

Building Educators' Understanding of Early Mathematics to Promote Students' Later Mathematics Success

REL West Webinar

Hosted by Dr. Rebecca Perry
October 22, 2015

REL WEST WestEd

Presenters

Dr. Rebecca Perry

- Senior Program Associate with WestEd's Innovation Studies program.
- Primarily works with the Math in Common project, a five-year initiative drawing togeth 10 California school districts as they are implementing the K-8 Common Core State Standards in mathematics.
- Works also with REL West on issues in early mathematics education

Goals for the Webinar

Highlight practical, research-based information about early mathematics learning with implications for instructional practice. Attendees will learn about:

1. **Practical, evidence-based recommendations for teaching math to children ages 3–5;**
2. **How these practices relate to the CCSS-M, especially for Grade K–2;**
3. **How these mathematical foundations support the success of students in later elementary and middle school grades**

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Agenda

1. Recommendations from the IES Early Mathematics Practice Guide – Dr. Douglas Frye
2. Bridging the IES Practice Guide and the Common Core State Standards – Dr. Akihiko Takahashi and Dr. Rebecca Perry
3. Q&A with Dr. Frye
4. Closing and Final Survey

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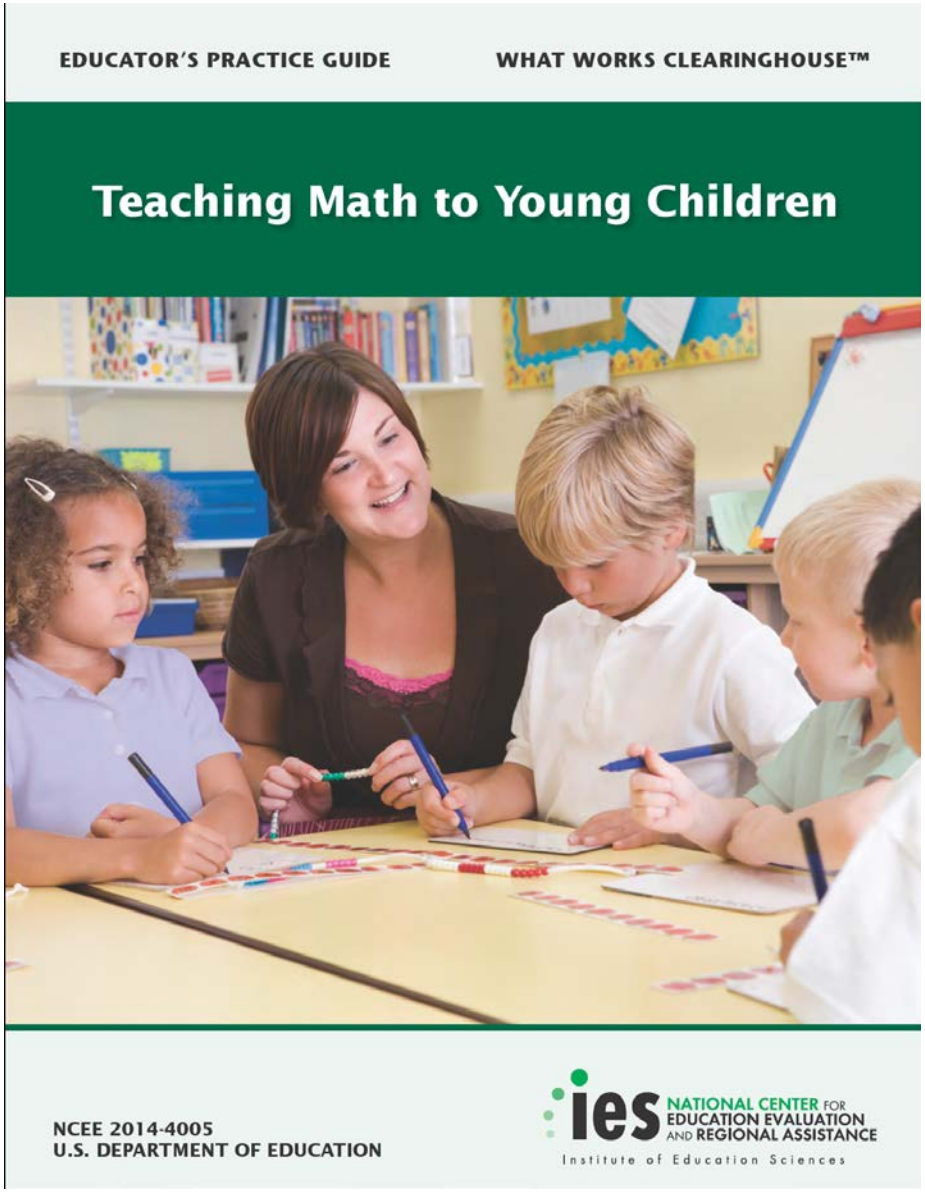
Presenters

