Institute of Education Sciences

Regional Educational Laboratory Midwest

at American Institutes for Research

Teaching Fractions Toolkit

Professional development for grade 6 teachers: Module 2 facilitator guide

Understanding fraction addition and subtraction

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Contents

Facilitator guide overview

Facilitator guide purpose

Each Teaching Fractions Toolkit professional development module has a facilitator guide for facilitators who are leading the toolkit's professional development for a group of grade 6 math teachers. The facilitator guides, along with the slide decks for each module, provide facilitators with the information and accompanying resources they need to prepare for and lead teacher learning experiences. With these resources, a facilitator can support teachers with applying evidence-based strategies to enhance grade 6 students' fraction understanding.

Facilitator guide components

Each facilitator guide includes information and resources for the module as a whole (the module introduction), detailed instructions for two facilitated meetings (Meeting A and Meeting B), and interim activities that participating teachers will complete individually between the meetings. The components that follow this overview are as follows.

Professional development modules summary

The professional development modules summary table summarizes the six professional development modules, including their schedule and content. Review this overview to put in context the module you are currently preparing to facilitate for teachers.

Module introduction

The module introduction includes a summary of the module content and activities, the module objectives, a review of which practice guide recommendations are the focus of the module, a review of the sequence of meetings and activities in the module, and an overview of preparation and resources needed to facilitate the module.

Information for facilitating each section of the module

Information about facilitating the module for a group of teachers is divided into three sections:

1. Preparing for and leading Meeting A.
2. Supporting teachers in their individual work on interim activities for the module.
3. Preparing for and leading Meeting B.

The sections about facilitating Meetings A and B for the module include agendas with links to resources, detailed instructions for the professional development activities, tips, and background and rationale information. The section focused on the interim activities provides an overview of self-paced professional development activities that teacher participants will complete individually between meetings and guidance about supporting teachers in that work.

Credits and references

Credits and references are provided for citations, photos, and/or resources referred to in this facilitator guide and the presentation slide decks for this professional development module.

Color-coding in the facilitator guides

In both this guide and the associated module workbook, pages have a right-side page edge color that indicates the module section:

A brown edge indicates **general introductory information** about the module (starts on page 6).

A purple edge indicates information about the facilitated **Meeting A** of the module (starts on page 9).

A green edge indicates the information about the module's **interim activities** to be completed by teachers between meetings (starts on page 15).

A blue edge indicates information about the facilitated **Meeting B** of the module (starts on page 16).

Professional development modules summary

The complete professional development sequence includes six modules, each with three sections. The authors recommended that the modules be completed in sequential order because the activities of later modules build on the activities completed during prior modules.

Module and title	Description of module content	Module sections
Module 1 Fractions on a number line	• Provides a background for the professional development, including why a focus on effective fractions instruction, key areas of emphasis, activities, tools, associated icons, and module structure. • Introduces the practice guide and Recommendation 2 and the importance of visual representations. • Explores the Recommendation 2 implementation steps, with an emphasis on number lines as a central representational tool for teaching fraction concepts, and provides opportunities to engage with associated math tasks. • Introduces the Teacher Reflection Tool and uses it to record a baseline assessment of understanding and use of Recommendations 2-4. • Incorporates additional activities: using online apps for locating fractions on a number line and comparing fractions, introducing toolkit formative assessments, analyzing student work, and completing a module reflection.	**Meeting 1A** (120 minutes) **Module 1 interim activities** (teacher self-paced; 120 minutes) **Meeting 1B** (60 minutes)
Module 2 Understanding fraction addition and subtraction	• Introduces Recommendation 3 and its implementation steps, with a focus on creating and using area models, number lines, and other visual representations to understand the addition and subtraction of fractions. • Implements a formative assessment with students to help identify and plan to address any common mistakes and misunderstandings related to adding and subtracting fractions. • Incorporates additional activities: reading from the practice guide, using an online app for fraction estimation, working on a fraction addition task, analyzing student work from a fraction addition and subtraction formative assessment, and completing a module reflection.	**Meeting 2A** (60 minutes) **Module 2 interim activities** (teacher self-paced; 120 minutes) **Meeting 2B** (60 minutes)

Module and title	Description of module content	Module sections
Module 3 Understanding fraction multiplication and division	• Explores Recommendation 3 implementation steps, with a focus on using real-world contexts with plausible numbers and the use of visual representations to improve students' understanding of procedures for multiplication and division with fractions. • Continues the focus on fraction division and Recommendation 3, with exploration of visual representations used for fraction division tasks and use of predicting and judging the reasonableness of answers for fraction division problems. • Reviews the implementation steps and potential roadblocks associated with Recommendation 3 and supports collaborative planning of a lesson that incorporates those implementation strategies to mitigate anticipated roadblocks. • Incorporates additional activities, including using an online app for dividing fractions, analyzing students' work and identifying strengths and roadblocks related to Recommendation 3, engaging with a sorting task while consolidating knowledge related to Recommendations 2 and 3, revisiting and updating the Teacher Reflection Tool, and completing a module reflection.	**Meeting 3A** (60 minutes) **Module 3 interim activities** (teacher self-paced; 120 minutes) **Meeting 3B** (60 minutes)
Module 4 Implementation of fraction computation	• Continues the focus on fraction computation and Recommendations 2 and 3, including activities focused on the relationship between fraction multiplication and fraction division. • Applies new classroom implementation strategies associated with implementation steps for Recommendations 2 and 3 and includes approaches to mitigate anticipated roadblocks. • Incorporates additional activities, including using an online app for multiplying fractions, reading about Recommendation 3 implementation steps, analyzing your students' work and identifying strengths and roadblocks related to Recommendations 2 and 3, and completing a module reflection.	**Meeting 4A** (60 minutes) **Module 4 interim activities** (teacher self-paced; 120 minutes) **Meeting 4B** (60 minutes)

