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Regional Educational Laboratory Midwest

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Teaching Fractions Toolkit

Professional development for grade 6 teachers: Module 1 facilitator guide

Fractions on a number line

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Facilitator guide overview

Facilitator guide purpose

Each Teaching Fractions Toolkit professional development module has a facilitator guide for facilitators who are leading the toolkit's professional development for a group of grade 6 math teachers. The facilitator guides, along with the slide decks for each module, provide facilitators with the information and accompanying resources they need to prepare for and lead teacher learning experiences. With these resources, a facilitator can support teachers with applying evidence-based strategies to enhance grade 6 students' fraction understanding.

Facilitator guide components

Each facilitator guide includes information and resources for the module as a whole (the module introduction), detailed instructions for two facilitated meetings (Meeting A and Meeting B), and interim activities that participating teachers will complete individually between the meetings. The components that follow this overview are as follows.

Professional development modules summary

The professional development modules summary table summarizes the six professional development modules, including their schedule and content. Review this overview to put in context the module you are currently preparing to facilitate for teachers.

Module introduction

The module introduction includes a summary of the module content and activities, the module objectives, a review of which practice guide recommendations are the focus of the module, a review of the sequence of meetings and activities in the module, and an overview of preparation and resources needed to facilitate the module.

Information for facilitating each section of the module

Information about facilitating the module for a group of teachers is divided into three sections:

1. Preparing for and leading Meeting A.
2. Supporting teachers in their individual work on interim activities for the module.
3. Preparing for and leading Meeting B.

The sections about facilitating Meetings A and B for the module include agendas with links to resources, detailed instructions for the professional development activities, tips, and background and rationale information. The section focused on the interim activities provides an overview of self-paced professional development activities that teacher participants will complete individually between meetings along with guidance about supporting teachers in that work.

Credits and references

Credits and references are provided for citations, photos, and/or resources referred to in this facilitator guide and the presentation slide decks for this professional development module.

Color coding in the facilitator guides

In both this guide and the associated module workbook, pages have a right-side page edge color that indicates the module section:

A brown edge indicates **general introductory information** about the module (starts on page 6).

A purple edge indicates information about the facilitated **Meeting A** of the module (starts on page 9).

A green edge indicates the information about the module's **interim activities** to be completed by teachers between meetings (starts on page 22).

A blue edge indicates information about the facilitated **Meeting B** of the module (starts on page 23).

Professional development modules summary

The complete professional development sequence includes six modules, each with three sections. The authors recommend that the modules be completed in sequential order because the activities of later modules build on the activities completed during prior modules.

Module and title	Description of module content	Module sections
Module 1 Fractions on a number line	• Provides a background for the professional development, including why a focus on effective fractions instruction, key areas of emphasis, activities, tools, associated icons, and module structure. • Introduces the practice guide and Recommendation 2 and the importance of visual representations. • Explores the Recommendation 2 implementation steps, with an emphasis on number lines as a central representational tool for teaching fraction concepts, and provides opportunities to engage with associated math tasks. • Introduces the Teacher Reflection Tool and uses it to record a baseline assessment of understanding and use of Recommendations 2-4. • Incorporates additional activities: using online apps for locating fractions on a number line and comparing fractions, introducing toolkit formative assessments, analyzing student work, and completing a module reflection.	**Meeting 1A** (120 minutes) **Module 1 interim activities** (teacher self-paced; 120 minutes) **Meeting 1B** (60 minutes)

Module and title	Description of module content	Module sections
Module 2 Understanding fraction addition and subtraction	• Introduces Recommendation 3 and its implementation steps, with a focus on creating and using area models, number lines, and other visual representations to understand the addition and subtraction of fractions. • Implements a formative assessment with students to help identify and plan to address any common mistakes and misunderstandings related to adding and subtracting fractions. • Incorporates additional activities: reading from the practice guide, using an online app for fraction estimation, working on a fraction addition task, analyzing student work from a fraction addition and subtraction formative assessment, and completing a module reflection.	**Meeting 2A** (60 minutes) **Module 2 interim activities** (teacher self-paced; 120 minutes) **Meeting 2B** (60 minutes)
Module 3 Understanding fraction multiplication and division	• Explores Recommendation 3 implementation steps, with a focus on using real-world contexts with plausible numbers and the use of visual representations to improve students' understanding of procedures for multiplication and division with fractions. • Continues the focus on fraction division and Recommendation 3, with exploration of visual representations used for fraction division tasks and use of predicting and judging the reasonableness of answers for fraction division problems. • Reviews the implementation steps and potential roadblocks associated with Recommendation 3 and supports collaborative planning of a lesson that incorporates those implementation strategies to mitigate anticipated roadblocks. • Incorporates additional activities, including using an online app for dividing fractions, analyzing students' work and identifying strengths and roadblocks related to Recommendation 3, engaging with a sorting task while consolidating knowledge related to Recommendations 2 and 3, revisiting and updating the Teacher Reflection Tool, and completing a module reflection.	**Meeting 3A** (60 minutes) **Module 3 interim activities** (teacher self-paced; 120 minutes) **Meeting 3B** (60 minutes)

Module and title	Description of module content	Module sections
Module 4 Implementation of fraction computation	• Continues the focus on fraction computation and Recommendations 2 and 3, including activities focused on the relationship between fraction multiplication and fraction division. • Applies new classroom implementation strategies associated with implementation steps for Recommendations 2 and 3 and includes approaches to mitigate anticipated roadblocks. • Incorporates additional activities, including using an online app for multiplying fractions, reading about Recommendation 3 implementation steps, analyzing your students' work and identifying strengths and roadblocks related to Recommendations 2 and 3, and completing a module reflection.	**Meeting 4A** (60 minutes) **Module 4 interim activities** (teacher self-paced; 120 minutes) **Meeting 4B** (60 minutes)
Module 5 Understanding ratio, rate, and proportion	• Introduces Recommendation 4, with a focus on Implementation Steps 1 and 2, associated Roadblocks 4.1 and 4.2, and activities to support understanding of proportional relations and using visual representations to solve ratio, rate, and proportion problems. • Uses an online app to explore and use visual representations, including tape diagrams, to solve ratio and rate problems. • Incorporates additional activities, including using visual representations to explore the multiplicative relations within and between equivalent ratios; using double number lines to solve ratio, rate, and proportion problems; making connections between visual representations and the cross-multiplication strategy; and completing a module reflection.	**Meeting 5A** (60 minutes) **Module 5 interim activities** (teacher self-paced; 120 minutes) **Meeting 5B** (60 minutes)
Module 6 Implementation of ratio, rate, and proportion	• Explores Recommendation 4 further, with a particular focus on Implementation Step 3 and associated Roadblock 4.3. • Applies and supports reflection on classroom implementation based on Recommendation 4 implementation steps as well as approaches for mitigating anticipated roadblocks. • Incorporates additional activities, including using the Teacher Reflection Tool, to support reflection on understanding and use of the practice guide, and on learning completed across the professional learning modules.	**Meeting 6A** (60 minutes) **Module 6 interim activities** (teacher self-paced; 120 minutes) **Meeting 6B** (60 minutes)

Note: The information about how to carry out the recommendation is in the form of steps for implementing the recommendation, which are referred to as "implementation steps" throughout this professional development.

Module 1 introduction

Module 1 introduces the structure and content of the practice guide, especially Recommendation 2, with particular attention to the background information, how to carry out the recommendation, and potential roadblocks and solutions. An introduction to Recommendation 2 describes the importance of using visual representations to support effective fractions instruction.

The module includes learning about and using number lines as a central representational tool for teaching fraction concepts, particularly locating and comparing fractions. Activities include engaging in math tasks to place fractions on a number line to determine the relationship between them and then exploring potential misunderstandings and roadblocks associated with implementing these strategies in the classroom. The Teacher Reflection Tool is introduced as a gauge for understanding and using Recommendations 2-4.

Additional activities include using online apps for locating fractions on a number line and comparing fractions, incorporating strategies to carry out Recommendation 2, introducing toolkit formative assessments, using a template for analyzing student work, and completing a module reflection.

Objectives for Module 1

By the end of Module 1, teachers will be able to do the following:

- Identify the structure and importance of the Developing Effective Fractions Instruction for Kindergarten Through 8th Grade practice guide.
- Explain the importance of student recognition of fractions as numbers.
- Identify how number lines represent and build an understanding of whole numbers, fractions, and decimals.
- Understand and use number lines to locate and compare fractions in instruction.
- Understand and use number lines and fraction strips to show fraction equivalence and density in instruction.
- Recognize misunderstandings in student work and apply strategies and approaches to remediate them.
- Use the Teacher Reflection Tool to self-assess baseline understanding and use of the practice guide recommendations.

Recommendation focus for Module 1

The Teaching Fractions Toolkit professional development focuses on practice guide Recommendations 2-4 in the context of grade 6 math content. The design of the professional development also is in line with Recommendation 5 and its implementation steps, which relate to professional development programs for teachers of students learning fractions content.

Module 1 specifically focuses on building knowledge related to Recommendation 2 in the context of grade 6 standards and foundational knowledge often taught in grades 4-5.

Recommendation 2: “Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.”

Sections of Module 1

This module includes an initial facilitated meeting (Meeting 1A), a set of self-paced activities for teachers to do between meetings (interim activities) that is estimated to take 2 hours, and a second facilitated meeting (Meeting 1B). Unlike the later modules, the design of Module 1 does not require any classroom implementation by participating teachers during the interim activities. So, Module 1 can be conducted prior to the start of the school year, and it is possible to combine the two meetings and the interim activities into one or two workshop days. In later modules, classroom implementation will be part of the interim activities, so the meetings will need to be spaced out over a couple of weeks.

