Institute of Education Sciences

Regional Educational Laboratory Midwest

at American Institutes for Research

Teaching Fractions Toolkit

Professional development for grade 6 teachers: Module 1 workbook

Fractions on a number line

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A Publication of the National Center for Education Evaluation and Regional Assistance at IES

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Contents

Introduction

The Developing Effective Fractions Instruction for Kindergarten Through 8th Grade Toolkit (Teaching Fractions Toolkit) was developed for math teachers, leaders, administrators, and others to support implementation of evidence-based recommendations for increasing student understanding of fractions. This toolkit is based on the recommendations of the What Works Clearinghouse practice guide Developing Effective Fractions Instruction for Kindergarten Through 8th Grade.

The six professional development modules in the Teaching Fractions Toolkit are designed to support grade 6 teachers in understanding and applying evidence-based recommendations to support effective fractions instruction. Four primary areas of focus are fraction computation and ratio, rate, and proportion problem solving; understanding why computational procedures with fractions work; understanding and using visual representations to support fraction computation and ratio, rate, and proportion problem solving; and understanding and overcoming common mistakes and misunderstandings and potential roadblocks to solving problems involving fractions. Activities in the modules include exploring math tasks, analyzing student work, planning and implementing lessons, using formative assessment resources, and reflecting on classroom practice. Each professional development module includes two synchronous meetings, along with asynchronous activities for teachers to complete between the meetings.

Module 1

Fractions on a Number Line focuses on Recommendation 2, with an emphasis on number lines.

Meeting 1A ➔ **Module 1 interim activities** ➔ **Meeting 1B**

The materials appear in the order in which they are introduced in the module. Pages with module section information have right-side page edge colors to indicate the section:

- A brown edge indicates **general information**; this section starts on this page.
- A purple edge indicates **Meeting 1A**; this section starts on page 5.
- A green edge indicates the **Module 1 interim activities**; this section starts on page 13.
- A blue edge indicates **Meeting 1B**; this section starts on page 18.

Module 1 overview

Module 1 introduces the structure and content of the practice guide, especially Recommendation 2, with particular attention to background information, how to carry out the recommendation, and potential roadblocks and solutions. An introduction to Recommendation 2 describes the importance of using visual representations to support effective fractions instruction.

The module includes learning about and using number lines as a central representational tool for teaching fraction concepts, particularly locating and comparing fractions. Activities include engaging in math tasks to place fractions on a number line to determine the relationship between them and then exploring potential misunderstandings and roadblocks associated with implementing these strategies in the classroom. The Teacher Reflection Tool is introduced as a gauge for understanding and using Recommendations 2-4.

Additional activities include using online apps to locate fractions on a number line and comparing fractions, incorporating strategies to carry out Recommendation 2, introducing toolkit formative assessments, using a template for analyzing student work, and completing a module reflection.

Objectives for Module 1

By the end of Module 1, you will be able to do the following:

- Identify the structure and importance of the Developing Effective Fractions Instruction for Kindergarten Through 8th Grade practice guide.
- Explain the importance of student recognition of fractions as numbers.
- Identify how number lines represent and build an understanding of whole numbers, fractions, and decimals.
- Understand and use number lines to locate and compare fractions in instruction.
- Understand and use number lines and fraction strips to show fraction equivalence and density in instruction.
- Recognize misunderstandings in student work and apply strategies and approaches to remediate them.
- Use the Teacher Reflection Tool to self-assess baseline understanding and use of the practice guide recommendations.

Recommendation focus for Module 1

The Teaching Fractions Toolkit professional development focuses on practice guide Recommendations 2-4 in the context of grade 6 math content. The design of the professional development also is in line with Recommendation 5 and its implementation steps, which relate to professional development programs for teachers of students learning fractions content.

Module 1 specifically focuses on building knowledge related to Recommendation 2 in the context of grade 6 standards and foundational knowledge often taught in grades 4-5.

Recommendation 2: “Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.”

Schedule and checklist for Module 1

Use the following organizer to keep track of your module schedule and progress. For some modules, including Module 1, interim activities include classroom implementation followed by student work analysis and/or reflections. Record your plan for completing the Module 1 interim activities in the table. All interim activities should be completed before Meeting 1B. Record any relevant guidance in the notes column.

Module 1 organizer

Section details	Notes	Completed
Meeting 1A (120 minutes) Date: Time: Location:		☐
Module 1 interim activities (teacher self-paced; 120 minutes) Plan for completion:		☐
Meeting 1B (60 minutes) Date: Time: Location:		☐

Meeting 1A

During Meeting 1A, you will learn about the practice guide and explore Recommendation 2. The importance of visual representations and the role of the number line as a central representational tool for teaching fraction concepts is emphasized. You also will locate fractions on number lines and compare fractions using interactive online apps. These activities set the foundation for the interim activities that you will explore between Meetings 1A and 1B.