Module and title	Description of module content	Module sections
Module 5 Understanding ratio, rate, and proportion	• Introduces Recommendation 4, with a focus on Implementation Steps 1 and 2, associated Roadblocks 4.1 and 4.2, and activities to support understanding of proportional relations and using visual representations to solve ratio, rate, and proportion problems. • Uses an online app to explore and use visual representations, including tape diagrams, to solve ratio and rate problems. • Incorporates additional activities, including using visual representations to explore the multiplicative relations within and between equivalent ratios; using double number lines to solve ratio, rate, and proportion problems; making connections between visual representations and the cross-multiplication strategy; and completing a module reflection.	**Meeting 5A** (60 minutes) **Module 5 interim activities** (teacher self-paced; 120 minutes) **Meeting 5B** (60 minutes)
Module 6 Implementation of ratio, rate, and proportion	• Explores Recommendation 4 further, with a particular focus on Implementation Step 3 and associated Roadblock 4.3. • Applies and supports reflection on classroom implementation based on Recommendation 4 implementation steps as well as approaches for mitigating anticipated roadblocks. • Incorporates additional activities, including using the Teacher Reflection Tool, to support reflection on understanding and use of the practice guide, and on learning completed across the professional learning modules.	**Meeting 6A** (60 minutes) **Module 6 interim activities** (teacher self-paced; 120 minutes) **Meeting 6B** (60 minutes)

Note: The information about how to carry out the recommendation is in the form of steps for implementing the recommendation, which are referred to as "implementation steps" throughout this professional development.

Module 2 introduction

Module 2 focuses on representing and understanding addition and subtraction with fractions by creating and using visual representations, such as number lines and area models. It starts by introducing Recommendation 3 and explores an online app for estimating fraction sums and differences on a number line. The readings are from the Recommendation 3 text in the practice guide.

The module includes content related to learning about and using a formative assessment for eliciting common mistakes and misunderstandings regarding adding and subtracting fractions. Teachers will implement the formative assessment with students between Meeting 2A and Meeting 2B. During Meeting 2B, teachers will analyze student responses to inform planning about how to address any identified common mistakes and misunderstandings.

Meeting 2B also includes using estimation and visual representations—both a written diagram and a model created by an online app—during work on a fraction computation task.

Objectives for Module 2

By the end of Module 2, teachers will be able to do the following:

- Create and use visual representations to represent and understand computational procedures with fractions.
- Explain how visual representations relate to computations with fractions and why they make sense.
- Explain how to use area models, number lines, and other visual representations to improve students' understanding of formal computational procedures.
- Identify and address common mistakes and misunderstandings related to computational procedures with fractions.

Recommendation focus for Module 2

The Teaching Fractions Toolkit professional development focuses on practice guide Recommendations 2-4 in the context of grade 6 math content. The design of the professional development also is in line with Recommendation 5 and its implementation steps, which relate to professional development programs for teachers of students learning fractions content.

Module 2 specifically focuses on building knowledge related to Recommendation 3 in the context of grade 6 standards and foundational knowledge often taught in grades 4-5.

Recommendation 3: "Help students understand why procedures for computations with fractions make sense."

Sections of Module 2

This module includes an initial facilitated meeting (Meeting 2A), a set of self-paced activities for teachers to do between meetings (interim activities) that is estimated to take 2 hours, and a second facilitated meeting (Meeting 2B).

Module 2

Understanding Fraction Addition and Subtraction focuses on Recommendation 3, including a consideration of common mistakes and misunderstandings and the role of visual representations for adding and subtracting fractions.

Meeting 2A ➔ **Module 2 interim activities** ➔ **Meeting 2B**

Preparation and resources for Module 2

Before leading Module 2, be sure to review all resources and facilitator instructions in this guide and in the slide decks, review pages 26-34 and 42-46 of the practice guide, explore the module activities yourself if you have not done so before, and anticipate what questions your participants might have. Make sure that a room is reserved with workspace for teachers to explore tasks and discuss ideas if meeting in person or ensure a videoconference setup if meeting online. Ensure that you will have internet access so you can access and demonstrate the online apps. Based on the size of your group, consider when you will use breakout groups to increase opportunities for engagement and discussion. Also consider how you will gauge participants' developing understanding. In larger groups you might consider using informal checks for understanding or exit tickets.

To facilitate this module, in addition to this facilitator guide, you will need the materials listed below that are also posted on the Teaching Fractions Toolkit website. It may be helpful to print copies of these to refer to when preparing for and facilitating the meetings so you do not have to view everything electronically when leading the meetings:

- **Practice guide:** The full practice guide that serves as the foundation for this professional development toolkit.
- **Slide deck:** The Meeting 2A slide deck and the Meeting 2B slide deck.
- **Participant workbook:** The Module 2 workbook that includes all handouts used in this module. Each participant will need access to an electronic copy or a print copy of the workbook. Participants may prefer a print copy so they can write on it.
- **Additional resources:** Module activities refer to online apps, readings, videos, and other resources. Links to these resources are summarized in the following table for your quick reference and embedded in the participant workbooks and on the Teaching Fractions Toolkit website.

Type	Resource	Link and description
Online app	Estimating Fraction Sums and Differences	https://www.geogebra.org/m/aaedp6mg Use number lines to estimate fraction sums and differences.
Optional online app	Fraction Addition	https://www.geogebra.org/m/npx3mctd Uses bar models to find sums of fractions.
Optional online app	Fraction Subtraction	https://www.geogebra.org/m/quk6d8gz Uses bar models to find differences between fractions.
Practice guide readings	Recommendation 3	Read pages 26-34 to learn about Recommendation 3.