Module 1

Fractions on a number line focuses on Recommendation 2, with an emphasis on number lines.

Meeting 1A ➔ **Module 1 interim activities** ➔ **Meeting 1B**

Preparation and resources for Module 1

Before leading Module 1, be sure to review all resources and facilitator instructions in this guide and in the slide decks, review pages 6-11 and 19-25 of the practice guide, explore the module activities yourself if you have not before, and anticipate what questions your participants might have. Make sure that a room is reserved with workspace for teachers to explore tasks and discuss ideas if meeting in person or ensure a videoconference setup if meeting online. Ensure that you will have internet access so you can access and demonstrate the online apps. Based on the size of your group, consider when you will use breakout groups to increase opportunities for engagement and discussion. Also consider how you will gauge participants' developing understanding. In larger groups you might consider using informal checks for understanding or exit tickets.

To facilitate this module, in addition to this facilitator guide, you will need the materials listed below that are also posted on the Teaching Fractions Toolkit website. It may be helpful to print copies of these to refer to when preparing for and facilitating the meetings so you do not have to view everything electronically when leading the meetings:

- **Practice guide:** The full practice guide that serves as the foundation for this professional development toolkit.
- **Slide deck:** The Meeting 1A slide deck and the Meeting 1B slide deck.
- **Participant workbook:** The Module 1 workbook that includes all handouts used in this module. Each participant will need access to an electronic copy or a print copy of the workbook. Participants may prefer a print copy so they can write on it.
- **Additional resources:** Module activities refer to online apps, readings, videos, and other resources. Links to these resources are summarized in the following table for your quick reference and also embedded in the participant workbook.

Type	Resource	Link and description
Online app	Comparing Rational Numbers–Fractions	https://www.geogebra.org/m/vcy4zn8e Use multiple models to compare two common fractions.
Online app	Locating Fractions on a Number Line (1)	https://www.geogebra.org/m/uchh5qmd Place common fractions on a number line and receive feedback about the locations.
Optional online app	Locating Fractions on a Number Line (2)	https://www.geogebra.org/m/xh7mtge2 This supplemental_resource is similar to Locating Fractions on a Number Line (1) app but with different common fractions to place on the number lines.
Optional online apps	Locating Decimals on a Number Line (1) and (2)	https://www.geogebra.org/m/yyqpcnxf https://www.geogebra.org/m/bmtt9cd5 These two apps are supplemental_resources and similar to Locating Fractions on a Number Line (1) but with decimal fractions to place on the number lines.
Practice guide readings	Introduction to the practice guide and Recommendation 2	Read pages 6-7 to learn about the practice guide and pages 19-25 to learn about Recommendation 2.

Meeting 1A

During Meeting 1A, introduce participants to the practice guide and support their exploration of Recommendation 2, including the use of number lines when teaching fraction concepts. Lead activities in which participants use online apps to locate fractions on number lines and compare fractions, laying the groundwork for the interim activities between Meetings 1A and 1B.

The agenda for Meeting 1A is as follows.

Agenda and resources for Meeting 1A

Minutes	Topic and resources
5	**Introductions and icebreaker** (slides 1-3)
10	**Background for professional development** (slides 4-16) • Module 1, Handout 1: Name the point exploration
15	**Practice guide overview** (slides 17-20) • Practice guide overview • Practice guide structure (page iii)
20	**Introduction to recommendations** (slides 21-28) • Recommendation 2 (page 19) and its implementation steps (page 21) • Potential roadblocks for Recommendation 2 (page 25)
5	**Break** (slide 29)
20	**Math tasks with number lines** (slides 30-38) • Module 1, Handout 2: Fractions on the number line
25	**Online apps** (slides 39-48) • Module 1, Handout 3: Locating Fractions on a Number Line (1) app instructions • Locating Fractions on a Number Line (1) app: https://www.geogebra.org/m/uchh5qmd • Module 1, Handout 4: Comparing Rational Numbers–Fractions app instructions • Comparing Rational Numbers–Fractions online app: https://www.geogebra.org/m/vcy4zn8e
10	**Introduction to the Teacher Reflection Tool** (slides 49-52)
10	**Wrap-up** (slides 53-59) • Instructions for Module 1 interim activities

Facilitator directions for Meeting 1A

The directions for how to facilitate each Meeting 1A agenda topic are in the following table.

Introductions and icebreaker (5 minutes, slides 2-3)	
1. **Review the agenda** (slide 2). 2. **Have participants introduce themselves and respond to the icebreaker prompt** about which fraction represents them (slide 3).	**Tip:** Each person can share their introduction out loud if you have a small group; if it is a larger group, then have everyone share on a shared online document or in a videoconference chat. **Tip:** Example icebreaker responses in the past included statements such as "$\frac{5}{4}$ because my to do list is overly full!" or "$\frac{1}{3}$ because I'm one of three children." **Tip:** Slides can be edited to align with local context. If participants know each other, introductions may not be needed.

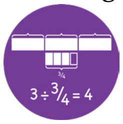
Background for professional development (10 minutes, slides 4-16)	
1. **Provide an overview of the Teaching Fractions Toolkit professional development and Module 1 content.** Share the following information (slides 4-8): • Although the importance of fractions is well known, students continue to have difficulties with understanding and applying fractions in problem solving in middle school math courses and beyond. Even after studying fractions and related topics for several years, U.S. students often lack a conceptual understanding of fractions (Siegler et al., 2010). Without an adequate understanding of fraction concepts, such as fraction magnitude, number line estimation, and magnitude comparisons, students are not prepared for problem solving with fractions (Barbieri et al., 2020; Liu, 2018), and misconceptions about operations with fractions have been repeatedly observed on postsecondary math placement tests (Lee & Boyadzhiev, 2020).	**Background:** The Fractions Content Progression document provides additional information about how content related to fractions develops across the grades. **Background:** Citations are included for your reference in the background information in case you or participants want to know more about where this research comes from. Full references for these citations can be found in the Credits and references section at the end of this document. **Background:** It is important for participants to understand that Teaching Fractions Toolkit professional development is not a curriculum, but rather a course for

- Understanding fractions, rates, and ratios in grade 6 builds to proportional reasoning content in the grade 7 standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), which, in turn, is associated with later math achievement and success in gatekeeping courses such as Algebra I (Empson et al., 2011). Computational skills with fractions underpin advanced math (Booth & Newton, 2012), are essential for success in high school math, and are a prerequisite for college-level math courses (Siegler & Lortie-Forgues, 2015).
- Students who have experienced opportunity gaps, such as multilingual learner students (Saxe & Sussman, 2019), students with math difficulties and disabilities (Barbieri et al., 2020; Butterworth, 2005; Butterworth & Reigosa-Crespo, 2007), and students of color (Minor, 2016), experience more difficulties with fraction problem solving.

2. **Review fractions professional development background and structure.**
   - Reinforce the overarching goals of the professional development (slides 7-8).
   - Review the icons for activities and tools. Share that the icons will be used on materials, including the presentation slides, to designate particular types of resources and tools, including math tasks, student work analysis, lesson planning and implementation, use of online interactive apps, use of the Teacher Reflection Tool, and use of formative assessments (slide 9).
   - Reinforce that each module consists of two synchronous meetings separated by some interim asynchronous activities (slide 10).
3. **Review the Module 1 objectives** (slides 11-12).
4. **Let participants know that they will now work on a brief math exploration** to start considering some ideas about number lines that will be further explored during this module.

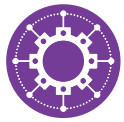
teachers to learn about evidence-based recommendations and strategies. In addition, it may be helpful to share with participants that this professional development Is one component of the Teaching Fractions Toolkit. The Teaching Fractions Toolkit supports the implementation of evidence-based recommendations for increasing student understanding of fractions from the What Works Clearinghouse practice guide Developing Effective Fractions Instruction for Kindergarten Through 8th Grade. The toolkit includes resources for administrators and leaders as they address fractions instruction in kindergarten through eighth grade. The toolkit professional development that you are facilitating focuses on grade 6 math content and has resources for teachers in addition to the resources for you as a facilitator. The Developing Effective Fractions Instruction for Kindergarten Through 8th Grade Toolkit: Overview briefly describes the key components of the toolkit.

The six teacher modules include the following:



Exploration of math tasks

Student work analysis



Lesson planning and implementation

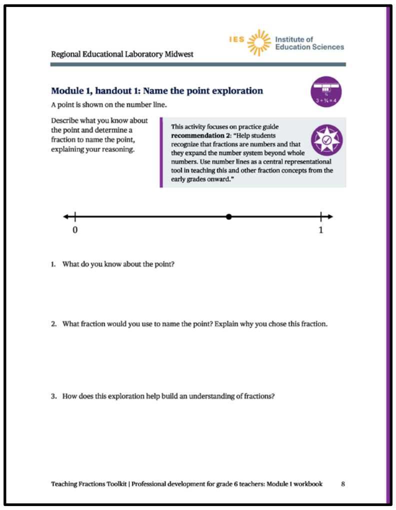
Use of online interactive apps

5. **Have participants work on questions 1 and 2 from Module 1, Handout 1,** share their thinking with the group, and then discuss how students might approach the problem (slide 13). Emphasize that they can estimate what fraction is indicated by the location of the point—they do not need to identify the exact fraction. After participants have completed the task as a learner, you will ask them, “What knowledge is required to complete this activity?” to help inform their practice with students.
6. **Show the number line with additional partitioning** and ask participants how the additional partitions support solving the problem (slides 14-15). Possible responses can include:
   - “The added partitions help to see that the point is halfway between $\frac{1}{2}$ and $\frac{3}{4}$.”
   - “I can now see the point as $\frac{5}{8}$ because $\frac{1}{2}$ is $\frac{4}{8}$ and it is $\frac{1}{8}$ more.”
7. **Support a discussion among participants about how the Name the point exploration can help students build their understanding of fractions** and uncover what they know about fractions (slide 16). Note the following:
   - This exploration connects to the practice guide and the recommendation that is the focus of this module.
   - The practice guide provides implementation steps to help enact the recommendations in the classroom.
   - For example, Recommendation 2, Implementation Step 2, states that teachers should “provide opportunities for students to locate and compare fractions on number lines.” Initially, students can use number lines on which fractions are already marked when locating and comparing fractions.
   - Possible responses could include: “Students need to know that numerous fractions exist on a number line between 0 and 1,” “Students need to know that you can make partitions of equal size on the number line to locate fractions,” “Students need to know that the number of equal parts that a number line is partitioned into is the same as the denominator of the unit fraction.”