The agenda for Meeting 1A is as follows.

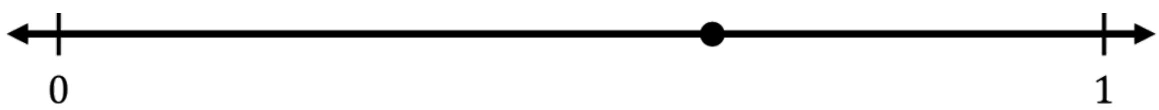
Agenda for Meeting 1A

Topic and resources
Introductions and icebreaker
Background for professional development • Module 1, Handout 1: Name the point exploration
Practice guide overview • Practice guide overview • Practice guide structure (page iii)
Introduction to recommendations • Recommendation 2 (page 19) and its implementation steps (page 21) • Potential roadblocks for Recommendation 2 (page 25)
Break
Math tasks with number lines • Module 1, Handout 2: Fractions on the number line
Online apps • Module 1, Handout 3: Locating Fractions on a Number Line (1) app instructions • Locating Fractions on a Number Line (1) app: https://www.geogebra.org/m/uchh5qmd Supplemental resources similar to the Locating Fractions on a Number Line (1) app: – Locating Fractions on a Number Line (2): https://www.geogebra.org/m/xh7mtge2 – Locating Decimals on a Number Line (1): https://www.geogebra.org/m/yyqpcnxf – Locating Decimals on a Number Line (2): https://www.geogebra.org/m/bmtt9cd5 • Module 1, Handout 4: Comparing Rational Numbers–Fractions app instructions • Comparing Rational Numbers–Fractions online app: https://www.geogebra.org/m/vcy4zn8e
Introduction to the Teacher Reflection Tool
Wrap-up • Instructions for Module 1 interim activities

Module 1, Handout 1: Name the point exploration

A point is shown on the number line. Describe what you know about the point and determine a fraction to name the point, explaining your reasoning.

> This activity focuses on practice guide **Recommendation 2**: “Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.”



1. What do you know about the point?

2. What fraction would you use to name the point? Explain why you chose this fraction.

3. How does this exploration help build an understanding of fractions?

IES Institute of Education Sciences

Developing effective fractions instruction for kindergarten through 8th grade

Practice guide overview

Regional Educational Laboratory Midwest

at American Institutes for Research

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This document provides an overview of the recommendations for fraction instruction presented in the What Works Clearinghouse practice guide *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*. The evidence-based recommendations are based on research for improving K-8 students' understanding of fractions, with the expectation that general education teachers, mathematics specialists and coaches, special educators, and administrators will use these resources to improve their teaching of fractions.

Recommendations[1] in the practice guide

1. Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts.
2. Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from early grades onward.
3. Help students understand why procedures for computations with fractions make sense.
4. Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems.
5. Professional development programs should place a high priority on improving teachers' understanding of fractions and of how to teach them.

The practice guide section for each recommendation includes the following:

The recommendation

Each actionable recommendation includes a summary of the research evidence that supports the recommendation and a description of how the recommended practice supports student learning.

Implementation steps

Implementation steps provide guidance about how to carry out each recommendation. This guidance is informed by studies that support the recommendation as well as the panel's expertise.

Potential roadblocks and solutions

A section on roadblocks and solutions explains potential obstacles related to the recommendation, along with advice from the expert panel about approaches for overcoming them.

Readers are encouraged to use the advice in ways that work best in their context. Users of this practice guide may decide to implement some or all recommendations. The complete practice guide is available on the What Works Clearinghouse website: https://ies.ed.gov/ncee/wwc/PracticeGuide/15

[1] Text for the recommendations is from the WWC practice guide.

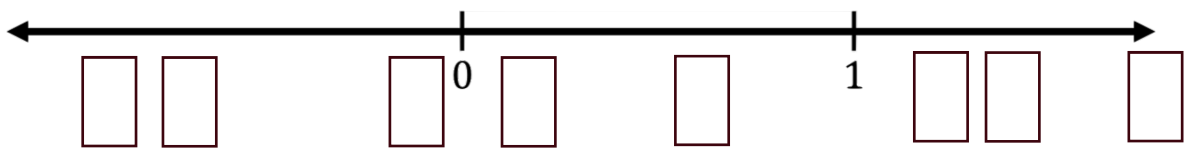
Module 1, Handout 2: Fractions on the Number Line

This task[2] focuses on placing positive and negative integers on a number line and then using that information to determine the relationship between two of the integers.

> This activity focuses on practice guide **Recommendation 2**: "Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward."