Meeting 2A

During Meeting 2A, introduce participants to Recommendation 3 in the context of adding and subtracting fractions, noting that Module 3 will continue this exploration in the context of multiplying and dividing fractions. Lead participants in activities in which they will make estimates about fraction sums and differences using an online app, review the Recommendation 3 implementation steps using a jigsaw approach, and make connections with the estimation activity. Then introduce a formative assessment focused on fraction addition and subtraction that participants will use with their students during the interim activities and discuss in Meeting 2B to further explore misunderstandings related to Recommendation 3, Implementation Step 3 and potential Roadblock 3.1.

The agenda for Meeting 2A is as follows.

Agenda and resources for Meeting 2A

Minutes	Topic and resources
5	**Introduction** (slides 1-3)
35	**Recommendation 3 exploration** (slides 4-12) • Module 2, Handout 1: Estimating Fraction Sums and Differences online app activity • Estimating Fraction Sums and Differences online app: https://www.geogebra.org/m/aaedp6mg • Module 2, Handout 2: Recommendation 3 jigsaw reading activity
15	**Fraction addition and subtraction formative assessment** (slides 13-19) • Module 2, Handout 3: Fraction addition and subtraction formative assessment activity
5	**Wrap-up** (slides 20-24) • Instructions for Module 2 interim activities

Facilitator directions for Meeting 2A

The directions for how to facilitate each Meeting 2A agenda topic are provided below. The left side includes the main instructions, and the right side includes background information and tips.

Introduction (5 minutes, slides 2-3)

1. **Review the agenda** (slide 2).
2. **Review the module objectives** (slide 3).
   - Module 2 introduces Recommendation 3 and supports the exploration of this recommendation through the lens of fraction addition and subtraction. Module 3 will continue exploration of Recommendation 3 in relation to fraction multiplication and division.
   - This module includes activities to focus on the role of visual representations in supporting fraction computation (Recommendation 3, Implementation Step 1) and on formative assessments and student work analyses to focus on the kinds of common mistakes and misunderstandings and understanding that students may have about fraction computation (Implementation Step 3).

Recommendation 3: "Help students understand why procedures for computations with fractions make sense."

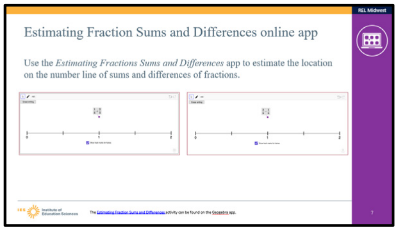
Recommendation 3 implementation steps:

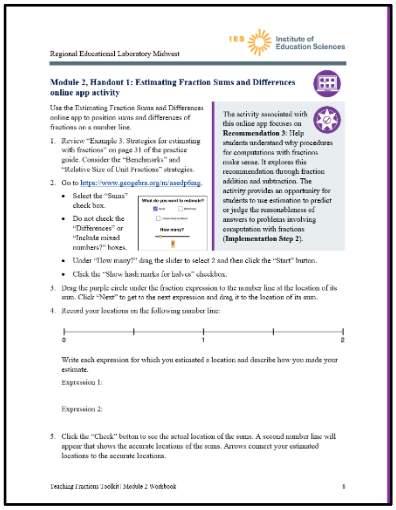
1. "Use area models, number lines, and other visual representations to improve students' understanding of formal computational procedures."
2. "Provide opportunities for students to use estimation to predict or judge the reasonableness of answers to problems involving computation with fractions."
3. "Address common [mistakes and misunderstandings] about computational procedures with fractions."
4. "Present real-world contexts with plausible numbers for problems that involve computing with fractions."

Recommendation 3 exploration (35 minutes, slides 4-12)

1. **Introduce Recommendation 3** by briefly reviewing both the recommendation itself and its four implementation steps (slides 5-6).
2. **Open the Estimating Fraction Sums and Differences online app** (https://www.geogebra.org/m/aaedp6mg) to introduce its key features; explain to participants that they will explore estimating fraction addition and subtraction problems using this online app to support later discussions about the implementation steps for Recommendation 3 (slide 7).
   - Users can set the number of expressions that will be provided; whether the expressions are sums, differences, or both; and whether the expressions will include mixed numbers.
   - Users of the app can drag and drop the purple dot for an expression to the number line to estimate the correct location on the number line, but it does not make an actual estimate until the check button is clicked.
   - A good starting point is to choose one or two sums and take time to model thinking and discuss strategies for placing the fractions on the number line.
   - Users can check the "Show halves" check box to add hash marks at the half locations on the number line (recommended).
3. **Demonstrate that users of the app can drag and drop the purple dot** for an expression to the number line to estimate the correct location on the number line. Model one example. Discuss your thinking. Generate the next expression. Ask a participant to share their thinking about where the purple dot representing the sum should go. Drag the point and discuss the solution after checking the estimate.
4. **Refer the participants to "Example 3. Strategies for estimating with fractions"** on page 31 of the practice guide. Ask them to consider "benchmarks" and "relative size of unit fractions" when using the app.
5. **Give participants time to work on Module 2, Handout 1** to make estimates for the locations of sums/differences presented in the app; then check the accuracy of their selections (slide 8).

Tip: If you feel comfortable doing so, you might choose to go directly to the app and hide the slides that include the app directions. These directions will be available for reference as needed.





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Module 2, Handout 1: Estimating Fraction Sums and Differences online app activity

Use the Estimating Fraction Sums and Differences online app to position sums and differences of fractions on a number line.