Diagnostic and reflection resources for teachers:

Teacher Reflection Tool

Formative assessments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Module 1, handout 1: Name the point exploration

A point is shown on the number line.

Describe what you know about the point and determine a fraction to name the point, explaining your reasoning.

This activity focuses on practice guide **recommendation 2:** “Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.”

1. What do you know about the point?
2. What fraction would you use to name the point? Explain why you chose this fraction.
3. How does this exploration help build an understanding of fractions?

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(Module 1, Handout 1, Workbook)

Background: Recommendation 2 identifies the use of number lines as a central representational tool for teaching fraction concepts from early grades onward. This exploration provides a concrete example for participants to experience and explore prior to digging into the practice guide structure and associated recommendations more fully.

Practice guide overview (15 minutes, slides 17-20)

1. **Explain that the professional development is based on the recommendations outlined in the What Works Clearinghouse practice guide [Developing Effective Fractions Instruction for Kindergarten Through 8th Grade](#).**
2. **Provide a brief overview of the practice guide and recommendations** (slide 18).
   - This practice guide presents five recommendations intended to help educators improve students' understanding of fractions.
   - The recommendations include strategies to develop young children's understanding of early fraction concepts and ideas for helping older children understand the meaning of fractions and the computations involved.
   - The practice guide also highlights ways to build on students' existing strategies to solve problems involving ratios, rates, and proportions.
   - For each recommendation, the practice guide includes the recommendation, guidance (how to carry out the recommendation), potential roadblocks (obstacles and advice from the expert panel about how to overcome them), and tools and resources (practical information and examples to help practitioners implement the recommendation).
3. **Explain that throughout the practice guide, and therefore throughout the Teaching Fractions Toolkit and these professional development modules, the term "fraction" refers to the full range of ways of expressing rational numbers, including decimals, percentages, and negative fractions** (slide 19).
4. **Provide an overview of the structure of the practice guide** (slide 20).
   - All five recommendations are summarized on page 1 of the practice guide. This summary is followed by an introduction that provides an overview of the importance of developing effective fractions instruction from kindergarten through grade 8 and provides the key parameters considered in developing the practice guide. It also summarizes the

Tip: The Practice Guide Overview document is a resource for understanding what the practice guide includes.

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Developing effective fractions instruction for kindergarten through 8th grade toolkit

Practice guide overview

Regional Educational Laboratory Midwest
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This document provides an overview of the recommendations for fraction instruction presented in the What Works Clearinghouse practice guide *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*. The recommendations are based on rigorous research on improving grades K-8 students' understanding of fractions and are intended to be used by general education teachers, math specialists and coaches, special educators, and administrators.

Recommendations[1] in this practice guide:

1. Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts.
2. Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers.
3. Help students understand why procedures for computations with fractions make sense.
4. Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems.
5. Professional development programs should place a high priority on improving teachers' understanding of fractions and how to teach them.

The practice guide section for each recommendation includes the following:

The Recommendation

Each actionable recommendation includes a summary of the research evidence that supports the recommendation, along with a description of how the recommended practice supports student learning.

Implementation steps

Implementation steps provide guidance about how to carry out each recommendation. This guidance is informed by studies that support the recommendation and by the panel's expertise.

Potential roadblocks and solutions

A section on roadblocks and solutions explains potential obstacles related to the recommendation and offers advice from the expert panel about approaches for overcoming them.

Readers are encouraged to use the provided advice in ways that work best in their context. Users of this practice guide may decide to implement some or all of the recommendations. The complete practice guide can be accessed on the What Works Clearinghouse website: https://ies.ed.gov/ncee/wwc/PracticeGuide/15

[1] Text for the recommendations is from the WWC practice guide.

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(Module 1, Practice guide overview)

Recommendation 1: "Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts."

Recommendation 2: "Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward."

Recommendation 3: "Help students understand why procedures for computations with fractions make sense."

recommendations and includes a discussion of the research supporting the guide.

- Five chapters each dive deeply into one recommendation.
- Strategies described for Recommendations 2-5 have been shown to be particularly beneficial for both students struggling in math and multilingual learner students.
- The guide also includes a glossary, appendices, references, and an index of key mathematical concepts.

5. **In line with this practice guide, this professional development ...**
   - Focuses on effective practices related to fraction computation and rate and ratio problem solving.
   - Deepens knowledge of why computational procedures with fractions work.
   - Reinforces understanding and use of a range of visual representations to support fraction. computation and ratio and rate problem solving.
   - Extends knowledge of common misunderstandings and roadblocks to solving problems with fractions and provides strategies to overcome them.

Recommendation 4: "Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems."

Recommendation 5: "Professional development programs should place a high priority on improving teachers' understanding of fractions and of how to teach them."

Introduction to recommendations (20 minutes, slides 21-28)

1. **Review the structure of the recommendation chapters of the practice guide** (slide 22).
2. **For each recommendation, you will find a detailed explanation of the recommendation,** information about how to carry out the recommendation (with examples), and potential roadblocks and solutions. The information about how to carry out the recommendation is in the form of steps for implementing the recommendation, which will be referred to as "implementation steps" throughout this professional development.
3. **Introduce Recommendation 2,** which starts on page 19 in the practice guide and explain that this recommendation will be the focus throughout this module (slide 23). Share the following:
   - Recommendation 1 focuses on ideas that are relevant in the elementary grades. Because this professional

Recommendation 2: "Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward."

Background: "Evidence for this recommendation primarily comes from studies demonstrating the usefulness of number lines for developing number sense with whole numbers. These studies used number line representations to teach development is designed for grade 6 teachers, it starts with Recommendation 2. Although some of the content of Recommendation 2 may be covered in late elementary school, this recommendation is still relevant in grade 6. Many common errors and misunderstandings stem from not understanding that fractions are numbers with magnitudes. Early fractions instruction generally focuses on the idea that fractions represent parts of a whole but does not convey the critical idea that fractions are numbers with magnitudes (values) that can be either ordered or considered equivalent.

4. **Ask participants what common mistakes and misunderstandings regarding fractions** they have seen in their teaching.
5. **Provide an overview of the implementation steps** that are provided for Recommendation 2 (slide 24). Emphasize the following:
   - Each implementation step provides a rationale as well as examples that can be used with students.
   - If we look at Implementation Step 2, we see that it focuses on providing opportunities for students to locate and compare fractions on number lines.
   - If we look at Implementation Step 3, we see that the focus is on fraction equivalence, fraction density (the concept that there are an infinite number of fractions between any two fractions), and negative fractions.
   - During Module 1, we will focus primarily on Implementation Steps 2 and 3. We will return to Implementation Steps 1 and 4 in later modules.
6. **Share an example of Implementation Step 3** (slide 25). Explain the following concepts:
   - Number lines can be used to help students understand that an infinite number of fractions exist between two fractions. Number lines also can be used to help students recognize equivalent fractions. They can see that more than one fraction can exist at the same location on the number line.
   - Looking at this example, students can see that $\frac{2}{4}$ is at the same location as $\frac{1}{2}$.
7. **Review the potential roadblocks for Recommendation 2** (slide 26). Explain the following concepts:

preschool and early elementary students about the magnitudes of whole numbers. An additional study showed how number lines can be used to teach decimals successfully. These findings indicate that number lines can improve the learning of fractions in elementary and middle school" (pp. 20-21).

Tip: Examples of misunderstandings that participants may raise include adding numerators and denominators, thinking that fractions are not numbers, not understanding that an infinite range of numbers exists between successive whole numbers or between any two fractions, and misunderstandings about how fractions are related to whole numbers. Other common misunderstandings will be explored further later in this module and in subsequent modules.

- Along with each roadblock in the practice guide is a suggested approach to address the roadblock. For example, a suggested approach to Roadblock 2.1 is that “teachers should demonstrate that inserting three equally spaced hash marks divides the space into four equal segments or fourths,” leading to the generalization that “dividing the number line into $\frac{1}{n}$ units requires drawing $n - 1$ hash marks between two whole numbers” (p. 25).
- Roadblock 2.2 can be addressed by presenting students with contrasting cases; for example, have students locate 3 and 4 on a 0–4 number line and then identify the fraction $\frac{3}{4}$ as a fraction between 0 and 1.
- Understanding the equivalence of two fractions on a number line can be supported by showing the equivalent sets of numerical fractions above and below the number line. For example, halves could be labeled above the number line and eighths below the number line.
- Roadblock 2.4 can be addressed adding number lines to your instruction, particularly unmarked number lines. This will avoid simply counting the parts without understanding.