A. Task

1. Find and label the numbers $\frac{4}{3}$, $\frac{5}{4}$, $-\frac{2}{3}$, and $-\frac{3}{4}$ on the number line.



2. For each of the following, identify which inequality is true. Use the number line diagram to help explain your answers.

   a. Is $\frac{4}{3} > \frac{5}{4}$ or is $\frac{4}{3} < \frac{5}{4}$?

   b. Is $-\frac{2}{3} > -\frac{3}{4}$ or is $-\frac{2}{3} < -\frac{3}{4}$?

   c. Is $-\frac{3}{4}$ closer to 0 or is $\frac{5}{4}$ closer to 0? Explain how you know.

[2] Adapted from the Fractions on the Number Line task from Illustrative Mathematics, which is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License.

B. Reflections

1. How was the number line useful when comparing the fractions?

2. How did the number line help you determine whether $-\frac{3}{4}$ or $\frac{5}{4}$ is closer to 0?

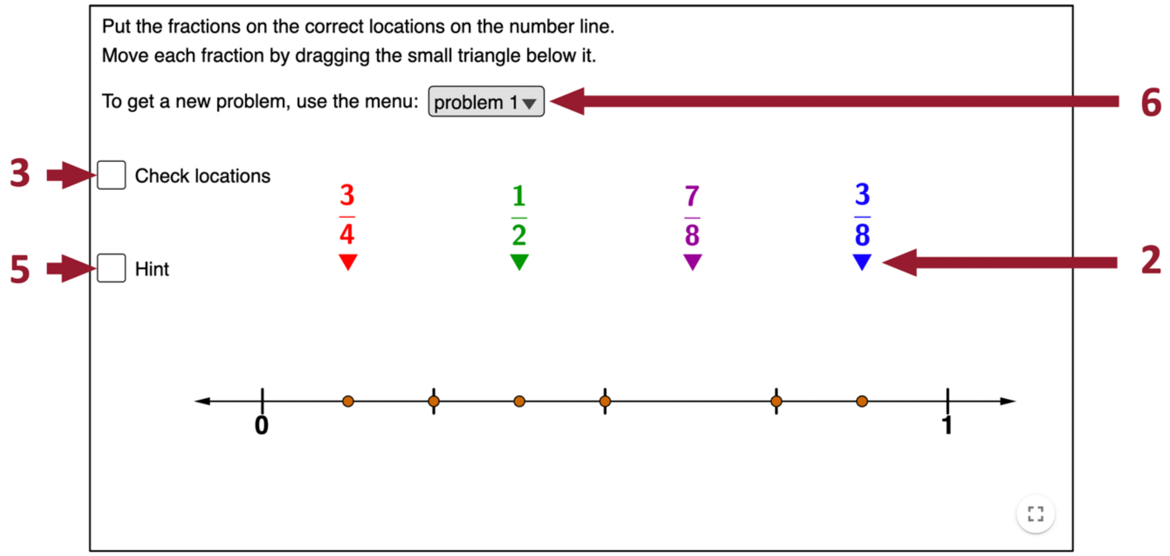
3. What do students need to understand to be successful with this task?

4. What can you do in the classroom to help students when comparing fractions using the number line?

Module 1, Handout 3: Locating Fractions on a Number Line (1) app instructions

Use the Locating Fractions on a Number Line (1) app to position fractions on a number line.

1. Go to https://www.geogebra.org/m/uchh5qmd.
2. Locate each fraction accurately on the number line by clicking and dragging the triangle below a fraction toward the number line until an "x" appears on the line and snaps to a point.
3. Check the accuracy of your locations by selecting the "Check locations" checkbox. Feedback is displayed above the fraction:
   a. "Correct!" means the location of the fraction is accurate.
   b. "Not Correct" means the location of the fraction is not accurate and the fraction must be moved to a different point on the number line.
4. If one or more fractions are incorrect, change their location(s) by unselecting the "Check locations" checkbox and dragging triangles to different points on the number line.
5. For help, select the "Hint" checkbox to show hash mark labels. When the "Hint" checkbox is selected, a "Hint 2" checkbox will appear that can be selected to show additional labels.
6. Select a new problem using the drop-down menu.

