# Planting the seeds for Common Core State Standards-Mathematics 

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## Common Core State Standards for Mathematics

- Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

- Standards for Mathematical Content

These Standards define what students should understand and be able to do in their study of mathematics. These Standards do not dictate curriculum or teaching methods.

## Emphasis on Problem Solving

## Standards and Focal Points, National Council of Teachers of Mathematics (NCTM)

- Problem solving means engaging in a task for which the solution is not known in advance.
- Good problems give students the chance to solidify and extend their knowledge and to stimulate new learning. Most mathematical concepts can be introduced through problems based on familiar experiences coming from students' lives or from mathematical contexts.
- Students need to develop a range of strategies for solving problems, such as using diagrams, looking for patterns, or trying special values or cases.

By early 1990s Japanese math textbooks, especially for elementary grades, using an approach based on Problem Solving (Teaching through Problem Solving).

Average Percentage of Trends in International Mathematics and Science Study (TIMSS) Mathematics Topics Taught in School and the Achievement (Average Scale Score) of the TIMSS 2003

Grade 4


## Standards for Mathematical Practice

Mathematically proficient students...

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## Developing these practices begins early

- Present problems in understandable contexts for the students so that the students can experience the progression from concrete, semi concrete to abstract.
- Give students opportunities to attack open-ended problems so that the students increase their confidence.
- Let students use manipulatives not only to find answers but to explain to others how to find answers.
- Help students learn to communicate how they solve problems using actions, verbal explanations, and equations.


## Present problems in understandable contexts

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reasoning abstractly and quantitatively


## Kindergarten: Operations and Algebraic (OA) Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction with objects, fingers, mental images, drawings2, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

(1)How many cats will there be ...

$5+3=8$

## 8 cats

## Sample Questions



Concrete

## Semi Concrete

Abstract

## Concrete



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## Open-ended problems

- Give students opportunities to attack openended problems so that the students increase their confidence.

> Make sense of problems and persevere construct viable arguments


IDEAS / QUESTIONS / PROBLEMS

## Construct viable arguments and critique the reasoning of others.

## Lesson Study Alliance

Helping teachers work together to improve teaching \& learning. http://www.LSAlliance.org


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## Workbook Example



## Use manipulatives to explain how to find answers.

- Let students use manipulatives not only to find answers but to explain to others how to find answers.

Make sense of problems
use tools strategically construct viable arguments

## Single digit addition

| $1+1=2$ | $2+1=3$ | $3+1=4$ | $4+1=5$ | $5+1=6$ | $6+1=7$ | $7+1=8$ | $8+1=9$ | $9+1=10$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $1+2=3$ | $2+2=4$ | $3+2=5$ | $4+2=6$ | $5+2=7$ | $6+2=8$ | $7+2=9$ | $8+2=10$ | $9+2=11$ |
| $1+3=4$ | $2+3=5$ | $3+3=6$ | $4+3=7$ | $5+3=8$ | $6+3=9$ | $7+3=10$ | $8+3=11$ | $9+3=12$ |
| $1+4=5$ | $2+4=6$ | $3+4=7$ | $4+4=8$ | $5+4=9$ | $6+4=10$ | $7+4=11$ | $8+4=12$ | $9+4=13$ |
| $1+5=6$ | $2+5=7$ | $3+5=8$ | $4+5=9$ | $5+5=10$ | $6+5=11$ | $7+5=12$ | $8+5=13$ | $9+5=14$ |
| $1+6=7$ | $2+6=8$ | $3+6=9$ | $4+6=10$ | $5+6=11$ | $6+6=12$ | $7+6=13$ | $8+6=14$ | $9+6=15$ |
| $1+7=8$ | $2+7=9$ | $3+7=10$ | $4+7=11$ | $5+7=12$ | $6+7=13$ | $7+7=14$ | $8+7=15$ | $9+7=16$ |
| $1+8=9$ | $2+8=10$ | $3+8=11$ | $4+8=12$ | $5+8=13$ | $6+8=14$ | $7+8=15$ | $8+8=16$ | $9+8=17$ |
| $1+9=10$ | $2+9=11$ | $3+9=12$ | $4+9=13$ | $5+9=14$ | $6+9=15$ | $7+9=16$ | $8+9=17$ | $9+9=18$ |

## Addition and Subtraction Calculations

Add and subtract within 20. (Standard 2.OA 2)

## Grade 2

- Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

K

- Calculate addition or subtraction to find the solution to the problems.
- Single digit addition and subtraction within ten.


## Grade 1

- Addition and subtraction with three numbers
- Single digit addition and subtraction using making ten strategies.