8. **Review Roadblock 2.1 in more depth by looking at potential solutions and an example** (slides 27-28).
   - Teach students to represent fraction units as **equal-size segments** on a number line.
   - Example: Fourths are represented by four equal-size segments between two whole numbers.
   - Demonstrate that drawing $\frac{1}{n}$ **units** requires using $n - 1$ **hash marks**.
   - Example: Fourths are represented by three equally spaced hash marks between two whole numbers; this creates four equal-size segments between the whole numbers.
   - Teach the student to draw $n - 1$ hash marks to create $\frac{1}{n}$ units; the student should use $4 - 1 = 3$ hash marks to make four equal-size segments between 0 and 1.

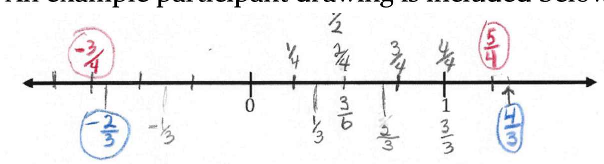
Background on Roadblock 2.1 example: Looking at two number lines partitioned differently, we can support understanding of what $\frac{1}{n}$ means. If you’re looking at four equal-size segments, the number of hash marks between the endpoints of the divided segment is three. If four hash marks are inserted between 0 and 1, then we are actually dividing the segment into five equal parts. So, the number of hash marks in between endpoints is one less than the number of equal parts needed.

Break (5 minutes)

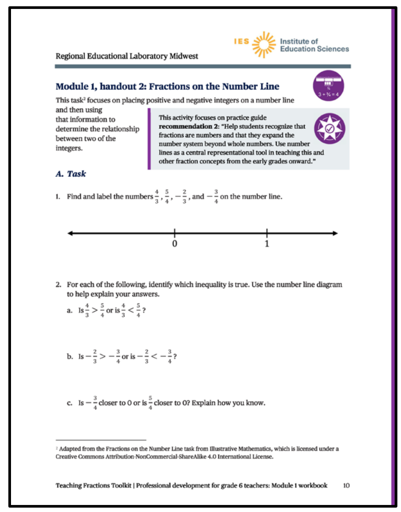
Remind participants to take care of their personal needs and supply the actual time to begin the next activity. Point out that the next activity will be engaging in a math task and will require Module 1, Handout 2 in their workbook.

Math tasks with number lines (20 minutes, slides 30-38)

1. **Have participants work on part A of Handout 2, the Fractions on the Number Line task,** individually for a few minutes and then share their strategies with the group (slides 31-32).
   - An example participant drawing is included below:



   - Notice in this example that the hash marks on the top of the number line are in intervals of $\frac{1}{4}$. Additional hash marks have been added and labeled in intervals of $\frac{1}{3}$ below the number line.
2. **Lead a discussion about the role of number lines in the task.** Prompt participants to consider how understanding the magnitude of fractions relates to this task (slide 33).
   - Number lines can be useful when comparing fractions because students can partition the number line and see relative size and location of fractions. If needed, two number lines can be drawn one above the other to compare fractions. (See figure 4, p. 23, in the practice guide for more information.)
   - Participants can use an understanding of unit fractions to answer reflection question 2. $-\frac{3}{4}$ is three $\frac{1}{4}$ to the left of 0, and $\frac{5}{4}$ is five $\frac{1}{4}$ to the right of 0.
   - Students need to understand the size of unit fractions and the relative relationship when the magnitude of the numerator or denominator changes.
3. **Participants can provide resources to students in the classroom, such as** a handout with stacked number lines (figure 4, p. 23, in the practice guide), Cuisenaire



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Module 1, handout 2: Fractions on the Number Line

This task[2] focuses on placing positive and negative integers on a number line and then using that information to determine the relationship between two of the integers.

This activity focuses on practice guide **recommendation 2:** "Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward."

A. Task

1. Find and label the numbers $\frac{4}{3}$, $\frac{5}{4}$, $-\frac{2}{3}$, and $-\frac{3}{4}$ on the number line.

2. For each of the following, identify which inequality is true. Use the number line diagram to help explain your answers.
   a. Is $\frac{4}{3} > \frac{5}{4}$ or is $\frac{4}{3} < \frac{5}{4}$?
   b. Is $-\frac{2}{3} > -\frac{3}{4}$ or is $-\frac{2}{3} < -\frac{3}{4}$?
   c. Is $-\frac{3}{4}$ closer to 0 or is $\frac{5}{4}$ closer to 0? Explain how you know.

[2] Adapted from the Fractions on the Number Line task from Illustrative Mathematics, which is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License.

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(Module 1, Handout 2, Workbook)

Background: Two insights support working on the problems on this handout: seeing the position of the fractions relative to 1 or -1 and seeing that thirds are bigger than fourths.

Tip: Encourage participants to use the relative size of the fractions to place the fractions on the number line rather than using common denominators. (Because $\frac{5}{4}$ is $\frac{1}{4}$ more than a whole and $\frac{4}{3}$ is $\frac{1}{3}$ more than a whole, and $\frac{1}{4}$ is less than $\frac{1}{3}$, $\frac{5}{4}$ is closer to 1 than $\frac{4}{3}$. Use the same relative size to place $-\frac{3}{4}$ and $-\frac{2}{3}$ as each fraction is a unit fraction away from -1.)

rods or multi-link fraction blocks, or online apps, to help reinforce the relative size of fractions. They can model their own thinking as they solve the problem, using a number line.

4. **Share the practice guide definition of magnitude and an overview of key points about fraction magnitude** (slide 34).
   - Fractions are numbers with magnitudes (values) that can be either ordered or considered equivalent.
   - Page 18 in the practice guide states: "fractions and equivalent magnitude differences (e.g., understanding that the difference between 0 and $\frac{1}{2}$ is the same as the difference between 1 and $1\frac{1}{2}$ or between 73 and $73\frac{1}{2}$)."
   - Page 20 in the practice guide states: "Number lines can clearly illustrate the magnitude of fractions; the relation between whole numbers and fractions; and the relations among fractions, decimals, and percents."
   - For any positive fraction, magnitude increases as the numerator increases, and magnitude decreases as the denominator increases. For example, if I have $\frac{1}{3}$ and the numerator increases to 2, then I have $\frac{2}{3}$ and the magnitude (value) of this fraction is greater than $\frac{1}{3}$. If I have $\frac{1}{3}$ and the denominator increases to 5, then I have $\frac{1}{5}$ and the magnitude (value) of this fraction is less than $\frac{1}{3}$.

Background: The practice guide uses the term "magnitude" to mean that fractions are numbers with numerical values that can be either ordered or considered equivalent.

5. **Share how number lines can support student understanding of magnitude and advanced fraction understanding** and note how the Fractions on the Number Line task illustrates that number lines can be used to understand and compare magnitudes of fractions (slides 35-36).
6. **Give participants time to reflect on and discuss one or two implications of Recommendation 2 for classroom instructional practice and one or two potential roadblocks** and suggested approaches for addressing them. Participants can write notes under question 4 on Handout 2 and share their ideas in the group (slide 37).

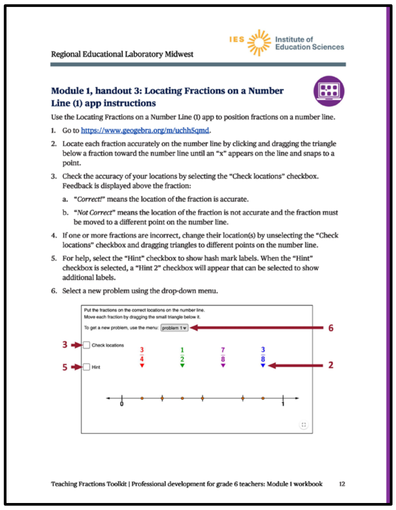
Tips: Potential implications for classroom practice could include using visual representations such as fraction strips, number lines, or hands-on manipulatives to illustrate fraction magnitude or using resources such as a handout of stacked number lines to review the relative size of fractions, show equivalent fractions, and demonstrate the density of fractions on a number line. Potential roadblocks and suggested approaches can include those outlined on page 25 of the practice guide. Potential misunderstandings

7. **Ask participants to discuss possible mistakes, misunderstandings, or roadblocks that may occur related to the Fractions on a Number Line task,** along with strategies and approaches they could implement to address the mistakes, misunderstandings, or roadblocks (slide 38).

specific to the task could include thinking that $\frac{5}{4}$ is greater that $\frac{4}{3}$ because 5 is greater than 4 or that the number line needs four partitions because one of the fractions is fourths.

Online apps (25 minutes, slides 39-48)

1. **Prepare participants for the Module 1 interim activities by providing an overview about the use of apps with dynamic visualizations** and specifically the features of the Locating Fractions on a Number Line (1) app. Stress the value of using apps to create accurate visual representations and rapid iteration, features that are particularly helpful when reinforcing fraction magnitude (slides 40-41).
2. **Go to the Locating Fractions on a Number Line (1) app** (https://www.geogebra.org/m/uchh5qmd) and demonstrate the app's features using problems 1 and 2. Give participants time for an initial exploration of the app in preparation for their work during the interim activities (slides 42-44).
   - Refer to the Module 1, Handout 3: Locating Fractions on a Number Line (1) app instructions, which provide step-by-step details about using the app.
   - Point out the key features of the app, including using the triangle below the fraction to move it to the number line, using the "Check locations" check box to check locations, and using the "Hint" checkbox to add scaffolding to support placement of the fractions.
   - Ask participants to share and discuss observations and questions as they explore the app.
   - You can review the Locating Fractions on a Number Line (1) app screencast to prepare for your app demonstration.
3. **Next, introduce the Comparing Rational Numbers—Fractions app** (https://www.geogebra.org/m/vcy4zn8e) and share that the app provides both an area model and a linked number line (slides 45-47).
   - This design supports students in bridging the part to whole understanding of fractions from elementary school to placement on a number line. It also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Module 1, handout 3: Locating Fractions on a Number Line (1) app instructions

Use the Locating Fractions on a Number Line (1) app to position fractions on a number line.