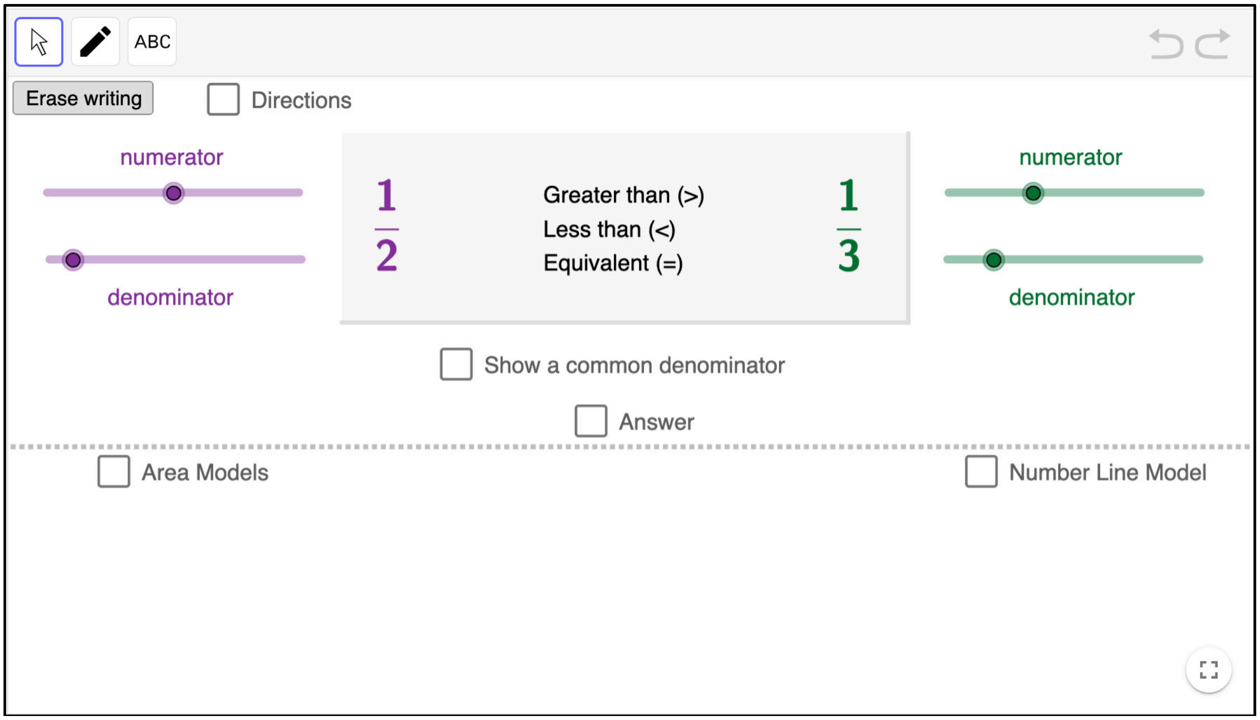


Module 1, Handout 4: Comparing Rational Numbers–Fractions app instructions

Use the Comparing Rational Numbers–Fractions online app to compare fractions using the visual representations of area models and number lines.

> The activity associated with this app focuses on practice guide **Recommendation 2**: "Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward."

1. Go to https://www.geogebra.org/m/vcy4zn8e.
2. Use the sliders to make your fractions.
3. Predict the relationship between the fractions.
4. Click in the checkboxes for the visual representations and observe the magnitude of each fraction.
   a. Area Models: This representation shows two rectangular area models so that you can compare the fractions. You can drag the diamond on the dotted line to compare the fractions more easily.
   b. Number Line Model: This representation shows a number line with the position of both fractions designated and labeled.
5. Use the representations to determine the relationship between the fractions.
6. Check your prediction using the app features.
   a. Click the "Show a common denominator" checkbox to compare using common denominators.
   b. Click the "Answer" checkbox to see the correct symbol showing the relationship between the fractions.
   c. Click the "In words" checkbox to see the relationship written out.
7. Try working with fractions that have different values and relationships.
8. Notice that this app shows fractions from only 0 to 1.


ABC
Erase writing
Directions
numerator
denominator
1/2
Greater than (>)
Less than (<)
Equivalent (=)
1/3
numerator
denominator
Show a common denominator
Answer
Area Models
Number Line Model

Module 1 interim activities

The interim activities between Meetings 1A and 1B include reading the description of Recommendation 2, using an app focused on locating fractions and an app focused on comparing fractions, and completing a self-assessment of your current understanding and application of implementation steps associated with practice guide Recommendations 2-4. For each recommendation, the tool includes a checklist for self-assessment and prompts for additional reflection. These activities will be discussed further in Meeting 1B.

Instructions for Module 1 interim activities

Complete these interim activities before Meeting 1B starts. The activities are designed to be done in order, starting with the activity in the second row of the following table. Track your progress by marking the checkbox in the rightmost column when you complete an activity.

Activity	Instructions and resources	Completed
Readings	Read the introduction on practice guide pages 6-7. Review practice guide pages 19-24 describing Recommendation 2 and its associated implementation steps. Review potential roadblocks and solutions related to Recommendation 2 (see practice guide page 25).	☐ ☐ ☐
Teacher Reflection Tool	Review and complete the Recommendation 2 checklist in the Teacher Reflection Tool. Then complete examples 2a-2c of the Recommendation 2 notes and reflections.	☐
Module 1, Handout 5	Complete Module 1, Handout 5, parts A and B. Reflect on how the content and structure of problems 3 and 5 support students when they are locating fractions. Complete part C and reflect on the practice guide implementation steps and potential roadblocks. Refer to Module 1, Handout 3, discussed during Meeting 1A to support your use of the app.	☐
Module 1, Handout 6	Complete Module 1, Handout 6, parts A and B. Refer to Module 1, Handout 4, discussed during Meeting 1A, to support your use of the app. Complete Module 1, Handout 6, part C to share your thoughts about how these activities support the practice guide implementation steps and can help address potential roadblocks.	☐