Dr. Douglas Frye

- Associate professor - University of Pennsylvania's Graduate School of Education
- Director of the Interdisciplinary Studies in Human Development program.
- Lead developer on the 2013 IES Educator's Practice Guide, Teaching Math to Young Children.



Using the *Teaching Math to Young Children* Practice Guide

Douglas Frye,

University of Pennsylvania

October 22, 2015

Need for Early Math Instruction

Compared to other countries, American children are less prepared for math learning at the beginning of school

Within the country, variations in math knowledge can found at entry to school

Better math understanding at school entry is associated with better math and reading achievement in 3rd grade (Duncan et al., 2007)

Increased Attention to Early Math

New curricula for preschool and kindergarten math have been developed and are being tested

- They include both self contained early math curricula (Appendix D.2) and comprehensive curricula that include early math (Appendix D.3)

IES Practice Guide to evaluate the evidence for different practices in early math instruction

Institute of Education Practice Guides

Systematic effort to evaluate and document evidence-based practices

Instructional practices collected from the scientific literature and the researcher-practitioner panel

Panel Members

Arthur J. Baroody
University of Illinois at Urbana-Champaign and University of Denver

Margaret Burchinal
University of North Carolina

Sharon M. Carver
Carnegie Mellon University Children's School

Douglas Frye (Chair)
University of Pennsylvania

Nancy C. Jordan
University of Delaware

Judy McDowell
School District of Philadelphia

Institute of Education Practice Guides

Studies collected from the literature and the evidence evaluated according to What Works Clearinghouse standards for:

- **Whether the practice has been shown to have a positive effect**
- **How general the outcome is for different settings and groups of children**

Teaching Math to Young Children (3–6 years) Recommendations

Recommendation 1. Teach number and operations using a developmental progression

Recommendation 2. Teach geometry, patterns, measurement, and data analysis using a developmental progression

Recommendation 3. Use progress monitoring to ensure that math instruction builds on what each child knows

Recommendation 4. Teach children to view and describe their world mathematically

Recommendation 5. Dedicate time each day to teaching math, and integrate math instruction throughout the school day

Ratings of Evidence for the Recommendations

Table 2. Recommendations and corresponding levels of evidence

	Levels of Evidence		
Recommendation	**Strong Evidence**	**Moderate Evidence**	**Minimal Evidence**
1. Teach number and operations using a developmental progression.		◆	
2. Teach geometry, patterns, measurement, and data analysis using a developmental progression.			◆
3. Use progress monitoring to ensure that math instruction builds on what each child knows.			◆
4. Teach children to view and describe their world mathematically.			◆
5. Dedicate time each day to teaching math, and integrate math instruction throughout the school day.			◆

Organization of the Recommendations

Early math content areas:

- Recommendation 1. Teach number and operations using a developmental progression
- Recommendation 2. Teach geometry, patterns, measurement, and data analysis using a developmental progression

How math can be integrated into early childhood education:

- Recommendation 3. Use progress monitoring to ensure that math instruction builds on what each child knows
- Recommendation 4. Teach children to view and describe their world mathematically
- Recommendation 5. Dedicate time each day to teaching math, and integrate math instruction throughout the school day

Integrating Math into Preschool

Sharon Carver's "Live Math Daily" as a guide to Recommendations 4 and 5:

Recommendation 5. Dedicate time each day to teaching math, and integrate math instruction throughout the school day

- Recommending a daily instructional time for early math as a part of circle time or activity stations
- Make math a part of classroom routines

Counting, patterns, and sets in putting out snacks or materials for activities

Combined literacy and numeracy content in selected books

Games, including board games, that require math skills

Integrating Math into Preschool

Recommendation 4. Teach children to view and describe their world mathematically

- Start with children's own math talk when they notice shapes, patterns and numbers of things
- Use open-ended questions to draw out math comments and suggest multiple representations and solutions

"How can you change this pattern to make a new one?"

- Begin to link the child's informal expressions to more formal ones and eventually to symbols

"and" or "more" -> "add"

Sharon's presentation for details and examples:
http://relsouthwest.sedl.org/bridge_events/2015-04-15_prekmath1/index.html

What Early Math Should be Taught?

Early math content areas:

- **Recommendation 1. Teach number and operations using a developmental progression**

- **Recommendation 2. Teach geometry, patterns, measurement, and data analysis using a developmental progression**

Common Core Standards for Kindergarten Math

Grade K Overview

Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- Work with numbers 11-19 to gain foundations for place value.

Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in each category

Geometry

- Identify and describe shapes..
- Analyze, compare, create, and compose shapes.

(http://corestandards.org/2010/math/content/6/EE/1)

What Do Children Have to Learn?

Please memorize this list of words*:*

- ***Wa’*, *cha’*, *wej*, *loS*, *vagh*, *jav*, *Soch*, *chorgh*, *Hut*, wa’maH**

What have you learned?

What more do you know when you know what you have memorized?

Counting and Cardinality

What you need to know beyond the memorized list of Klingon count word:

- Counting is placing the ordered list of count words in one-to-one correspondence with a set of items
- Cardinality is total number of items in the set as shown by the last count word used

Developmental Progression of Number Understanding

Table 3. Examples of a specific developmental progression for number knowledge

Developmental Progression

Subitizing (small-number recognition)	Subitizing refers to a child's ability to immediately recognize the total number of items in a collection and label it with an appropriate number word. When children are presented with many different examples of a quantity (e.g., two eyes, two hands, two socks, two shoes, two cars) labeled with the same number word, as well as non-examples labeled with other number words (e.g., three cars), children construct precise concepts of one, two, and three.
	A child is ready for the next step when, for example, he or she is able to see one, two, or three stickers and immediately—without counting—state the correct number of stickers.
Meaningful object counting	Meaningful object counting is counting in a one-to-one fashion and recognizing that the last word used while counting is the same as the total (this is called the cardinality principle).
	A child is ready for the next step when, for example, if given five blocks and asked, "How many?" he or she counts by pointing and assigning one number to each block: "One, two, three, four, five," and recognizes that the total is "five."
Counting-based comparisons of collections larger than three	Once children can use small-number recognition to compare small collections, they can use meaningful object counting to determine the larger of two collections (e.g., "seven" items is more than "six" items because you have to count further).
	A child is ready for the next step when he or she is shown two different collections (e.g., nine bears and six bears) and can count to determine which is the larger one (e.g., "nine" bears is more).
Number-after knowledge	Familiarity with the counting sequence enables a child to have number-after knowledge—i.e., to enter the sequence at any point and specify the next number instead of always counting from one.
	A child is ready for the next step when he or she can answer questions such as, "What comes after five?" by stating "five, six" or simply "six" instead of, say, counting "one, two, ... six."
Mental comparisons of close or neighboring numbers	Once children recognize that counting can be used to compare collections and have number-after knowledge, they can efficiently and mentally determine the larger of two adjacent or close numbers (e.g., that "nine" is larger than "eight").
	A child has this knowledge when he or she can answer questions such as, "Which is more, seven or eight?" and can make comparisons of other close numbers.
Number-after equals one more	Once children can mentally compare numbers and see that "two" is one more than "one" and that "three" is one more than "two," they can conclude that any number in the counting sequence is exactly one more than the previous number.
	A child is ready for the next step when he or she recognizes, for example, that "eight" is one more than "seven."