1. Go to https://www.geogebra.org/m/uchh5qmd.
2. Locate each fraction accurately on the number line by clicking and dragging the triangle below a fraction toward the number line until an "x" appears on the line and snaps to a point.
3. Check the accuracy of your locations by selecting the "Check locations" checkbox. Feedback is displayed above the fraction:
   a. "*Correct!*" means the location of the fraction is accurate.
   b. "*Not Correct*" means the location of the fraction is not accurate and the fraction must be moved to a different point on the number line.
4. If one or more fractions are incorrect, change their location(s) by unselecting the "Check locations" checkbox and dragging triangles to different points on the number line.
5. For help, select the "Hint" checkbox to show hash mark labels. When the "Hint" checkbox is selected, a "Hint 2" checkbox will appear that can be selected to show additional labels.
6. Select a new problem using the drop-down menu.

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(Module 1, Handout 3, Workbook)

Tip: If you feel comfortable, you might choose to go directly to the app and hide the slides that include the app directions. These directions will be available for reference as needed.

Tip: If time allows you can also introduce two additional apps to locate decimals on a number line. These apps include the Locating Decimals on a Number Line (1) (https://www.geogebra.org/m/jfbqrhrg#material/yyqpcnxf) and Locating Decimals on a Number Line (2) (https://www.geogebra.org/m/jfbqrhrg#material/bmtt9cd5).

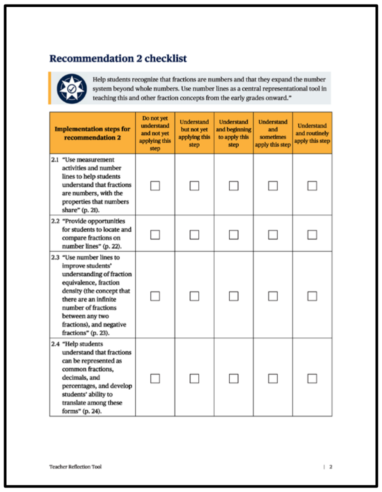
reinforces fraction magnitude concepts needed to compare and locate fractions.

- Review the app features then use the app to demonstrate the sliders and checkboxes (slide 46).
- Show unit and equivalent fraction examples such as $\frac{1}{3}$ and $\frac{1}{5}$ and also show $\frac{1}{3}$ and $\frac{4}{12}$. To demonstrate the relationships, click on the "Answer" checkbox, and then on the "In words" checkbox when it appears.

4. **Give participants time for an initial exploration of this app using Module 1, Handout 4: Comparing Rational Numbers—Fractions app.** After participants explore the app, ask them to compare $\frac{7}{9}$ and $\frac{3}{5}$. After they have represented the fractions and observed, ask participants to share and discuss observations and questions (slides 47-48).

Teacher Reflection Tool (10 minutes, slides 49-52)

1. **Introduce the Teacher Reflection Tool** (slide 50).
2. **Ask participants to reflect on checklist item 2.1 in the Teacher Reflection Tool as an example;** let them know they will reflect on other items during the interim activities (slide 51).
    - Review the five levels of implementation in the checklist.
    - Discuss the descriptors, including the distinctions between awareness, understanding, and applying the strategies associated with the recommendations.
3. **Review the reflection questions after the checklists that teachers will respond to during the interim activities** (slide 52).
    - Reinforce that the notes and reflections are for their self-reflection but can be helpful as participants move through the modules. Participants can be encouraged to share questions and challenges with facilitators so that focused support can be provided as needed throughout the professional development.
    - Point out that there will be multiple opportunities to use the Teacher Reflection Tool during the professional development, and their degree of implementation can be revised across time.



Recommendation 2 checklist

Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward."

Implementation steps for recommendation 2	Do not yet understand and not yet applying this step	Understand but not yet applying this step	Understand and beginning to apply this step	Understand and sometimes apply this step	Understand and routinely apply this step
2.1 "Use measurement activities and number lines to help students understand that fractions are numbers, with the properties that numbers share" (p. 21).	☐	☐	☐	☐	☐
2.2 "Provide opportunities for students to locate and compare fractions on number lines" (p. 22).	☐	☐	☐	☐	☐
2.3 "Use number lines to improve students' understanding of fraction equivalence, fraction density (the concept that there are an infinite number of fractions between any two fractions), and negative fractions" (p. 23).	☐	☐	☐	☐	☐
2.4 "Help students understand that fractions can be represented as common fractions, decimals, and percentages, and develop students' ability to translate among these forms" (p. 24).	☐	☐	☐	☐	☐

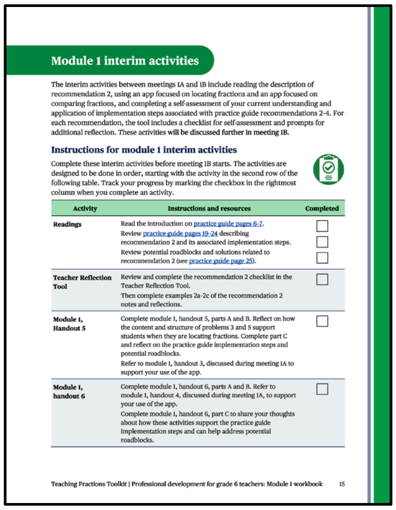
Teacher Reflection Tool | 2

(Module 1, Teacher Reflection Tool, Recommendation 2 checklist)

Tip: The Teacher Reflection Tool is introduced here, partway through Module 1, as a first opportunity for participants to reflect on their understanding and practice, rather than before participants started Module 1, because having some background understanding of what the practice guide and recommendations are as context will support teachers in using the tool.

Wrap-up (10 minutes, slide 53-59)

1. **Review the list of Module 1 interim activities that participants should complete before Meeting 1B** and answer any questions from participants (slide 54).
2. **Make sure everyone knows the time and location for Meeting 1B** (slide 55).



Module 1 interim activities

The interim activities between meetings 1A and 1B include reading the description of recommendation 2, using an app focused on locating fractions and an app focused on comparing fractions, and completing a self-assessment of your current understanding and application of implementation steps associated with practice guide recommendations 2-4. For each recommendation, the tool includes a checklist for self-assessment and prompts for additional reflection. These activities **will be discussed further in meeting 1B.**

Instructions for module 1 interim activities

Complete these interim activities before meeting 1B starts. The activities are designed to be done in order, starting with the activity in the second row of the following table. Track your progress by marking the checkbox in the rightmost column when you complete an activity.

Activity	Instructions and resources	Completed
Readings	Read the introduction on practice guide pages 6-7. Review practice guide pages 19-24 describing recommendation 2 and its associated implementation steps. Review potential roadblocks and solutions related to recommendation 2 (see practice guide page 25).	☐ ☐ ☐
Teacher Reflection Tool	Review and complete the recommendation 2 checklist in the Teacher Reflection Tool. Then complete examples 2a-2c of the recommendation 2 notes and reflections.	☐
Module 1, Handout 5	Complete module 1, handout 5, parts A and B. Reflect on how the content and structure of problems 3 and 5 support students when they are locating fractions. Complete part C and reflect on the practice guide implementation steps and potential roadblocks. Refer to module 1, handout 3, discussed during meeting 1A to support your use of the app.	☐
Module 1, handout 6	Complete module 1, handout 6, parts A and B. Refer to module 1, handout 4, discussed during meeting 1A, to support your use of the app. Complete module 1, handout 6, part C to share your thoughts about how these activities support the practice guide implementation steps and can help address potential roadblocks.	☐

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(Module 1, interim activities, Workbook)

Tip: Let participants know that the interim activities include the use of some handouts in their workbooks that they started working on during Meeting 1A.

Module 1 interim activities

Between Meetings 1A and 1B, participants will complete interim activities, which should take approximately 2 hours. These activities include reading the description of Recommendation 2, using the apps introduced during Meeting 1A that focus on locating fractions and comparing fractions, and completing the Teacher Reflection Tool self-assessment of their current understanding and application of implementation steps associated with practice guide Recommendations 2-4. Refer to the information for Meeting 1B for additional details about the interim activities.

Activity	Participants will ...
Readings	Read the introduction on practice guide pages 6-7. Review practice guide pages 19-24 describing Recommendation 2 and its associated implementation steps. Review potential roadblocks and solutions related to Recommendation 2 (see practice guide page 25).
Teacher Reflection Tool	Review and complete the Recommendation 2 checklist in the Teacher Reflection Tool. Then complete examples 2a-2c of the Recommendation 2 notes and reflections.
Module 1, Handout 5	Complete Module 1, Handout 5, parts A and B. Reflect on how the content and structure of problems 3 and 5 support students when they are locating fractions. Complete part C and reflect on the practice guide implementation steps and potential roadblocks. Refer to Module 1, Handout 3, discussed during Meeting 1A, to support their use of the app.
Module 1, Handout 6	Complete Module 1, Handout 6, parts A and B. Refer to Module 1, handout 4, discussed during Meeting 1A, to support their use of the app. Complete Module 1, Handout 6, part C to share their thoughts about how these activities support the practice guide implementation steps and can help address potential roadblocks.

Interim activities facilitator role

In the participant workbooks, participants have directions and all the resources they need to complete the interim activities between the two meetings. As a facilitator, you may wish to send a reminder midway between the two meetings to remind participants that they should be working on the interim activities and check in to see if any participants have questions or need support concerning the interim activities.