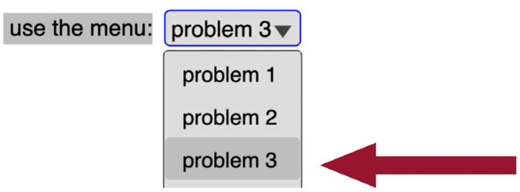
Module 1, Handout 5: Locating Fractions on a Number Line (1) app activity

Use the Locating Fractions on a Number Line (1) app at https://www.geogebra.org/m/uchh5qmd to do the problems and answer the questions.

This activity focuses on practice guide **Recommendation 2**: "Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward."

A. Problem 3

Select problem 3 using the drop-down menu:



Answer the following questions while working on problem 3.

1. How did you use the hash marks to help you determine the fraction locations?

2. How could locating $\frac{3}{12}$ or the other fractions prompt students to consider fractions as the relationship between the numerator and the denominator, rather than treat the numbers in the fraction as if they are whole numbers?

3. In what ways do the hints support locating the given fractions? If you didn't use the hints, do the problem again with the "Hint" and "Hint 2" checkboxes selected.

B. Problem 5

Select problem 5 using the menu. Answer the following question while working on problem 5.

How does problem 5 support students recognizing the unit as the distance from 0 to 1 on the number line, instead of treating the whole number line as the unit (based on Roadblock 2.2)?

C. Reflection

Think about your work on problems 3 and 5 as you answer the following questions about the practice guide implementation steps and potential roadblocks.

1. How could working with this app (or on similar tasks) support student understanding that fractions are numbers, with properties that numbers share (based on Recommendation 2, Implementation Step 1)?

2. How could working with this app (or on similar tasks) support student understanding of fraction equivalence (based on Recommendation 2, Implementation Step 3)?

3. How could working with this app (or on similar tasks) help identify student errors or misunderstandings related to partitioning and/or treating the numbers in a fraction as whole numbers (based on Roadblocks 2.1 and 2.2)?

Module 1, Handout 6: Comparing Rational Numbers–Fractions activity

A. Compare fractions

Compare the following pairs of fractions before engaging with the online app.

Is one fraction greater? If yes, which one? Show or explain your thinking in the space provided.

This activity focuses on practice guide **Recommendation 2**: "Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward."

$\frac{5}{6}$ and $\frac{7}{8}$	$\frac{5}{12}$ and $\frac{3}{5}$	$\frac{2}{5}$ and $\frac{4}{10}$

B. Explore with the app

Use the Comparing Rational Numbers–Fractions app at https://www.geogebra.org/m/vcy4zn8e to represent the problems in part A. Reflect on your work with the app and answer the questions that follow.

1. How does the app support the development of a conceptual understanding of fraction magnitude?

2. The app offers both an area model option and a number line model option. What would each model help students understand or see?

3. For each pair of fractions, identify the understandings or misunderstanding(s) that these particular fraction pairs might elicit.

$\frac{5}{6}$ and $\frac{7}{8}$	$\frac{5}{12}$ and $\frac{3}{5}$	$\frac{2}{5}$ and $\frac{4}{10}$

C. Reflection

Think about your work on the problems in part A as you answer the following questions about the practice guide implementation steps and potential roadblocks.

1. How could working with this app (or on similar tasks) support students in understanding that fractions are numbers, with properties that numbers share (based on Recommendation 2, Implementation Step 1)?

2. How could working with this app (or on similar tasks) support student understanding of fraction equivalence (based on Recommendation 2, Implementation Step 3 and Roadblock 2.3)?

Meeting 1B

Meeting 1B activities include introducing toolkit formative assessments, debriefing and deepening the use of online apps for locating fractions on a number line and comparing fractions, incorporating strategies to carry out Recommendation 2 and addressing potential roadblocks, using a template for analysis of student work, and completing a module reflection.

The agenda for Meeting 1B is as follows.

Agenda for Meeting 1B

Topic and resources
Icebreaker • Formative assessment exploration
Online apps focused on locating and comparing fractions • Module 1, Handout 5: Locating fractions on a number line activity using the Locating Fractions on a Number Line (1) app • Locating Fractions on a Number Line (1) app: https://www.geogebra.org/m/uchh5qmd • Module 1, Handout 6: Comparing Rational Numbers–Fractions activity • Comparing Rational Numbers–Fractions app: https://www.geogebra.org/m/vcy4zn8e
Locating fractions formative assessment • Module 1, Handout 7: Locating Fractions on a Number Line formative assessment activity
Analysis of student work • Module 1, Handout 8: Analyzing student responses to the Locating Fractions on a Number Line formative assessment
Wrap-up • Module 1, Handout 9: Module 1 reflection