Steps in the Progression

Subitizing

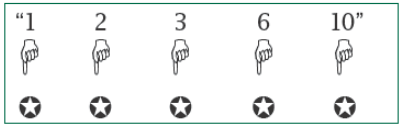
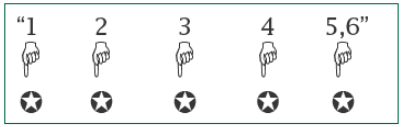
- Recognition without counting of the number of items in small collections of 1–3 things.
- Can be in answer to a “How many?” question

Object counting

- Counting as illustrated before
- Can be different objects and order not important
- Followed by the understanding of cardinality

Counting Errors

Table 4. Common counting errors

Type of Counting Error	Example	Remedy
SEQUENCE ERROR		
Saying the number sequence out of order, skipping numbers, or using the same number more than once.	"1 2 3 6 10" 	Practice reciting (or singing) the single-digit sequence, first focusing on one to ten, then later moving on to numbers greater than ten.
Struggling with the count sequence past twelve.	Skips 15: "1…13, 14, 16, 17, 18." Uses incorrect words: "1…13, 14, fiveteen." "1…18, 19, 10-teen" or "1…29, 20-ten, 20-eleven." Stops at a certain number: "1…20" (stops) "1…20" (starts from 1 again)	Highlight and practice exceptions, such as *fif + teen*. Fifteen and thirteen are commonly skipped because they are irregular. Recognize that a nine signals the end of a series and that a new one needs to begin (e.g., nineteen marks the end of the teens). Recognize that each new series (decade) involves combining a decade and the single-digit sequence, such as twenty, twenty plus one, twenty plus two, etc. Recognize the decade term that begins each new series (e.g., twenty follows nineteen, thirty follows twenty-nine, and so forth). This involves both memorizing terms such as ten, twenty, and thirty by rote and recognizing a pattern: "add *-ty* to the single-digit sequence" (e.g., *six + ty*, *seven + ty*, *eight + ty*, *nine + ty*).
COORDINATION ERROR		
Labeling an object with more than one number word.	"1 2 3 4 5,6" 	Encourage the child to slow down and count carefully. Underscore that each item needs to be tagged only once with each number word.
Pointing to an object but not counting it.	 "1 2 3 4"	Same as above.

Steps in the Progression (continued)

Comparisons of collections

- With small collections, the comparison can be made with subitizing
- Larger collections require counting with counting farther revealing more in the collection

Number-after knowledge

- Start counting with any familiar number and be able to give the next number

Similarities in Other Aspects of Math

- **Recommendation 2. Teach geometry, patterns, measurement, and data analysis using a developmental progression — e.g., for geometry:**

  Be able to name simple shapes (triangle, rectangle, square)

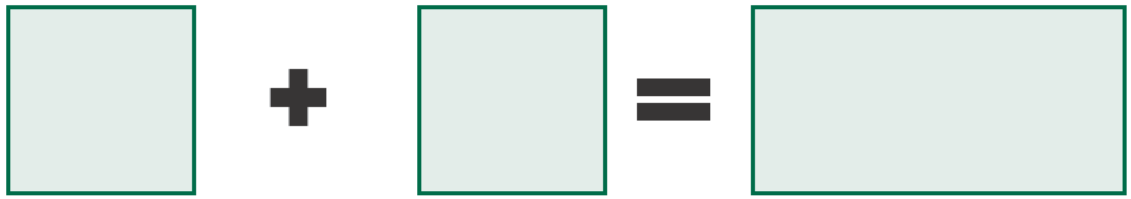
  Show *what is* and *what is not* the shape to work towards a definition (e.g., a triangle has 3 sides and 3 points versus a rectangle that has 4 of each)