The activity associated with this online app focuses on **Recommendation 3**: Help students understand why procedures for computations with fractions make sense. It explores this recommendation through fraction addition and subtraction. The activity provides an opportunity for students to use estimation to predict or judge the reasonableness of answers to problems involving computation with fractions (**Implementation Step 2**).

1. Review "Example 3. Strategies for estimating with fractions" on page 31 of the practice guide. Consider the "Benchmarks" and "Relative Size of Unit Fractions" strategies.
2. Go to https://www.geogebra.org/m/aaedp6mg.
   - Select the "Sums" check box.
   - Do not check the "Differences" or "Include mixed numbers?" boxes.
   - Under "How many?" drag the slider to select 2 and then click the "Start" button.
   - Click the "Show hash marks for halves" checkbox.
3. Drag the purple circle under the fraction expression to the number line at the location of its sum. Click "Next" to get to the next expression and drag it to the location of its sum.
4. Record your locations on the following number line:

   0 1 2

   Write each expression for which you estimated a location and describe how you made your estimate.

   Expression 1:

   Expression 2:
5. Click the "Check" button to see the actual location of the sums. A second number line will appear that shows the accurate locations of the sums. Arrows connect your estimated locations to the accurate locations.

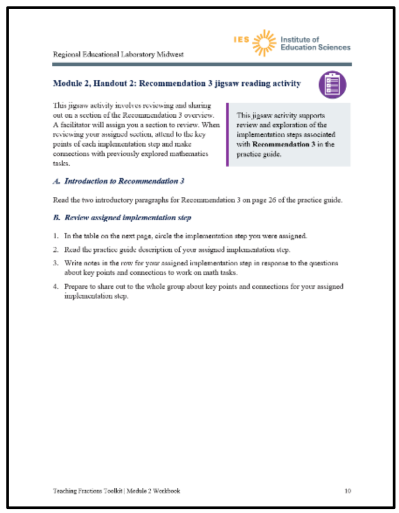
Teaching Fractions Toolkit | Module 2 Workbook 8

(Module 2, Handout 1, Workbook)

Background: When estimating the location of sums or differences of fractions, it is important for

6. **Have participants share the strategies they used and the challenges they encountered** when making estimates about the locations of the sums and differences; then discuss the strategies they used to locate the results of the computations (slide 9).
7. **Discuss what participants anticipate** the understandings, strategies, struggles, and supports might be for students working on an estimating task such as this one (slide 10).
8. **Tell participants that they will now look at Recommendation 3 and its implementation steps more carefully.** Make connections with the estimation task they just completed and discussed.
9. **Explain the directions for doing a jigsaw activity** to explore Recommendation 3, assign each participant one of the four implementation steps to review, and then give participants time to read and reflect using Module 2, Handout 2 (slide 11).
10. **Have participants report on and discuss the key points and connections** they identified for each implementation step (slide 12). Share the following possible connections to the task if participants do not mention them:
    - Implementation Step 1: Use number lines to support visualization of estimates of sums and differences of fractions.
    - Implementation Step 2: Attempt to estimate the location of fractions using reasoning before performing computation to find the sum or difference.
    - Implementation Step 3: Using the "Check" feature in the app may support students in addressing their misunderstandings about how fractions are added or subtracted or what the magnitude of a fraction is.
    - Implementation Step 4: Ask students to give a real-world context related to the fraction computation problems provided by the app.

students to use strategies other than finding common denominators. The practice guide emphasizes that "estimation requires students to use reasoning skills and thus leads them to focus on the meaning of procedures for computing with fractions" (p. 30) and "students can use these estimates to judge the reasonableness of their answers" (p. 30). For example, students might consider how close a fraction is to a benchmark such as $\frac{1}{2}$ or 1 and then consider the relative size of the second fraction. Then they can estimate the position of the sum or difference in relation to the original fraction.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Module 2, Handout 2: Recommendation 3 jigsaw reading activity

This jigsaw activity involves reviewing and sharing out on a section of the Recommendation 3 overview. A facilitator will assign you a section to review. When reviewing your assigned section, attend to the key points of each implementation step and make connections with previously explored mathematics tasks.

This jigsaw activity supports review and exploration of the implementation steps associated with **Recommendation 3** in the practice guide.

A. Introduction to Recommendation 3

Read the two introductory paragraphs for Recommendation 3 on page 26 of the practice guide.

B. Review assigned implementation step

1. In the table on the next page, circle the implementation step you were assigned.
2. Read the practice guide description of your assigned implementation step.
3. Write notes in the row for your assigned implementation step in response to the questions about key points and connections to work on math tasks.
4. Prepare to share out to the whole group about key points and connections for your assigned implementation step.

Teaching Fractions Toolkit | Module 2 Workbook 10

(Module 2, Handout 2, Workbook)

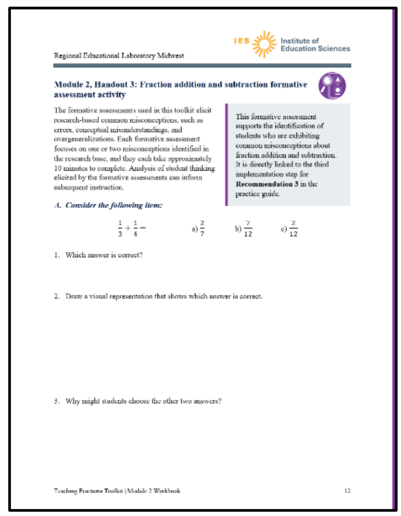
Tip: When it is time to assign implementation steps for the jigsaw activity, have participants count off by fours if you are meeting in person, ask for volunteers for each implementation step, or create breakout rooms for numbers 1-4 and assign participants to them if meeting by videoconference.