## Single digit addition

| $1+1=2$ | $2+1=3$ | $3+1=4$ | $4+1=5$ | $5+1=6$ | $6+1=7$ | $7+1=8$ | $8+1=9$ | $9+1=10$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $1+2=3$ | $2+2=4$ | $3+2=5$ | $4+2=6$ | $5+2=7$ | $6+2=8$ | $7+2=9$ | $8+2=10$ | $9+2=11$ |
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| $1+6=7$ | $2+6=8$ | $3+6=9$ | $4+6=10$ | $5+6=11$ | $6+6=12$ | $7+6=13$ | $8+6=14$ | $9+6=15$ |
| $1+7=8$ | $2+7=9$ | $3+7=10$ | $4+7=11$ | $5+7=12$ | $6+7=13$ | $7+7=14$ | $8+7=15$ | $9+7=16$ |
| $1+8=9$ | $2+8=10$ | $3+8=11$ | $4+8=12$ | $5+8=13$ | $6+8=14$ | $7+8=15$ | $8+8=16$ | $9+8=17$ |
| $1+9=10$ | $2+9=11$ | $3+9=12$ | $4+9=13$ | $5+9=14$ | $6+9=15$ | $7+9=16$ | $8+9=17$ | $9+9=18$ |

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| $1+3=4$ | $2+3=5$ | $3+3=6$ | $4+3=7$ | $5+3=8$ | $6+3=9$ | $7+3=10$ | $8+3=11$ | $9+3=12$ |
| $1+4=5$ | $2+4=6$ | $3+4=7$ | $4+4=8$ | $5+4=9$ | $6+4=10$ | $7+4=11$ | $8+4=12$ | $9+4=13$ |
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## Kindergarten: Operations and Algebraic Thinking

Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 $=2+3$ and $5=4+1$ ).

Using manipulative to develop the relationships among numbers

- Cuisenaire Rods
- Number Blocks and Counters


## Add and subtract within 20

- Standard 1.OA.6:

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10 . Use strategies such as counting on; making ten (e.g., $8+6=8+2+$ $4=10+4=14$ ); decomposing a number leading to a ten (e.g., 13-4 = 13-3-1 = $10-1$ = 9); using the relationship between addition and subtraction (e.g., knowing that $8+4=12$, one knows $12-8=4$ ); and creating equivalent but easier or known sums (e.g., adding $6+7$ by creating the known equivalent $6+6+$ $1=12+1=13$ ).

## Making Ten Strategy

Yuka collected 9 acorns and Hiroshi collected 4.
How many did they collect altogether?

## Making Ten Strategy

(1) Write the math sentence.


I wonder if there are more than 10?

$1, \ldots, 9,10,11,12,13$.
Altogether there are 13 blocks.


## Making Ten Strategy



## Making Ten Strategy

## How to calculate $9+4$

(1) 9 needs 1 more to make 10 .
(2) Split 4 into 1 and 3.
(3) Add I to 9 and make 10 .
4. 10 and 3 makes 13 .

## $9+4$ <br> (10). <br> 3

## Making Ten Strategy

4 Explain how to calculate $8+5$.


## Decomposing a Number Leading to a Ten Strategy


(I) Write the math sentence.

```
Math
Sentence
```


## Decomposing a Number Leading to a Ten Strategy



$$
\begin{aligned}
& 1,2, \ldots, 9.1,2,3, \\
& 4 \text {, are left, so, } 4 \text { is } \\
& \text { the answer. }
\end{aligned}
$$



## Decomposing a Number Leading to a Ten Strategy



We can't subtract 9 from 3 so...


## Decomposing a Number Leading to a Ten Strategy

## How to calculate 13-9

(1) You can't subtract 9 from 3.
2) Split 13 into 10 and 3.
(3) Subtract 9 from 10 and get 1 .
4. 1 and 3 make 4 .

## Decomposing a Number Leading to a Ten Strategy

4 Explain how to calculate II-8.


## Communication

- Help students learn to communicate how they solve problems using actions, verbal explanations, and equations.

Construct viable arguments

## Tom buys a chocolate for $25 ¢$ and candy for 14 C . How much is it going to cost?

1 Write a math sentence.


I wonder about how much it is going to be.

Math
Sentence


Can you explain why the math sentence is written like that?


## Workbook Example



$$
25+14=39
$$

Answer 39 yen

3 Explain Hiroki's and Kaori's methods.

## Workbook Example



7 first-graders and 12 second-graders are playing on the playground. 8 second-graders came and joined them. How many students are on the playground now?

## 2

Miho and Takumi each expressed this story using one math sentence as shown below. Describe their thinking by looking at the math sentences.


## Workbook Example



Let's think about ways to find the answer to $7 \times 4$.

We can use the properties of multiplication that we learned in 2 nd grade...

The multiplication math sentence that has the same answer as $7 \times 4$ is...

## Workbook Example

Kaori
The answer to

than $7 \times 3$.


## Workbook Example

The answer to


## Workbook Example

Shinji
The answer to $7 \times 4$ is as same as the answer to


## Workbook Example

## 5 Find different ways to find the answer to $12 \times 4$.



## Workbook Example



## Workbook Example



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- Let students use manipulatives not only to find answers but to explain to others how to find answers.
- Help students learn to communicate how they solve problems using actions, verbal explanations, and equations.
- Supporting students to organize notes is a key to develop mathematical practice.

