Appendix A: Proof of Equation (29)

Following Hedges (2007), note that $n(m-1)S_W^2 / \sigma_W^2$ has an approximate chi-squared distribution with $n(m\text{-}1)$ degrees of freedom. Thus $AsyVar(S_W^2) = 2\sigma_W^4 / n(m-1)$. Similarly, $(n-2)S_B^2 / E(S_B^2)$ has an approximate chi-squared distribution with $n(m\text{-}1)$ degrees of freedom. Thus,

$$AsyVar(S_B^2) = 2(\sigma_B^2 + \{\sigma_W^2 / m\})^2 /(n-2).$$

Using (26), we find then that:

$$(A.1) \quad AsyVar(S_y^2) = \frac{2(\sigma_B^2 + \{\sigma_W^2 / m\})^2}{(n-2)} + \frac{[(m-1)/m]^2 2\sigma_W^4}{n(m-1)}.$$

To obtain a variance expression for S_y in terms of the variance expression for S_y^2 in (A.1), we apply a Taylor series expansion of S_y around σ_y:

$$(A.2) \quad S_y - \sigma_y \approx \frac{(S_y^2 - \sigma_y^2)}{2\sigma_y}.$$

Because S_y^2 is asymptotically normal, the delta method implies that S_y is asymptotically normal with the following asymptotic variance:

$$(A.3) \quad AsyVar(S_y) \approx AsyVar(S_y^2) / 4\sigma_y^2.$$

After some algebra, (29) follows after inserting (A.1) into (A.3) and replacing σ_B^2, σ_W^2, and σ_y^2 by their estimators.