Fraction addition and subtraction formative assessment (15 minutes, slides 13-19)

1. **Review the purpose and structure of formative assessments** provided in this professional development (slide 14).
2. **Give participants a few minutes to work on one example item** ($\frac{1}{3}+\frac{1}{4}$) from the formative assessment on part A of Module 2, Handout 3; then have them share their thinking (slide 15). Ideas to highlight or share during the discussion include the following:
   - It is not necessary to do computation to solve this; instead, a solver can consider the reasonableness of answers, which relates to Recommendation 3, Implementation Step 2.
   - For the first answer choice: Considering the location of $\frac{2}{7}$ on the number line can support understanding that $\frac{2}{7}$ is less than the sum in question.
   - For the third answer choice: $\frac{2}{12}$ is equivalent to $\frac{1}{6}$, which is smaller than either fraction in the sum in question.
3. **Review foundational ideas that students need to understand related to what the formative assessment elicits.** This foundational understanding is what students need to develop to overcome common mistakes and misunderstandings (Recommendation 3, Implementation Step 3), and visual representations can support this understanding (Recommendation 3, Implementation Step 1; slide 16).
   - To understand part-whole representations of fractions, students must understand that a fraction can represent a part of some whole. The numerator represents the number of parts, and the denominator represents the size of those parts.
   - Student understanding also includes viewing each operation as adding or subtracting the numerators (the number of parts) but not the denominators (because they represent the size of the parts).
   - Student understanding includes learning to represent fractions with different visual models and to connect parts of the visual models with the numerator or the denominator.

Tip: Participants saw an earlier formative assessment in Module 1 that focused on locating fractions on a number line.

Tip: The same ideas outlined on slide 16 are reviewed in part B of Module 2, Handout 3, which participants will review during the interim activities.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Module 2, Handout 3: Fraction addition and subtraction formative assessment activity

The formative assessments used in this toolkit elicit research-based common misconceptions, such as errors, conceptual misunderstandings, and overgeneralizations. Each formative assessment focuses on one or two misconceptions identified in the research base, and they each take approximately 10 minutes to complete. Analysis of student thinking elicited by the formative assessments can inform subsequent instruction.

This formative assessment supports the identification of students who are exhibiting common misconceptions about fraction addition and subtraction. It is directly linked to the third implementation step for **Recommendation 3** in the practice guide.

A. Consider the following item:

$\frac{1}{3}+\frac{1}{4}=$ a) $\frac{2}{7}$ b) $\frac{7}{12}$ c) $\frac{2}{12}$

1. Which answer is correct?
2. Draw a visual representation that shows which answer is correct.
3. Why might students choose the other two answers?

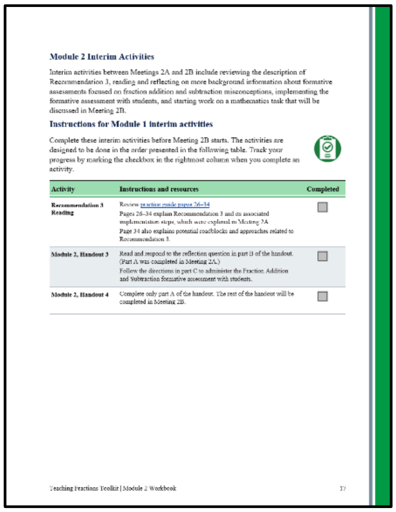
Teaching Fractions Toolkit | Module 2 Workbook 12

(Module 2, Handout 3, Workbook)

4. **Share two examples of misunderstandings that relate to Recommendation 3, Implementation Step 3** using the example item that participants explored (slides 17–18).
5. **Explain to participants that during the interim activities before Meeting 2B, they should take about 10 minutes to administer the fraction addition and subtraction formative assessment** to some students and then bring copies of student work to Meeting 2B to analyze (slide 19). Tell participants that the appendix in the Module 6 workbook includes three versions of the formative assessment in Module 2, Handout 3. These versions are formatted to be used with students, and they are written in English, Spanish, and Arabic.

Wrap-up (5 minutes, slides 20–24)

6. **Review the list of Module 2 interim activities** that participants should complete before Meeting 2B and answer any questions from participants (slide 21). It may be helpful to ask participants to submit all student work samples from the formative assessment to you for review prior to Meeting 2B. Next, explain the following:
   - For Module 2, Handout 3, they already completed part A during this meeting, but they will read and reflect on part B and use part C to administer the formative assessment to students.
   - For Module 2, Handout 4, they need to complete only part A. Part B and use of another online app will be explored during Meeting 2B.
7. **Verify that everyone knows the time and the location for Meeting 2B.**


Module 2 Interim Activities

Interim activities between Meetings 2A and 2B include reviewing the description of Recommendation 3, reading and reflecting on more background information about formative assessments focused on fraction addition and subtraction misconceptions, implementing the formative assessment with students, and starting work on a mathematics task that will be discussed in Meeting 2B.

Instructions for Module 1 interim activities

Complete these interim activities before Meeting 2B starts. The activities are designed to be done in the order presented in the following table. Track your progress by marking the checkbox in the rightmost column when you complete an activity.

Activity	Instructions and resources	Completed
Recommendation 3 Reading	Review practice guide pages 26–34. Pages 26–34 explain Recommendation 3 and its associated implementation steps, which were explored in Meeting 2A. Page 34 also explains potential roadblocks and approaches related to Recommendation 3.	☐
Module 2, Handout 3	Read and respond to the reflection question in part B of the handout. (Part A was completed in Meeting 2A.) Follow the directions in part C to administer the Fraction Addition and Subtraction formative assessment with students.	☐
Module 2, Handout 4	Complete only part A of the handout. The rest of the handout will be completed in Meeting 2B.	☐

Teaching Fractions Toolkit | Module 2 Workbook

(Module 2 interim activities, Workbook)

Tip: Let participants know that the interim activities include some of the handouts in their workbooks that they started working on during Meeting 2A.