Meeting 1B

During Meeting 1B, introduce the Teaching Fractions Toolkit formative assessments, support participants to debrief and deepen their use of online apps for locating fractions on a number line and comparing fractions (that they worked on during the interim activities), facilitate discussions about strategies to carry out Recommendation 2 and to address potential roadblocks, facilitate analysis of student work, and have participants complete a module reflection.

The agenda for Meeting 1B is as follows.

Agenda for Meeting 1B

Minutes	Topic and resources
5	**Icebreaker** (slide 3) • Formative assessment exploration
20	**Online apps focused on locating and comparing fractions** (slides 4-18) • Module 1, Handout 5: Locating Fractions on a Number Line (1) app activity • Locating Fractions on a Number Line (1) app: https://www.geogebra.org/m/uchh5qmd • Module 1, Handout 6: Comparing Rational Numbers–Fractions activity • Comparing Rational Numbers–Fractions app: https://www.geogebra.org/m/vcy4zn8e
15	**Locating fractions formative assessment** (slides 19-25) • Module 1, Handout 7: Locating Fractions on a Number Line formative assessment activity
15	**Analysis of student work** (slides 26-29) • Module 1, Handout 8: Analyzing student responses to the Locating Fractions on a Number Line formative assessment
5	**Wrap-up** (slides 30-35) • Module 1, Handout 9: Module 1 reflection

Facilitator directions for Meeting 1B

The directions for how to facilitate each Meeting 1B agenda topics are in the following table.

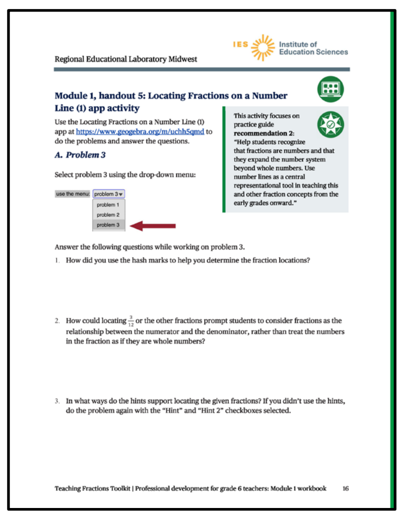
Icebreaker (5 minutes, slides 2-3)

1. **Review the agenda** (slide 2).
2. **Ask participants to complete the example formative assessment item shown on the slide** and be prepared to explain why they chose the fraction they selected (slide 3).
   - Ask participants why the choice of $\frac{7}{8}$ is provided. (This is a common misunderstanding in which students just count the partitions without noticing the length of the segment being partitioned.)
   - Ask participants to consider why $\frac{8}{9}$ might be included as a response. (Some students count the partition at zero. There are eight lines in all on this number line.)
3. **Ask participants to discuss what possible misunderstandings this item may elicit.**

Tip: This quick exploration is meant as an icebreaker to engage participants in content that will be explored more deeply during the meeting. If you display slide 3 as participants are arriving, they can start working on it as a "Do Now."

Online apps focused on locating and comparing fractions (20 minutes, slides 4-18)

1. **Review the Implementation Steps 2 and 3 for Recommendation 2 (practice guide, pp. 22-23).** Explain that participants can use the online apps that they explored during the Module 1 interim activities to help carry out these implementation steps (slide 5).
2. **Have participants refer to the Module 1, Handout 5:** Locating Fractions on a Number Line app activity that they worked on during the interim activities.
3. **Launch the Locating Fractions on a Number Line (1) app and display problem 3.** Review handout items 1-3 about problem 3 and ask participants to share their decision-making for placing fractions on the number line and demonstrate placement in the app.
4. **Debrief participants' work on problems 4-6 with participants sharing their thinking about fraction placement.** Encourage the strategies that use estimating such as those outlined in example 3 on page 31 of the practice guide.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Module 1, handout 5: Locating Fractions on a Number Line (1) app activity

Use the Locating Fractions on a Number Line (1) app at https://www.geogebra.org/m/uchh5qmd to do the problems and answer the questions.

A. Problem 3

Select problem 3 using the drop-down menu:

use the menu: problem 3
problem 1
problem 2
problem 3

This activity focuses on practice guide **recommendation 2:** "Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward."

Answer the following questions while working on problem 3.

1. How did you use the hash marks to help you determine the fraction locations?
2. How could locating $\frac{3}{12}$ or the other fractions prompt students to consider fractions as the relationship between the numerator and the denominator, rather than treat the numbers in the fraction as if they are whole numbers?
3. In what ways do the hints support locating the given fractions? If you didn't use the hints, do the problem again with the "Hint" and "Hint 2" checkboxes selected.

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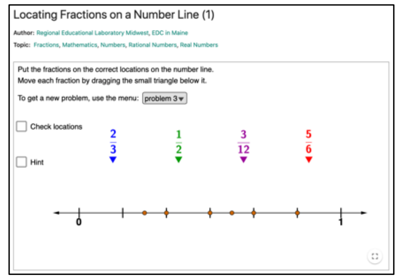
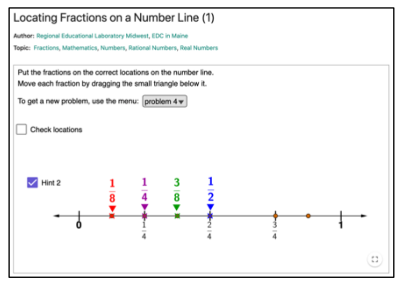
(Module 1, Handout 5, Workbook)

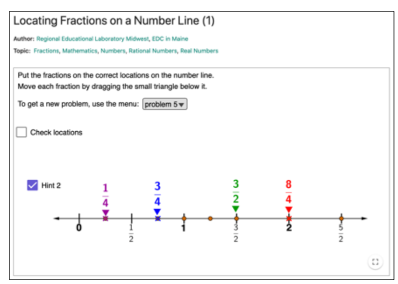
Tip: If this is a virtual session, participants can share their thinking while sharing their screen with the app open.

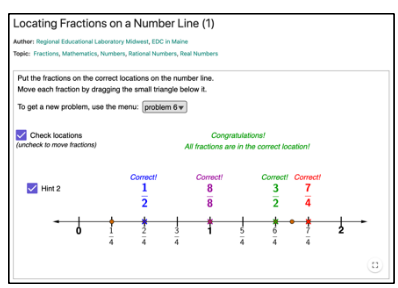
5. **While debriefing problems 3-6 or after participants have finished sharing their thinking about these problems**, share connections to Recommendation 2, Implementation Steps 2 and 3 from this work (slides 6-9).
   - Problem 3: Activities should include fractions in a variety of forms (proper, improper, mixed, and whole). Students can initially locate and compare fractions on number lines with the fractions already marked using the hints in the app. Pre-segmented number lines avoid the difficulty students have in accurately partitioning the number line.
   - Problem 4: Number lines are useful for locating and comparing fractions whose locations are indicated (such as $\frac{1}{4}$), fractions whose denominator is a factor of the unit fractions shown on the number line (such as $\frac{1}{2}$), as well as fractions with other denominators (e.g., $\frac{1}{7}$, $\frac{3}{5}$). For example, students might compare the locations of $\frac{7}{8}$ and $\frac{3}{4}$ on a number line marked with eighths.
   - Problem 5: Activities should include opportunities for students to locate whole numbers on the number line and compare their locations to those of fractions, including ones equivalent to whole numbers. Number lines are useful for comparing fractions of varying sizes to whole numbers greater than one, and teachers can label number lines with one fractional-unit sequence above the number line.
   - Problem 6: Teachers should provide students with opportunities to locate and compare fractions on number lines that are minimally labeled; for example, ones with the labels 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, and 2. This encourages students to think about the location of fractions relative to the labeled landmarks. In addition, number lines can be valuable for teaching equivalent fractions, negative fractions, and fraction density. For example, number lines can be used to illustrate that equivalent fractions describe the same magnitude. Teachers can divide a 0-to-1 number line into halves and quarters and show that $\frac{1}{2}$ and $\frac{2}{4}$ occupy the same, or equivalent, point on the number line.

Tip: If you feel comfortable, you might choose to go directly to the app and hide the slides that include the app directions. These directions will be available for reference as needed.

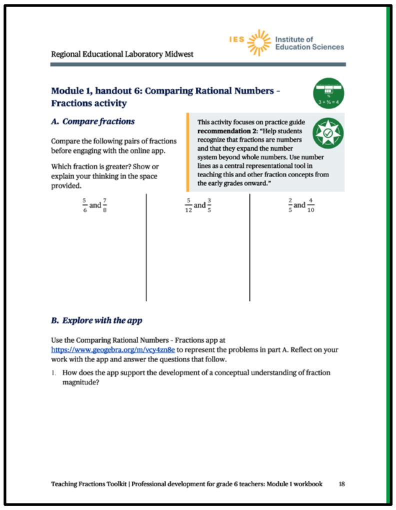








6. **Have participants share their ideas about the reflection questions from parts B and C of Handout 5** (slide 10).
7. **Next, transition to the second online app that participants worked on during the interim activities.** Ask participants to refer to Module 1, Handout 6: Comparing Rational Numbers–Fractions activity.
8. **Have participants share how they compared the three pairs of fractions shown in part A on Handout 6 before using the online app** (slide 11).
9. **Display the app and discuss how the app can be used to address common mistakes and misunderstandings for the pair of fractions $\frac{5}{6}$ and $\frac{7}{8}$ from Handout 6.** Have participants discuss what misunderstanding this pair of fractions might support students to address. Show a few different examples of fractions that are a unit fraction away from a whole, having participants predict each time which is bigger then check in the app. Share the following if it does not come up in discussion (slides 12-14):
   - Using fractions such as fractions $\frac{5}{6}$ and $\frac{7}{8}$, students can confront the common misunderstanding that these are equal because they are each lacking "one" to be a whole. Students often overgeneralize from $\frac{1}{8}$ being bigger than $\frac{1}{6}$ because 8 is bigger than 6.
10. **Share how the app can be used to address common mistakes and misunderstandings for the other pairs of fractions from Handout 6** (slides 15-16).
    - Using fractions such as $\frac{2}{5}$ and $\frac{4}{10}$, students can confront the common misunderstanding that a fraction is larger just because its numerator and/or denominator is larger; for example, $\frac{4}{10}$ is larger because 4 is greater than 2 and 10 is greater than 5.
    - Using fractions such as $\frac{5}{12}$ and $\frac{3}{5}$, students can learn to use benchmarks on the number line, noticing that $\frac{5}{12}$ is less than $\frac{6}{12}$ (or half) and $\frac{3}{5}$ is greater than $\frac{1}{2}$, and if needed they can think of $\frac{3}{5}$ as $\frac{6}{10}$ and because $\frac{5}{10}$ is $\frac{1}{2}$, $\frac{6}{10}$ (or $\frac{3}{5}$) is greater than $\frac{1}{2}$ so $\frac{3}{5}$ is greater than $\frac{5}{12}$.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Module 1, handout 6: Comparing Rational Numbers - Fractions activity

A. Compare fractions

Compare the following pairs of fractions before engaging with the online app.