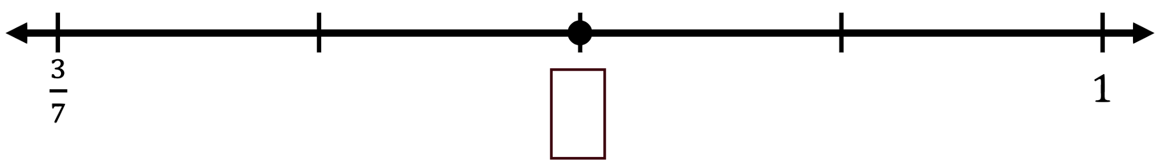
Module 1, Handout 7: Locating Fractions on a Number line formative assessment activity

The formative assessments in the professional development elicit common misunderstandings, errors, and overgeneralizations. Each formative assessment focuses on one or two common misunderstandings identified in the research base, and each takes approximately 10 minutes to complete. Analysis of student thinking elicited by the formative assessments can inform subsequent instruction.

> The formative assessment (in section D of this handout) supports identification of students who are exhibiting common mistakes and misunderstandings about locating fractions on a number line. It is directly linked to the second implementation step for Recommendation 2 in the practice guide.

A. Example item

1. Which fraction names this point on the number line?



a. $\frac{3}{5}$ b. $\frac{2}{7}$ c. $\frac{1}{2}$ d. $\frac{5}{7}$

2. Explain why you chose this fraction.

3. Why might students choose the other answers?

B. Read and reflect

The number line is an essential model used in developing fraction concepts. It supports the critical understanding that each fraction is one number with a single value that falls on a specific location on a number line. Number lines clearly illustrate the magnitude of fractions; the relationships between fractions and whole numbers; and the relationships among fractions, decimals, and percents.

Students need to have a foundational set of conceptual understandings about fractions to locate them on a number line; for example, consider the following:

1. Each fraction has a specific location on a number line.
2. Although the starting and ending points of number lines may vary, the value of any one fraction will be at only one location on the number line.

   For example, in figure A, the number line shown ranges from 0 to 1, and in figure B the number line ranges from −1 to 1. The location of $\frac{1}{2}$ remains the same—equidistant between 0 and 1.

Figure A

Figure B

3. The starting and ending points of the number line determine the value at each partition after the number line has been divided.
4. The number and distribution of partitions indicated by hash marks help students find the length of an interval between two hash marks.
5. There are always fractions located between any two points or hash marks on a number line (the density property).

Students can develop flawed understandings that may lead to misconceptions about fractions and number lines if left unaddressed. The Locating Fractions on a Number formative assessment focuses on Roadblocks 2.1-2.3 in the practice guide and on two other misunderstandings noted in the research on students' mathematical reasoning.

C. Potential roadblocks and misunderstandings

- **Roadblock 2.1.** "Students try to partition the number line into fourths by drawing four hashmarks rather than three, or they treat the whole number line as the unit."
- **Roadblock 2.2.** "When students locate fractions on the number line, they treat numbers in the fraction as whole numbers (e.g., placing $\frac{3}{4}$ between 3 and 4)."
- **Roadblock 2.3.** "Students have difficulty understanding that two equivalent fractions are the same point on a number line."
- **Misunderstanding 1: Counting interval hash marks and/or points.** Some students consistently overgeneralize from whole number reasoning by counting the number of hash marks and/or points not located right on a hash mark without considering which fractions the hash marks represent.
- **Misunderstanding 2: Treating the whole line as one unit, ignoring the starting and ending values.** Students with this misunderstanding tend to disregard the starting and ending values of a number line, as well as the size of the intervals represented by each hash mark. Instead, they assume that the number line corresponds to a fraction bar of length 1 and focus on the relative position of the fraction along the number line.

1. Which, if any, of these misunderstandings or roadblocks have you seen in your classroom? (Please describe)

2. What strategies have you used in the past to support student understanding related to these misunderstandings or roadblocks?

D. Review the formative assessment

Review the Locating Fractions on a Number Line formative assessment (on the next two pages). Complete items 6 and 8.

- While working on the formative assessment items, be sure to show or explain how you know the answer you choose is correct.
- After you complete each item, review the alternative selected response choices and be prepared to discuss why those choices may have been included.
    - What misunderstandings might each item elicit?
    - What do you notice about the selected response item choices?

Locating Fractions on a Number Line

Name: ____________________

Directions: Solve each problem. Select your answer from the choices provided.

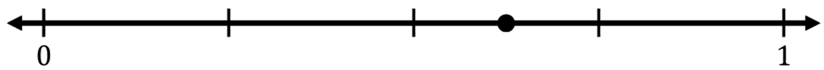
1. Which fraction can name this point?