Module 2 interim activities

Between Meetings 2A and 2B, participants will complete interim activities, which should take approximately 2 hours. These activities include reading about Recommendation 3, reviewing and reflecting on information about the fraction addition and subtraction formative assessment and having some students work on it, and completing Part A of the Jog-A-Thon task. Refer to the information for Meeting 2B for additional details about the interim activities.

Activity	Participants will . . .
Reading	Review practice guide pages 26-34. Pages 26-34 explain Recommendation 3 and its associated implementation steps, which were explored during Meeting 2A. Page 34 also explains potential roadblocks and approaches related to Recommendation 3.
Module 2, Handout 3	Read and respond to the reflection question in part B of the handout. (Part A was completed during Meeting 2A.) Follow the directions in Part C of the handout to administer the fraction addition and subtraction formative assessment with students.
Module 2, Handout 4	Complete only Part A of the handout. The rest of the handout will be completed during Meeting 2B.

Interim activities facilitator role

In the participant workbooks, participants have directions and all the resources they need to complete the interim activities between the two meetings. As a facilitator, you might wish to send a reminder midway between the two meetings to remind participants that they should be working on the interim activities and check in to see if any participants have questions or need support concerning the interim activities. For this module, it is especially important to remind participants to either submit the student work to you prior to the start of Meeting 2B or bring student work from the formative assessment to Meeting 2B. It may be helpful to ask participants to submit work ahead of time, so that you can review the work and/or share the work more easily at meeting 2B.

Meeting 2B

During Meeting 2B, support participants to consider common mistakes and misunderstandings related to Recommendation 3 by examining provided examples of student work and work from their own classrooms on the fraction addition and subtraction formative assessment. Then lead participants in exploring ideas about the role of visual representations in adding and subtracting fractions related to Recommendation 3 by engaging with the Jog-A-Thon task they started during the interim activities using an online app. End the meeting by having participants complete a module reflection.

The agenda for Meeting 2B is as follows.

Agenda for Meeting 2B

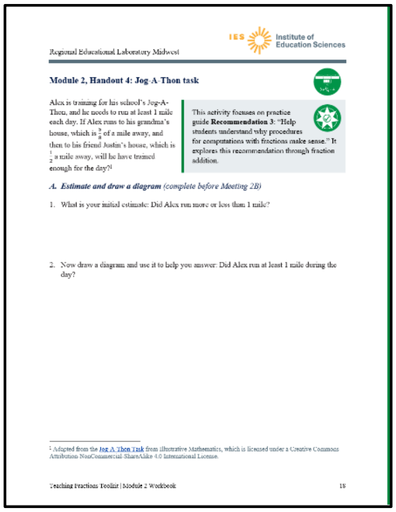
Minutes	Topic and resources
5	**Introduction** (slides 1-2)
25	**Analysis of student thinking and misunderstandings** (slides 3-10) • Module 2, Handout 5: Analyzing student responses to the fraction addition and subtraction formative assessment
20	**Exploration of online apps in line with Recommendation 3** (slides 11-15) • Module 2, Handout 4: Jog-A-Thon task (part B) • Fraction Addition online app: https://www.geogebra.org/m/npx3mctd • Optional: Fraction Subtraction online app: https://www.geogebra.org/m/quk6d8gz
10	**Wrap-up** (slides 16-21) • Module 2, Handout 6: Module 2 reflection

Facilitator directions for Meeting 2B

The directions for how to facilitate each section of the Meeting 2B agenda are in the following table.

Introduction (5 minutes, slides 1-2)	
1. **Review the agenda** (slide 2).	
Analysis of student thinking and misunderstandings (25 minutes, slides 3-10)	
1. **Introduce the core ideas behind how participants will analyze student work** in this professional development (slide 4). Share that participants will: • attend to evidence to support inferences, • seek seeds of understanding even while identifying evidence of common mistakes and misunderstandings, and • take an inquiry stance to consider what we can learn about students' thinking from their work. 2. **Give participants time to analyze the four sample student responses** shown on Module 2, Handout 5, following the directions for part A on the handout (slide 5). 3. **Lead a discussion about what participants noticed in the example student responses.** It should focus on which students showed evidence of a misunderstanding, how the various approaches are similar or different, possible next steps or questions for these students, and anything else participants noticed or wondered about during their analysis (slide 6). 4. **Next, tell participants to select four of their own students' responses** to items on the formative assessment, following the directions for part B of Module 2, Handout 5 (slide 7). 5. **Lead a discussion about what participants noticed in their own students' work.** It should focus on evidence of a misunderstanding, how the various approaches are similar or different, possible next steps or questions for the students, and anything else participants noticed or wondered (slide 8).	**Background:** "Attending to evidence to support inferences" means paying attention to what is visible in the work (and/or audible in a recording) to identify the evidence before interpreting what it means about student understanding. It is sometimes helpful to provide an example of evidence for inference that you can show, such as "We can see that my bottle of water is almost empty; we could infer that I was thirsty and so I drank most of the water already, or I was running late and didn't have time to fill it up before starting today," and so on. "Seeking seeds of understanding" is important for recognizing the building blocks of student understanding that can support next steps. "Taking an inquiry stance" may require a shift in mindset about looking at student work. Teachers often are under so much pressure to make quick decisions based on what students do. Encourage participants to consider this an opportunity to step back and investigate student work in more depth.