Which fraction is greater? Show or explain your thinking in the space provided.

This activity focuses on practice guide **recommendation 2**: "Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward."

$\frac{5}{6}$ and $\frac{7}{8}$ | $\frac{5}{12}$ and $\frac{3}{5}$ | $\frac{2}{5}$ and $\frac{4}{10}$

B. Explore with the app

Use the Comparing Rational Numbers - Fractions app at https://www.geogebra.org/m/vcy4zn8e to represent the problems in part A. Reflect on your work with the app and answer the questions that follow.

1. How does the app support the development of a conceptual understanding of fraction magnitude?

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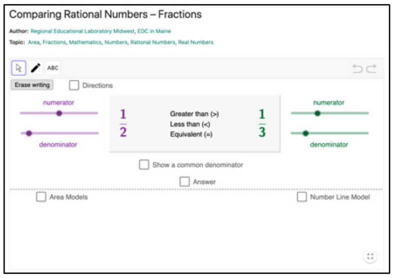
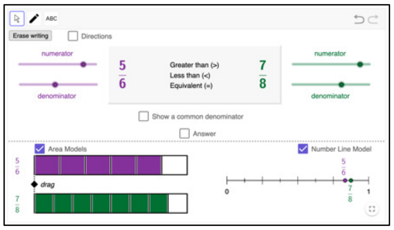
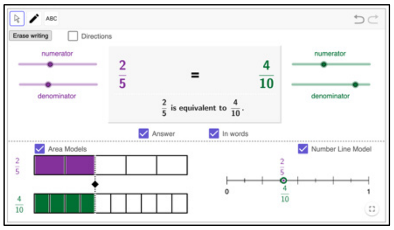
(Module 1, Handout 6, Workbook)

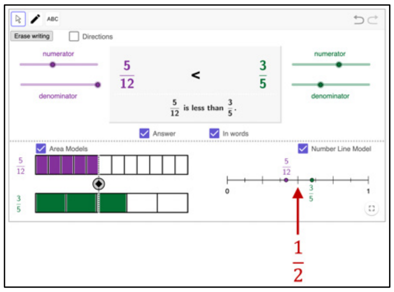


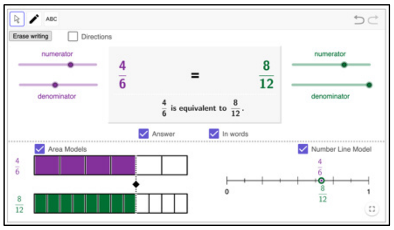




Tip: You can turn off one or the other visual model in the app while you are demonstrating what different pairs of fractions look like.

11. **Next, show a series of pairs of fractions that are not on Handout 6,** all using the same visuals, and ask participants to look for patterns: $\frac{4}{6}$ and $\frac{8}{12}$, $\frac{3}{6}$ and $\frac{6}{12}$, $\frac{2}{6}$ and $\frac{4}{12}$, and $\frac{1}{6}$ and $\frac{2}{12}$ all using the same visuals (slide 17).
12. **Explain some key points about the app and how it relates to the implementation steps** (slide 17 continued):
    - The app relates to Recommendation 2, Implementation Step 2 because it supports students to compare fractions on number lines that are minimally labeled.
    - It relates to Recommendation 2, Implementation Step 3 because it is useful for comparing positive fraction magnitudes and valuable for teaching equivalent fractions, number lines can be used to illustrate that equivalent fractions describe the same magnitude, and teachers can divide a 0-to-1 number line into halves and quarters and show that $\frac{1}{2}$ and $\frac{2}{4}$ occupy the same, or equivalent, point on the number line.
    - It is good for having students transition from or look at the comparative language. When students click on the answer in this app, it gives area models and number line models for the same fraction comparison, which is an essential scaffolding for students in upper elementary moving into middle school when transitioning from area models to number lines.
13. **Lead a discussion about how the app may help address some of the potential roadblocks for Recommendation 2** (slide 18).





Locating fractions formative assessment (15 minutes, slides 19-25)

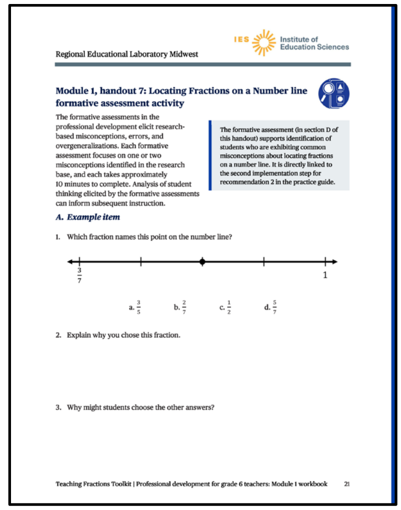
1. **Set the stage for using formative assessment items to elicit understanding, misunderstandings, and common mistakes** (slides 20-22). Explain the following concepts:
    - The short formative assessments included in the Teaching Fractions Toolkit are explicitly designed to elicit what students understand or misunderstand about fractions based on the research literature,

Recommendation 5, Implementation Step 3 states that professional development programs should "develop teachers' ability to assess students' understandings and misunderstandings of fractions" (practice guide, p. 44).

allowing teachers to see what students think about targeted content.

- Knowing the types of mistakes that students most often make when working with fractions, and understanding the underlying misunderstandings that cause them, are important to support effective fractions learning.
- Examining student work associated with common misunderstandings can provide insight into student thinking and inform subsequent learning activities.
- A few different examples of these formative assessments will be included throughout the Teaching Fractions Toolkit modules.

2. **Provide a brief overview of the Locating Fractions on a Number Line formative assessment (Module 1, Handout 7)** and then give participants some time to work on the questions in part A. Then discuss the participants' responses as well as the other answer choices that they did not select (slide 23).
    - The item in part A shows a number line that starts at $\frac{3}{7}$ rather than zero.
    - Students may just count the hash marks and not consider the starting point when determining their answer.
    - Each hash mark represents $\frac{1}{7}$.
    - The correct answer is choice d ($\frac{5}{7}$).
    - Students might choose a, $\frac{3}{5}$, because the point is on the third of five hash marks. Students may choose b, $\frac{2}{7}$, because the point is on the second hash mark. Students might choose c, $\frac{1}{2}$, because the point is halfway on the provided line.
3. **Review potential roadblocks from Recommendation 2** and other common misunderstandings that students may encounter when locating fractions on a number line (slide 24). Share the following:
    - Students can develop flawed understandings that lead to misconceptions about fractions and number lines if common errors and misunderstandings are not uncovered and addressed. The Locating Fractions on a Number Line formative assessment focuses on Roadblocks 2.1-2.3 in the practice guide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Module 1, handout 7: Locating Fractions on a Number line formative assessment activity

The formative assessments in the professional development elicit research-based misconceptions, errors, and overgeneralizations. Each formative assessment focuses on one or two misconceptions identified in the research base, and each takes approximately 10 minutes to complete. Analysis of student thinking elicited by the formative assessments can inform subsequent instruction.

The formative assessment (in section D of this handout) supports identification of students who are exhibiting common misconceptions about locating fractions on a number line. It is directly linked to the second implementation step for recommendation 2 in the practice guide.

A. Example item

1. Which fraction names this point on the number line?

$\frac{3}{7}$ 1

a. $\frac{3}{5}$ b. $\frac{2}{7}$ c. $\frac{1}{2}$ d. $\frac{5}{7}$

2. Explain why you chose this fraction.

3. Why might students choose the other answers?

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(Module 1, Handout 7, Workbook)

Potential roadblocks and other common misunderstandings

- **Roadblock 2.1.** "Students try to partition the number line into fourths by drawing four hash marks rather than three, or they treat the whole number line as the unit."
- **Roadblock 2.2.** "When students locate fractions on the number line, they treat numbers in the fraction as whole numbers (e.g., placing $\frac{3}{4}$ between 3 and 4)."
- **Roadblock 2.3.** "Students have difficulty understanding that two equivalent fractions are the same point on a number line."
- Students overgeneralize from whole number reasoning and count the hash marks (including at 0) or points without considering which fractions the hash marks represent.
- Students treat the whole line as one unit, ignoring the starting and ending values.

and on two other misconceptions noted in the research on students' mathematical reasoning.

