shallow questions.

[**Slide:** *Let's Practice, Step 1* (1 of 5)]

So, let's practice together. We're going to start with an objective here that I'm giving us, which is to simplify an expression by combining like terms. So, let's take a minute and think about simplifying an expression by combining like terms. So let's say its 4x minus 7. What kinds of things might a student do wrong when they try to simplify 4x minus 7? Actually, this would be easier if we think about it in terms of 4x minus 7 plus 2x. Anybody have any ideas? Combine all numbers and add an x; seen that all the time. Anybody else seen that?

[**Slide:** *Let's Practice, Step 2* (2 of 5)]

So, another thing that, that students do frequently besides combine the numbers and add an x—or, by the way, sometimes they combine all the numbers and then they take the x away—it's sort of whatever is most convenient for them at the moment. They oftentimes do not work with the negative signs correctly in, in an expression or equation. So they often don't see the negative sign as part of the term so they think they can sort of move it around as need be or delete it and it doesn't change the meaning of the problem.

[**Slide:** *Let's Practice, Step 3* (3 of 5)]

[**Slide:** *Let's Practice, Step 4* (4 of 5)]

So, if we choose this misconception for this problem, we'll then work to create an incorrect worked example using the misconception, again marking it as incorrect and using a student name. So here is an example where we have the quantity 5 minus 4x plus 12x, and the student—Joseph in this case—is trying to simplify this expression and applying the associative property but is not considering the negative sign as part of a term. They are…he is just allowing that to stand alone and moving the parentheses so that he can then combine 4x and 12x.

[**Slide:** *Let's Practice, Step 4* (4 of 5)]

When it comes to having a student look at that…so that you can have a student examine this incorrect example and, and stop there, and they're not necessarily going to engage in it in a way that helps to fix their misconception. What we find is that using these self-explanation prompts can really help them focus their attention on the pieces that you want to address.

[**Slide:** *Let's Practice, Step 5* (5 of 5)]

So when it comes to writing the self-explanation prompt focusing on that misconception, you, we like to be specific about what we're asking them. So in this particular case you see we have, “Why can’t Joseph just move the parentheses in this problem?” So, I mean, you could start by just, by saying, “What did he do? What did he do wrong?” But if you do that, then they’re just going to give you a procedural answer; they’re not going to be thinking about the concepts. So we give them the procedure here and say, “Yeah, obviously he moved the parentheses. Why can’t he do that? Why isn’t that okay?” And then we ask what he should have written in order to get the answer in simplest form, so we do give them an opportunity to write it correctly. You don’t have to do this for all incorrect examples, but it gives them another opportunity for practice once, hopefully, that misconception is addressed.

So you want to make sure that your questions are very targeted to try to get them to focus on the piece that you really want to fix in that example, and not be tempted to try to fix multiple things in one piece because that will not be as successful.

[**Slide:** *Q&A*]

JODI DAVENPORT

Okay, so I think that I’ll answer…first take the most straightforward question and so, I guess, Julie, could you say something about have you seen this…at what age or grade levels you’ve seen this work and done it?

JULIE BOOTH

So, across a wide variety of grade levels, the idea of presenting incorrect worked examples and, and having students consider them can be effective. I’ve seen it done all the way down to about first or second grade and then all the way, you know, a lot of the work was done through college. However, the way that you need to do it looks a lot different if you’re working with really young children. And so, I’ve done these kinds of in-depth curriculum development projects for algebra, for middle grades mathematics in general, and I’m now working on creating materials for fourth and fifth grade math. And it’s a very different approach because you’re dealing with children that, you know, have different literary skills first of all, language skills at that point, and so you want to make sure that the self-explanation questions that you’re asking, and the context you set up, are very clear and easily, easy to, to follow, so that they’re not just kind of caught up in the language.

The examples themselves are obviously simpler whenever you are dealing with younger kids because the mathematics is simpler. But I have seen some pretty good evidence that at least down to about 4th grade, something that resembles this kind of activity where you’re changing up even their math practice and having them work with these examples can be effective. For younger kids, I think that, you know, they’re, they’re not necessarily doing homework problems themselves anyway, but I think that activities could be focused around errors in the classroom where they sit and talk about, “Hey, what could be, what could be wrong about this? What could we do differently? Why might this not work?” I think that with that kind of scaffolding that the younger kids could benefit from it as well.

JODI DAVENPORT

Okay, and we have a couple more questions. And, you know, I think that maybe either Cathy or Julie could speak to this one. It's about, would worked examples from your students be more powerful than made-up students?

JULIE BOOTH

So, Bob…actually, yes, it could be very beneficial, and the, this approach originates from math classrooms in Asian countries—Singapore, China, Japan—where they would have somebody…you know, have students write their answers on the board and if it was wrong, they would not erase it very quickly like we often do here. They would take 20 minutes and discuss it, and it can be extremely powerful but it can also be…if you, if you, if you think about doing that in your own classroom here in the states, oftentimes it's looked upon as something that might embarrass the student that did it incorrectly in the first place. So, it's, it…it's powerful because it's in the moment and it's powerful because it's relevant, and clearly it's something that, that your students are doing wrong. But there's a fine line between setting up the right kind of culture where that wouldn't be embarrassing or wouldn't be picking on that particular student. That makes it a little bit more difficult. So we were really trying to get in the back door here by providing these kinds of opportunities without having to put American students on the spot in ways that they aren't used to having that happen.

CATHY CARROLL

Right, that can be a little bit tricky but, I wanted to just add that I've been doing some work with teachers that have been like, if they work on a task one day, then looking through the student work that night and finding examples to use, the next day without students' names on them but to highlight some of the same kinds of ideas of pulling that, the, you know, the incorrect work and then trying to make some sense of it. So there is, then, the work from their own students but there's not names attached to it that would create that issue of, you know.

JULIE BOOTH

Yeah, although, I would wager that it's probably no more effective. It's easier for you because you, then, as a teacher, don't have to come up with the examples.

CATHY CARROLL

Right, right.

JULIE BOOTH

But I think that, that it really…it's not that it's more powerful in those situations; it's just making it easier to put errors, common errors, in front of the students. And you would still have to think about what kinds of prompts to ask them, what kinds of questions to ask, so that they're not just focused on procedural items and they're really digging in.

CATHY CARROLL

Exactly. Right, right.

JODI DAVENPORT

So related, we had a question about whether it would be a good idea to introduce worked examples to your whole class before putting them into homework?

JULIE BOOTH

Ah, I think it's an excellent idea and especially with the younger kids. I think it's, it's completely necessary because oftentimes they don't, they don't know what to do with it; it looks so different. They're not used to seeing things worked out for them and they're especially not used to seeing errors. So we've done things in the past where, you know, it's sort of even little scripted things where the, the teacher can walk them through a basic incorrect worked example and how, how you might react to it and how you might elicit responses to it. Just sort of to, to help the students know what to do with it and also to be comfortable pointing out mistakes. Most teachers say that once the, the, the ability to point out mistakes is sort of made available to them, and it becomes okay to, to, to find mistakes, that students are very good at it and very happy to share their, their knowledge of what they see the teacher do that isn't correct.