   ○ $\frac{3}{4}$

   ○ $\frac{5}{8}$

   ○ $\frac{1}{2}$

   ○ $\frac{4}{5}$



Explain why you chose this fraction.

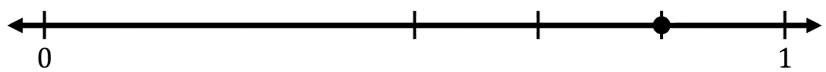
2. Which fraction can name this point?

   ○ $\frac{7}{8}$

   ○ $\frac{3}{4}$

   ○ $\frac{5}{6}$

   ○ $\frac{4}{5}$



Explain why you chose this fraction.

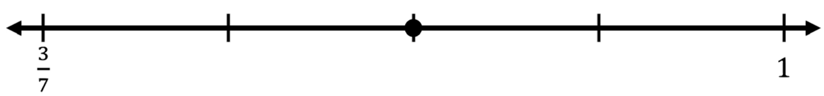
3. Which fraction can name this point?

   ○ $\frac{3}{5}$

   ○ $\frac{2}{7}$

   ○ $\frac{1}{2}$

   ○ $\frac{5}{7}$



Explain why you chose this fraction.

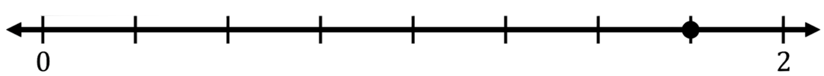
4. Which fraction can name this point?

   ○ $\frac{8}{9}$

   ○ $\frac{7}{4}$

   ○ $\frac{13}{7}$

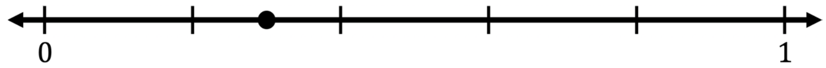
   ○ $\frac{7}{8}$



Explain why you chose this fraction.

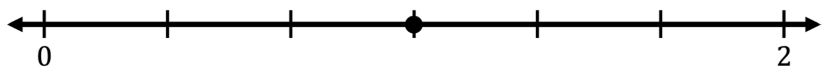
Locating Fractions on a Number Line (continued)

5. Which fraction can name this point?
   - ○ $\frac{3}{10}$
   - ○ $\frac{1}{4}$
   - ○ $\frac{3}{6}$
   - ○ $\frac{2}{5}$



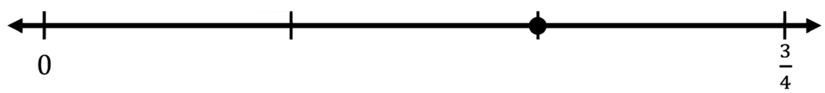
Explain why you chose this fraction.

6. Which fraction can name this point?
   - ○ $\frac{4}{7}$
   - ○ $\frac{4}{3}$
   - ○ $\frac{1}{2}$
   - ○ $\frac{1}{1}$



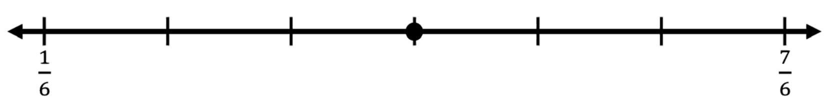
Explain why you chose this fraction.

7. Which fraction can name this point?
   - ○ $\frac{5}{8}$
   - ○ $\frac{1}{2}$
   - ○ $\frac{1}{4}$
   - ○ $\frac{2}{3}$



Explain why you chose this fraction.

8. Which fraction can name this point?
   - ○ $\frac{1}{2}$
   - ○ $\frac{4}{7}$
   - ○ $\frac{2}{3}$
   - ○ $\frac{5}{6}$



Explain why you chose this fraction.

Module 1, Handout 8: Analyzing student responses to the Locating Fractions on a Number Line formative assessment

A. Directions

Review the student responses to formative assessment items on the following page. Consider the evidence you see to determine whether either of the identified misunderstandings are present or if other mathematical understandings exist.

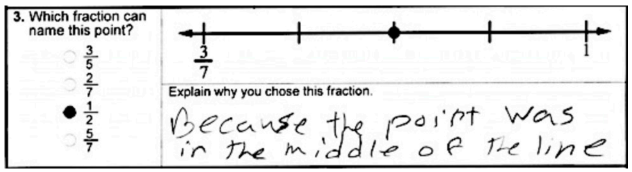
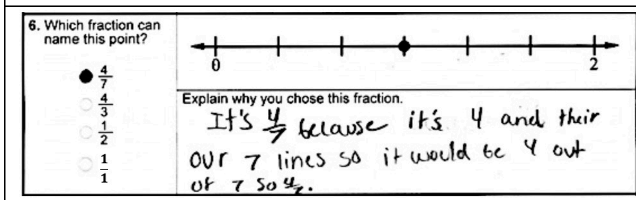
This analysis can support understanding student thinking and misunderstandings in line with the **second implementation step for Recommendation 2**. Analysis of student work with a focus on separating what evidence you see from inferences about that evidence[1] is at the core of this activity.