6. **Make connections between the misunderstandings that participants have been exploring through the formative assessment and Recommendation 3, Implementation Step 3** (address common mistakes and misunderstandings regarding computational procedures with fractions; slide 9). Share the following with participants:
   - Implementation Step 3 includes three examples of misunderstandings about adding and subtracting fractions.
   - These three misunderstandings are on the slide. The first aligns with the first misunderstanding from the formative assessment, the second aligns with the second misunderstanding from the formative assessment, and the third is relevant when students work with fractions greater than 1.
   - The formative assessment and careful analysis of student work can support teachers in noticing, identifying, and addressing these kinds of misunderstandings with students.
7. **Wrap up this section of the meeting by making connections to one of the potential roadblocks** listed for Recommendation 3: Roadblock 3.1 (slide 10). Share the following with participants:
   - Each recommendation has a set of potential roadblocks to implementation and possible solutions.
   - In some cases, the potential roadblocks to implementation are student misunderstandings.
   - Roadblock 3.1 is a student error or misunderstanding that relates directly to Recommendation 3 implementation steps that the group has been exploring through the formative assessment and student work analysis.
   - Teachers can use visual representations to help students understand the need for adding similar units, which leads to finding a common denominator. For example, using representations that hold units constant, such as a measuring tape with marked units (or a number line with marked units) can help students see the need for common unit fractions when adding to fractions.
   - Teachers can also demonstrate how common units are used to add fractions and how that relates to finding common denominators using the Fraction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Module 2, Handout 4: Jog-A-Thon task

Alex is training for his school's Jog-A-Thon, and he needs to run at least 1 mile each day. If Alex runs to his grandma's house, which is $\frac{5}{8}$ of a mile away, and then to his friend Justin's house, which is $\frac{1}{2}$ a mile away, will he have trained enough for the day?[1]

This activity focuses on practice guide **Recommendation 3**: "Help students understand why procedures for computations with fractions make sense." It explores this recommendation through fraction addition.

A. Estimate and draw a diagram *(complete before Meeting 2B)*

1. What is your initial estimate: Did Alex run more or less than 1 mile?

2. Now draw a diagram and use it to help you answer: Did Alex run at least 1 mile during the day?

[1] Adapted from the Jog A Thon Task from Illustrative Mathematics, which is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License.

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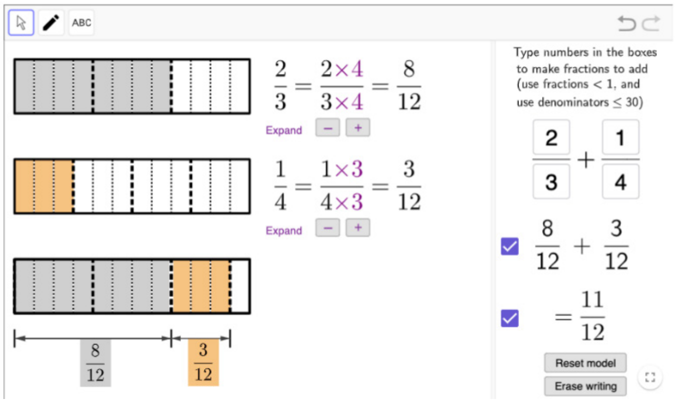
(Module 2, Handout 4, Workbook)

Tip: If any participants did not bring student work for analysis, they can pair up with colleagues to look at the work.

Tip: Misunderstandings related to multiplication and division of fractions will be explored in Module 3.

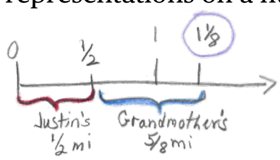
Background: Roadblock 3.1 is explained on page 34 in the practice guide.

Addition app (https://www.geogebra.org/m/jfbqrhrg#material/npx3mctd). With this app students can engage with making equal-size partitions in tape diagrams representing two different fractions, which then shows visually how the fractions can be added, and links this partitioning process to the use of a common denominator.

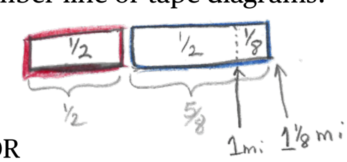


Exploration of online apps in line with Recommendation 3 (20 minutes, slides 11-15)

1. **Tell participants that we will now transition** from a focus on misunderstandings related to Recommendation 3, Implementation Step 3 to a focus on visual representations in line with Implementation Step 1, including how these visual recommendations can support students who have the misunderstandings that we explored (slide 12).
2. **Ask participants to share and discuss the visual representations they drew during the interim activities** for Part A of the Jog-A-Thon task (Module 2, Handout 4), as well as what different strategies and challenges they anticipate students might have when solving this task.
   - Examples of participant representations could include representations on a number line or tape diagrams:



OR



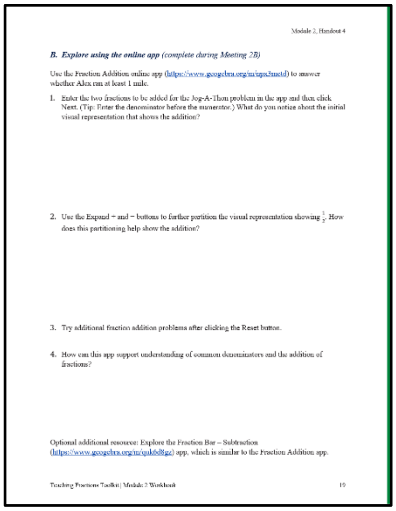
   - Highlight that Implementation Step 3 focuses on using visual representations to support students' understanding.

Background: Implementation Step 1 for Recommendation 3 is to "Use area models, number lines, and other visual representations to improve students' understanding of formal computational procedures."

Tip: If participants need help navigating the Fraction Addition app, then model how to enter the two fractions for the Jog-A-Thon problem in the app and how to use the "Expand" + and – buttons to change the partitioning of the visual representation and the Reset button to start again with a new fraction addition problem.