4. **Have participants review the formative assessment included in Module 1, Handout 7, part D.** The formative assessment is designed to elicit students' understanding and misunderstandings about locating fractions on a number line. Have participants complete items 6 and 8 and reflect on the questions provided (slide 25).
   - What misunderstandings might each item elicit?
   - What do you notice about the selected response item choices?
   - Key points to emphasize if they don't come up in the conversation:
     - Item 6 is based on the common misunderstanding that a point halfway on the number line automatically has a magnitude of $\frac{1}{2}$, ignoring the starting and ending values.
     - Item 8 also is related to treating the whole line as one unit, ignoring the starting and ending values.

Professional development activities with teachers should emphasize how students develop an understanding of fractions and the obstacles they face in learning about them.

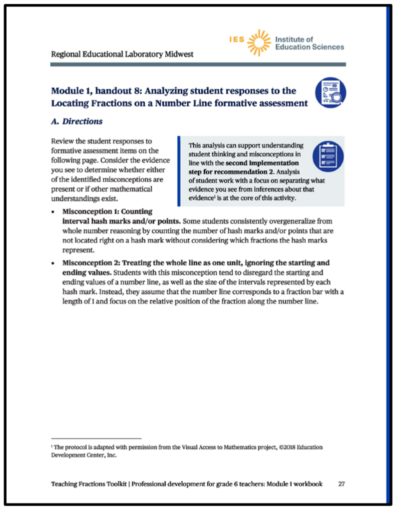
One method that is useful for meeting this goal is to provide teachers with opportunities to analyze and critique student thinking about fractions.

Teachers should know the types of mistakes students most often make when working with fractions and understand the underlying misconceptions that cause them.

Analyzing students' work is a useful way to identify problem areas and gain insight into students' thought processes. To be most effective, teachers must know how to design problems that diagnose the source of errors (practice guide, pp. 44-46).

Analysis of student work (15 minutes, slides 26-29)

1. **Introduce the protocol for looking at student work, Module 1, Handout 8:** Analyzing student responses to the Locating Fractions on a Number Line formative assessment. Provide the following instructions (slide 27):
   - Review the student's response to each item.
   - When analyzing the work, focus on the evidence you see instead of making inferences. Support participants to apply an assets-based perspective. What is the evidence of understanding? What is the evidence of partial understanding?
   - Consider evidence of the targeted misunderstandings and of other mathematical understandings.
   - Consider the questions you would ask the student to elicit their thinking and build from what they do understand.



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Module 1, handout 8: Analyzing student responses to the Locating Fractions on a Number Line formative assessment

A. Directions

Review the student responses to formative assessment items on the following page. Consider the evidence you see to determine whether either of the identified misconceptions are present or if other mathematical understandings exist.

This analysis can support understanding student thinking and misconceptions in line with the **second implementation step for recommendation 2.** Analysis of student work with a focus on separating what evidence you see from inferences about that evidence[1] is at the core of this activity.

- **Misconception 1: Counting interval hash marks and/or points.** Some students consistently overgeneralize from whole number reasoning by counting the number of hash marks and/or points that are not located right on a hash mark without considering which fractions the hash marks represent.
- **Misconception 2: Treating the whole line as one unit, ignoring the starting and ending values.** Students with this misconception tend to disregard the starting and ending values of a number line, as well as the size of the intervals represented by each hash mark. Instead, they assume that the number line corresponds to a fraction bar with a length of 1 and focus on the relative position of the fraction along the number line.

[1] The protocol is adapted with permission from the Visual Access to Mathematics project, ©2018 Education Development Center, Inc.

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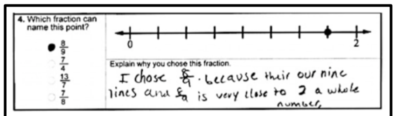
(Module 1, Handout 8, Workbook)

2. **Before starting work on the protocol for the work shown on Handout 8, look at one piece of work together as a group**—a response to item 4 from the formative assessment—and then discuss the potential understanding seen in that student's work, evidence of misunderstandings, and questions that could be asked of the student (slide 28).
   - What did students write or draw that is evidence of understanding?
   - If any misunderstandings exist, describe evidence in the student's work
   - What questions do you want to ask the student or what are the next steps you want to try with the student?
3. **Give participants time to briefly reflect individually on the other student work examples** drawn from the formative assessment shown on Handout 8 and then have them choose one of the students' responses to discuss (slide 29).
   - What strengths are evident?
     Note: The student recognized the number of partitions and was able to represent eight out of nine partitions as a fraction.
   - What possible misunderstanding does this student's work reveal?
     Note: The student did not identify the full distance shown as two units long. The point is located at $1\frac{3}{4}$.
   - How might you address this lack of understanding? Consider strategies we have discussed in Module 1 and you have read about in the practice guide. (Refer to the guidance in the practice guide on pages 22-25.)
     Note: One strategy is to use a number line greater that one unit long and use fraction strips to partition the number line. You can have students measure and name the starting and ending point and what each partition represents.



"I chose 8/9 because their our [there are] nine lines and 8/9 is very close to 2 a whole number."

Background: The three protocol columns focus on student potential, common mistakes and misunderstandings, and questions and strategies.

- **Student potential:** What did students write or draw that is evidence of understanding?
- **Identify the misunderstanding:** If any exist, describe evidence of the misunderstanding in the student's work.
- **Questions and strategies:** What questions do you want to ask the student or what are the next steps you want to try?

Wrap-up (5 minutes, slides 30-35)	
1. **Have participants write reflections on Module 1, Handout 9** about one strategy or idea from the Module 1 activities, practice guide readings, and discussions related to Recommendation 2 and how they will incorporate those ideas into their practice (slide 31). 2. **Check-in with participants about any questions and review the Module 2 schedule** (slide 32-33).	**Tip:** Because this is the end of the first module working together, you may want to gather some quick feedback about what is working well and what could support the participants to support you in planning for Module 2. Example questions include the following: • What worked well for you in Module 1? • What questions do you have after Module 1? **Tip:** If there is time, ask teachers to share out about their reflections. If not, consider asking them to share their written reflections with you.

Credits and references

Information is provided about photos and other resources used in Module 1.

Photos

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Resources

The toolkit is based on the recommendations of the What Works Clearinghouse practice guide Developing Effective Fractions Instruction for Kindergarten Through 8th Grade.

The Comparing Rational Numbers–Fractions app was originally created by EDC in Maine using GeoGebra and licensed under a Creative Commons Attribution-ShareAlike 3.0 Unported International License (CC BY-SA 3.0).

The formative assessment resources were adapted with permission from the Eliciting Mathematical Meaning project, © 2015 Education Development Center, Inc.

The Fractions on a Number Line task was adapted from the Illustrative Mathematics Fractions on the Number Line, which is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License (CC BY-NC-SA 4.0).

The Locating Fractions on a Number Line (1) online app was originally created by EDC in Maine using GeoGebra and licensed under a Creative Commons Attribution-ShareAlike 3.0 Unported International License (CC BY-SA 3.0).

The student work analysis protocol was excerpted and adapted from the Visual Access to Mathematics project, © 2018 Education Development Center, Inc.

References

Barbieri, C. A., Rodrigues, J., Dyson, N., & Jordan, N. C. (2020). Improving fraction understanding in sixth graders with mathematics difficulties: Effects of a number line approach combined with cognitive learning strategies. *Journal of Educational Psychology,* 112(3), 628-648. http://eric.ed.gov/?ID=EJ1247111

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology,* 37(4), 247-253. http://eric.ed.gov/?ID=EJ977998

Butterworth, B. (2005). The development of arithmetical abilities. *Journal of Child Psychology and Psychiatry,* 46, 3-18. http://doi.org/10.1111/j.1469-7610.2004.00374.x

Butterworth, B., & Reigosa-Crespo, V. (2007). Information processing deficits in dyscalculia. In D. B. Berch & M. M. Mazzocco (Eds.), Why is math so hard for some children? The nature and origins or mathematical learning difficulties and disabilities (pp. 65-81). Brookes.

Empson, S. B., Levi, L., & Carpenter, T. P. (2011). The algebraic nature of fractions: Developing relational thinking in elementary school. In J. Cai & E. Knuth (Eds.), Early algebraization. Advances in mathematics education (pp. 409-428). Springer. https://doi.org/10.1007/978-3-642-17735-4_22

Lee, H., & Boyadzhiev, I. (2020). Underprepared college students' understanding of and misconceptions with fractions. *International Electronic Journal of Mathematics Education,* 15(3), Article em0583. https://eric.ed.gov/?id=EJ1249568

Liu, Y. (2018). Fraction magnitude understanding and its unique role in predicting general mathematics achievement at two early stages of fraction instruction. *British Journal of Educational Psychology,* 88(3), 345-362. https://eric.ed.gov/?id=EJ1187824

Minor, E. C. (2016). Racial differences in mathematics test scores for advanced mathematics students. *The High School Journal,* 99(3), 193-210. https://eric.ed.gov/?id=EJ1104153

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common core state standards: Mathematics. http://www.corestandards.org/assets/CCSSI_Math Standards.pdf

Saxe, G. B., & Sussman, J. (2019). Mathematics learning in language inclusive classrooms: Supporting the achievement of English learners and their English proficient peers. *Educational Researcher,* 48(7), 452-465. https://doi.org/10.3102/0013189X19869953

Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., Thompson, L., & Wray, J. (2010). Developing effective fractions instruction for kindergarten through 8th grade: A practice guide (NCEE 2010-4039). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. https://eric.ed.gov/?id=ED512043

Siegler, S. F., & Lortie-Forgues, H. (2015). Conceptual knowledge of fraction arithmetic. *Journal of Educational Psychology,* 107(3), 909-918. https://eric.ed.gov/?id=EJ1071576

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