  Similar to what we saw with number, memorization is just a beginning. It is also important to move to relations and what can be done with shapes

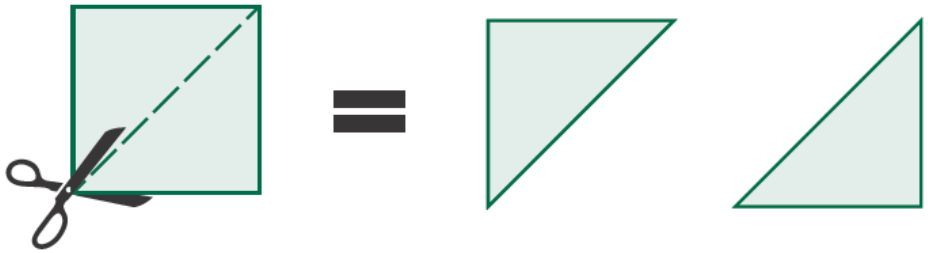
Relations Among Shapes

Figure 4. Combining and separating shapes

Two identical squares can be combined to form a rectangle.



A square can be cut along the diagonal to form two triangles.



Value of Developmental Progressions

Order that math skills are typically understood

Makes it possible to give instruction at children's level so that easier steps are learned before harder, more advanced ones

They also make it possible to use *progress monitoring* to assess where children are in the progression

Characteristics of Developmental Progressions

Some steps in developmental progressions must follow in a given order:

- **For example, being able to understand and solve "5 + 3 =__" requires understanding cardinality**

Other steps describe the *typical* order that children follow but the order could be different and individual children may follow a different path

Comparison Step in the Progression

Counting-based comparisons of collections larger than three	Once children can use small-number recognition to compare small collections, they can use meaningful object counting to determine the larger of two collections (e.g., "seven" items is more than "six" items because you have to count further).
	A child is ready for the next step when he or she is shown two different collections (e.g., nine bears and six bears) and can count to determine which is the larger one (e.g., "nine" bears is more).

Conclusion

Because math is partially progressive, the main steps occur in order

The order or sequence allows:

- **Assessing approximately where children are in the sequence**
- **Arranging instruction so that it anticipates the sequence and targets the appropriate level**

And it implies that aligned early math instruction can serve as the foundation for math learning in school

Availability of the Practice Guide

http://ies.ed.gov/ncee/wwc/PracticeGuide.aspx?sid=18

Presenters

Dr. Akihiko Takahashi

- Associate elementary mathematics professor at DePaul University.
- Since 2000, has worked with schools, school districts, universities, and foundations in the U.S. to design and conduct professional development to improve mathematics teaching and learning.
- Training in education and practical experience in both Japan and the U.S.

Video 1

Planting the Seeds for Common Core State Standards in Mathematics: An Emphasis on Problem Solving

https://youtu.be/jCOPbGCvA1I

Standards for Mathematical Practice

Mathematically proficient students:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Developing these practices begins early

1. Present problems in understandable contexts for the students so that the students can experience the progression from concrete, semi concrete to abstract.
2. Give students opportunities to attack open-ended problems so that the students increase their confidence.
3. Let students use manipulatives not only to find answers but to explain to others how to find answers.
4. Help students learn to communicate how they solve problems using actions, verbal explanations, and equations.