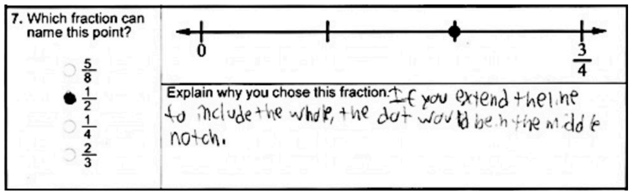
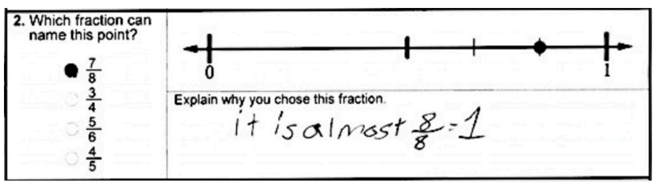
- **Misunderstanding 1: Counting interval hash marks and/or points.** Some students consistently overgeneralize from whole number reasoning by counting the number of hash marks and/or points that are not located right on a hash mark without considering which fractions the hash marks represent.
- **Misunderstanding 2: Treating the whole line as one unit, ignoring the starting and ending values.** Students with this misunderstanding tend to disregard the starting and ending values of a number line, as well as the size of the intervals represented by each hash mark. Instead, they assume that the number line corresponds to a fraction bar with a length of 1 and focus on the relative position of the fraction along the number line.

[1] The protocol is adapted with permission from the Visual Access to Mathematics project, ©2018 Education Development Center, Inc.

B. Analyses of example work

Examine each student's work and record your ideas in the following table.

Item	**Student potential** **What did students write or draw that is evidence of understanding?**	**Identify any common errors or misunderstandings and describe the evidence** **1. counting interval hash marks and/or points** **2. treating the line as one unit**	**Questions and strategies** **What questions do you want to ask the student or what are next steps you want to try with the student?**
 Answer: $\frac{1}{2}$ Explanation: "Because the point was in the middle of the line"			
 Answer: $\frac{4}{7}$ Explanation: "It's $\frac{4}{7}$ because it's 4 and their our 7 [there are] lines so it would be 4 out of 7 so $\frac{4}{7}$."			

Item	Student potential What did students write or draw that is evidence of understanding?	Identify any common errors or misunderstandings and describe the evidence 1. counting interval hash marks and/or points 2. treating the line as one unit	Questions and strategies What questions do you want to ask the student or what are next steps you want to try with the student?
 Answer: $\frac{1}{2}$ Explanation: “If you extend the line to include the whole, the dot would be in the middle notch.”			
 Answer: $\frac{7}{8}$ Explanation: “it is almost $\frac{8}{8} = 1$”			

C. Discussion questions

1. What might you ask these students to further understand or clarify their thinking?

2. What might you do next with these students? Why?

3. What else did you notice or wonder?

Module 1, Handout 9: Module 1 reflection

Module 1 focused on Recommendation 2, particularly on locating and comparing fractions using number lines. The handout questions support reflection on the implementation steps, the roadblocks, and approaches for **Recommendation 2**: "Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward."

Consider the activities and tasks from Module 1 as you reflect on your learning related to Recommendation 2.

1. Choose a strategy identified under one of the four implementation steps for Recommendation 2 in the practice guide and explain how you will use that strategy to support students' understanding of locating and comparing fractions.

   Implementation steps for Recommendation 2:

   - "Use measurement activities and number lines to help students understand that fractions are numbers, with all the properties that numbers share" (practice guide, p. 21).
   - "Provide opportunities for students to locate and compare fractions on number lines" (practice guide, p. 22).
   - "Use number lines to improve students' understanding of fraction equivalence, fraction density (the concept that there are an infinite number of fractions between any two fractions), and negative fractions" (practice guide, p. 23).
   - "Help students understand that fractions can be represented as common fractions, decimals, and percentages, and develop students' ability to translate among these forms" (practice guide, p. 24).

2. As students transition from the use of whole numbers to fractions, they may overgeneralize ideas about magnitude and other properties of whole numbers to their use of fractions. (See Roadblock 2.2 on practice guide page 25 for more information.)

   Describe how you would help students to overcome those types of misunderstandings using strategies from your readings from the practice guide, activities, and discussions during Module 1.

Credits

Information is provided about photos and other resources used in Module 1.

Photos

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Resources

The toolkit is based on the recommendations of the What Works Clearinghouse practice guide Developing Effective Fractions Instruction for Kindergarten Through 8th Grade.

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The Locating Fractions on a Number Line (1) online app was originally created by EDC in Maine using GeoGebra and licensed under a Creative Commons Attribution-ShareAlike 3.0 Unported International License (CC BY-SA 3.0).

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