Tip: If you feel comfortable doing so, you might choose to go directly to the app and hide the slides that include the app directions. These directions will be available for reference as needed.

3. **Show participants the Fraction Addition app (https://www.geogebra.org/m/npx3mctd) that they will use to continue exploring visual representations** in the context of the Jog-A-Thon task and point them to the directions for using the app embedded in part B of Handout 4 (slide 13).
4. **Give participants time to work on part B of Handout 4 using the app; then lead a discussion** about what the initial visual representation shown in the app for the Jog-A-Thon problem shows, what the Expand buttons that control partitioning could help students see, and how the visual representations shown in the app support understanding of common denominators (slide 14).
5. **Lead a discussion to debrief how the addition app,** and the subtraction app (https://www.geogebra.org/m/quk6d8gz) if the participants had time to explore it, support the implementation steps for Recommendation 3 (slide 15). Highlight or summarize connections to the implementation steps during the discussion:
   - The visual representations in the apps can support student understanding of procedures for fraction addition and subtraction (related to Implementation Step 1). Note that other visual representations might be most supportive in other scenarios. An example from the practice guide is that many students learn to represent decimals using base-10 blocks or 100 grids (10 by 10 squares, with each square representing $\frac{1}{100}$ and the whole square representing 1). Familiarity with this representation also might help students understand adding and subtracting decimal and common fractions. For example, 100 grids can be used to illustrate that adding 2.34 + 1.69 is the same as adding $\frac{234}{100} + \frac{169}{100}$.
   - The visual representations can be integrated with opportunities for students to predict or judge the reasonableness of solutions (Implementation Step 2).
   - The visual representations can encourage understanding that moves past common mistakes and misunderstandings (related to Implementation Step 3).
   - The apps can be used with problems that are set in real-world contexts with plausible numbers (related to Implementation Step 4).


Module 2, Handout 4

B. Explore using the online app *(complete during Meeting 2B)*

Use the Fraction Addition online app (https://www.geogebra.org/m/npx3mctd) to answer whether Alex ran at least 1 mile.

1. Enter the two fractions to be added for the Jog-A-Thon problem in the app and then click Next. (Tip: Enter the denominator before the numerator.) What do you notice about the initial visual representation that shows the addition?
2. Use the Expand + and − buttons to further partition the visual representation showing $\frac{1}{2}$. How does this partitioning help show the addition?
3. Try additional fraction addition problems after clicking the Reset button.
4. How can this app support understanding of common denominators and the addition of fractions?

Optional additional resource: Explore the Fraction Bar – Subtraction (https://www.geogebra.org/m/quk6d8gz) app, which is similar to the Fraction Addition app.

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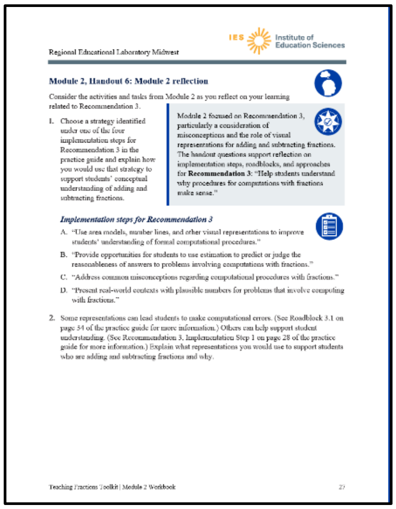
(Module 2, Handout 4, Workbook)

Tip: If you have enough time, encourage participants to explore subtraction problems in the Fraction Subtraction app as well.

Wrap-up (10 minutes, slides 16-21)

1. **Tell participants to write reflections on Module 2, Handout 6** about a strategy identified in one of the four implementation steps for Recommendation 3: How would they use that strategy to support students with adding or subtracting fractions? What representations would they use to support students who are adding and subtracting fractions (slide 17)?
2. **Check in with participants about any questions and review the Module 3 schedule** (slides 18-19).



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Module 2, Handout 6: Module 2 reflection

Consider the activities and tasks from Module 2 as you reflect on your learning related to Recommendation 3.

Module 2 focused on Recommendation 3, particularly a consideration of misconceptions and the role of visual representations for adding and subtracting fractions. The handout questions support reflection on implementation steps, roadblocks, and approaches for **Recommendation 3**: "Help students understand why procedures for computations with fractions make sense."

1. Choose a strategy identified under one of the four implementation steps for Recommendation 3 in the practice guide and explain how you would use that strategy to support students' conceptual understanding of adding and subtracting fractions.

Implementation steps for Recommendation 3

A. "Use area models, number lines, and other visual representations to improve students' understanding of formal computational procedures."

B. "Provide opportunities for students to use estimation to predict or judge the reasonableness of answers to problems involving computations with fractions."

C. "Address common misconceptions regarding computational procedures with fractions."

D. "Present real-world contexts with plausible numbers for problems that involve computing with fractions."

2. Some representations can lead students to make computational errors. (See Roadblock 3.1 on page 34 of the practice guide for more information.) Others can help support student understanding. (See Recommendation 3, Implementation Step 1 on page 28 of the practice guide for more information.) Explain what representations you would use to support students who are adding and subtracting fractions and why.

(Module 2, Handout 6, Workbook)

Credits

Information is provided about photos and other resources used in Module 2.

Photos

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Resources

The toolkit is based on the recommendations of the What Works Clearinghouse practice guide Developing Effective Fractions Instruction for Kindergarten Through 8th Grade.

The Estimating Fraction Sums and Differences online app was adapted from an app originally created by John Ulbright using GeoGebra and licensed under a Creative Commons Attribution-ShareAlike 3.0 Unported International License (CC BY-SA 3.0).

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