Video 2

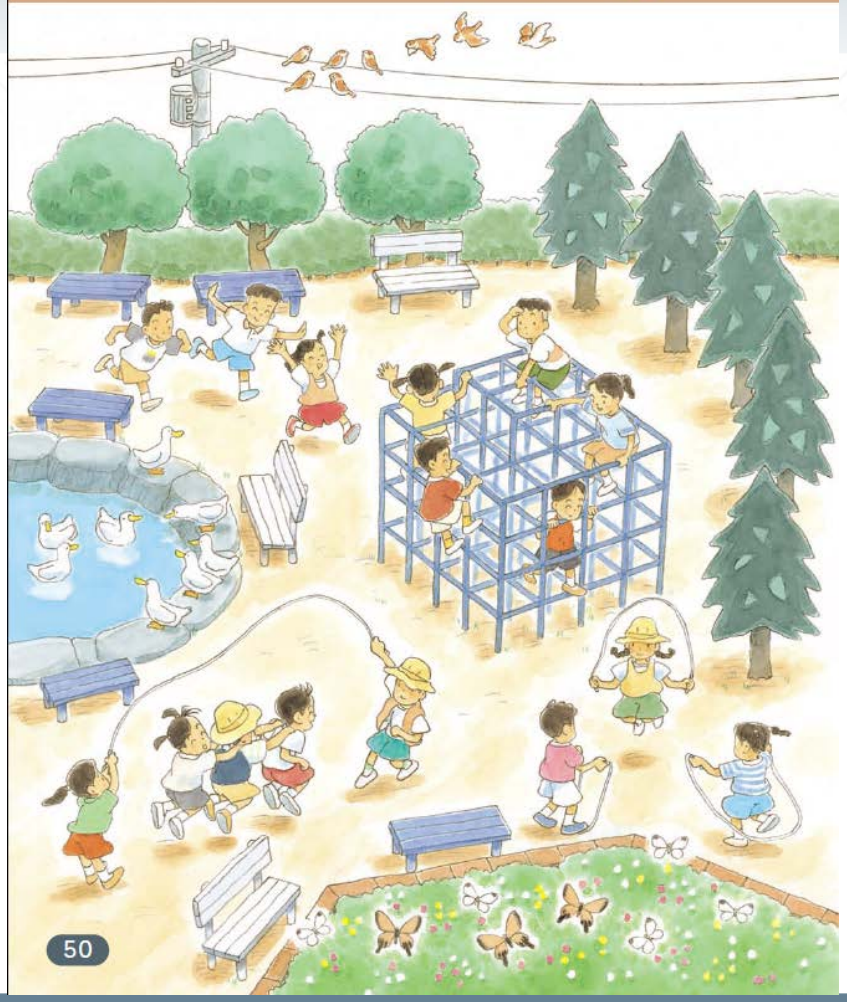
Four Recommendations for Addressing the Standards for Mathematical Practice

https://youtu.be/ipmIeYaSmdI

Recommendation #2

Give students opportunities to attack open-ended problems so that the students increase their confidence.

