Appendix A: Proof of Equation (29)

Following Hedges (2007), note that \( n(m - 1)S_w^2 / \sigma_w^2 \) has an approximate chi-squared distribution with \( n(m-1) \) degrees of freedom. Thus \( \text{AsyVar}(S_w^2) = 2\sigma_w^4 / n(m - 1) \). Similarly, \( (n - 2)S_B^2 / E(S_B^2) \) has an approximate chi-squared distribution with \( n(m-1) \) degrees of freedom. Thus,

\[
\text{AsyVar}(S_B^2) = 2(\sigma_B^2 + \{\sigma_w^2 / m\})^2 / (n-2).
\]

Using (26), we find then that:

\[
(A.1) \quad \text{AsyVar}(S_y^2) = \frac{2(\sigma_B^2 + \{\sigma_w^2 / m\})^2}{(n-2)} + \frac{[(m-1)/m]^2 2\sigma_w^4}{n(m-1)}.
\]

To obtain a variance expression for \( S_y \) in terms of the variance expression for \( S_y^2 \) in (A.1), we apply a Taylor series expansion of \( S_y \) around \( \sigma_y \):

\[
(A.2) \quad S_y - \sigma_y \approx \frac{(S_y^2 - \sigma_y^2)}{2\sigma_y}.
\]

Because \( S_y^2 \) is asymptotically normal, the delta method implies that \( S_y \) is asymptotically normal with the following asymptotic variance:

\[
(A.3) \quad \text{AsyVar}(S_y) \approx \text{AsyVar}(S_y^2) / 4\sigma_y^2.
\]

After some algebra, (29) follows after inserting (A.1) into (A.3) and replacing \( \sigma_B^2, \sigma_w^2, \) and \( \sigma_y^2 \) by their estimators.