10 Make story problems for the math sentence 5+3.



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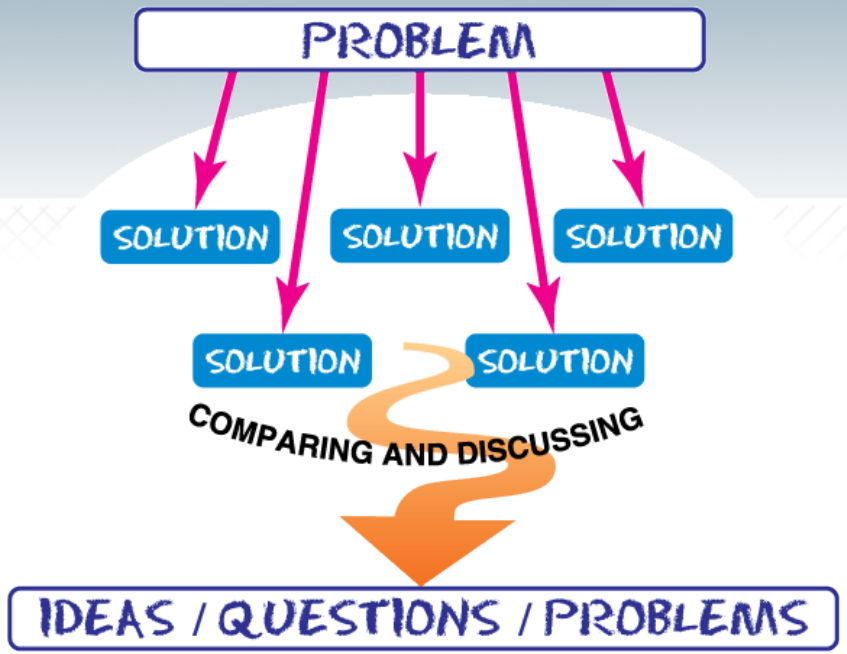
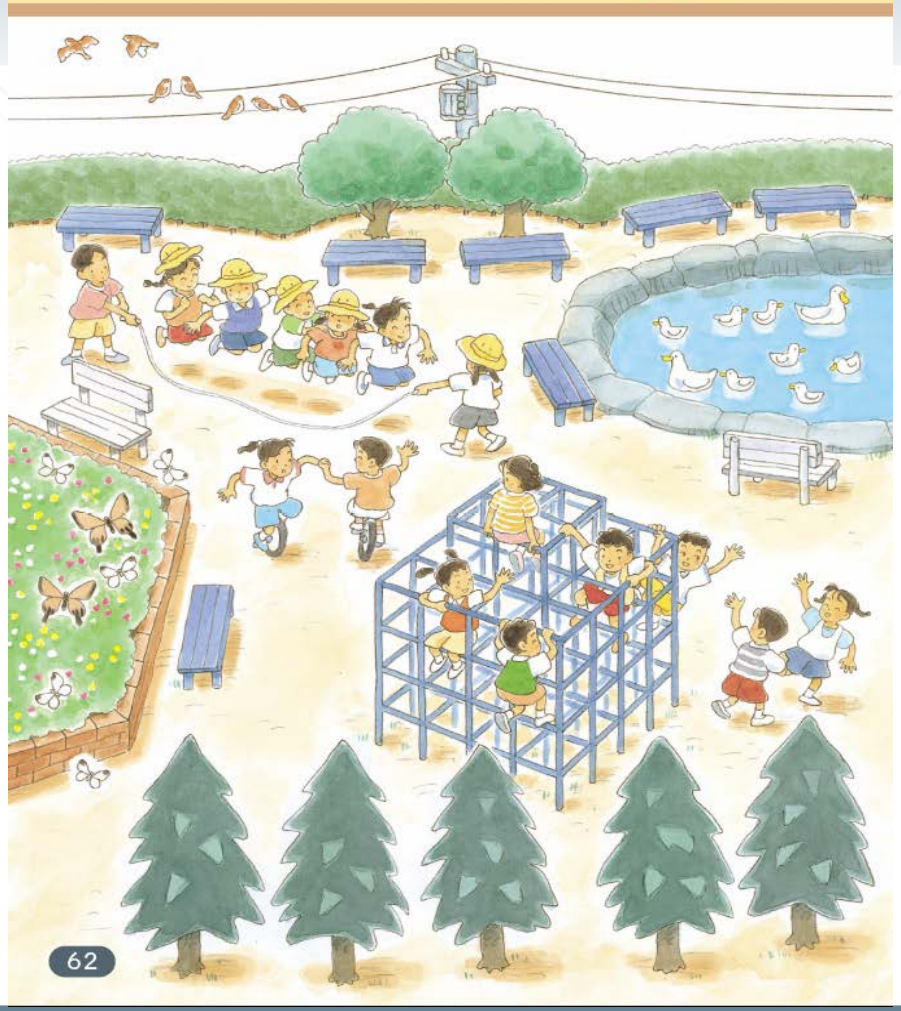


Construct viable arguments and critique the reasoning of others.

7 Make story problems for the math sentence 7−2.



Video 3

Thinking Mathematically through Number Composition and Decomposition

https://youtu.be/kOVdzD3bB6k

Thank You

Additional Materials from this webinar can be found at:
https://relwest.wested.org/events/327