Institute of Education Sciences

Regional Educational Laboratory Midwest

at American Institutes for Research

Teaching Fractions Toolkit

Professional development for grade 6 teachers: Module 5 facilitator guide

Understanding ratio, rate, and proportion

REL 2025-018
U.S. DEPARTMENT OF EDUCATION

A Publication of the National Center for Education Evaluation and Regional Assistance at IES

Contents

Facilitator guide overview 1
Facilitator guide purpose 1
Facilitator guide components 1
Color-coding in the facilitator guides 2
Professional development modules summary 3
Module 5 Introduction 6
Objectives for Module 5 6
Recommendation focus for Module 5 6
Sections of Module 5 7
Preparation and resources for Module 5 7
Meeting 5A 9
Agenda and resources for Meeting 5A 9
Facilitator directions for Meeting 5A 10
Module 5 interim activities 18
Interim activities facilitator role 18
Meeting 5B 19
Agenda for Meeting 5B 19
Facilitator directions for Meeting 5B 20
Credits 28
Photos 28
Resources 28

Facilitator guide overview

Facilitator guide purpose

Each Teaching Fractions Toolkit professional development module has a facilitator guide for facilitators who are leading the toolkit's professional development for a group of grade 6 math teachers. The facilitator guides, along with the slide decks for each module, provide facilitators with the information and accompanying resources they need to prepare for and lead teacher learning experiences. With these resources, a facilitator can support teachers with applying evidence-based strategies to enhance grade 6 students' fraction understanding.

Facilitator guide components

Each facilitator guide includes information and resources for the module as a whole (the module introduction), detailed instructions for two facilitated meetings (Meeting A and Meeting B), and interim activities that participating teachers will complete individually between the meetings. The components that follow this overview are as follows.

Professional development modules summary

The professional development modules summary table summarizes the six professional development modules, including their schedule and content. Review this overview to put in context the module you are currently preparing to facilitate for teachers.

Module introduction

The module introduction includes a summary of the module content and activities, the module objectives, a review of which practice guide recommendations are the focus of the module, a review of the sequence of meetings and activities in the module, and an overview of preparation and resources needed to facilitate the module.

Information for facilitating each section of the module

Information about facilitating the module for a group of teachers is divided into three sections:

1. Preparing for and leading Meeting A.
2. Supporting teachers in their individual work on interim activities for the module.
3. Preparing for and leading Meeting B.

The sections about facilitating Meetings A and B for the module include agendas with links to resources, detailed instructions for the professional development activities, tips, and background and rationale information. The section focused on the interim activities provides an overview of self-paced professional development activities that teacher participants will complete individually between meetings and guidance about supporting teachers in that work.

Credits and references

Credits and references are provided for citations, photos, and/or resources referred to in this facilitator guide and the presentation slide decks for this professional development module.

Color-coding in the facilitator guides

In both this guide and the associated module workbook, pages have a right-side page edge color that indicates the module section:

A brown edge indicates **general introductory information** about the module (starts on page 6).

A purple edge indicates information about the facilitated **Meeting A** of the module (starts on page 9).

A green edge indicates the information about the module's **interim activities** to be completed by teachers between meetings (starts on page 18).

A blue edge indicates information about the facilitated **Meeting B** of the module (starts on page 19).

Professional development modules summary

The complete professional development sequence includes six modules, each with three sections. The authors recommended that the modules be completed in sequential order because the activities of later modules build on the activities completed during prior modules.

Module and title	Description of module content	Module sections
Module 1 Fractions on a number line	• Provides a background for the professional development, including why a focus on effective fractions instruction, key areas of emphasis, activities, tools, associated icons, and module structure. • Introduces the practice guide and Recommendation 2 and the importance of visual representations. • Explores the Recommendation 2 implementation steps, with an emphasis on number lines as a central representational tool for teaching fraction concepts, and provides opportunities to engage with associated math tasks. • Introduces the Teacher Reflection Tool and uses it to record a baseline assessment of understanding and use of Recommendations 2-4. • Incorporates additional activities: using online apps for locating fractions on a number line and comparing fractions, introducing toolkit formative assessments, analyzing student work, and completing a module reflection.	**Meeting 1A** (120 minutes) **Module 1 interim activities** (teacher self-paced; 120 minutes) **Meeting 1B** (60 minutes)
Module 2 Understanding fraction addition and subtraction	• Introduces Recommendation 3 and its implementation steps, with a focus on creating and using area models, number lines, and other visual representations to understand the addition and subtraction of fractions. • Implements a formative assessment with students to help identify and plan to address any common mistakes and misunderstandings related to adding and subtracting fractions. • Incorporates additional activities: reading from the practice guide, using an online app for fraction estimation, working on a fraction addition task, analyzing student work from a fraction addition and subtraction formative assessment, and completing a module reflection.	**Meeting 2A** (60 minutes) **Module 2 interim activities** (teacher self-paced; 120 minutes) **Meeting 2B** (60 minutes)

Module and title	Description of module content	Module sections
Module 3 Understanding fraction multiplication and division	• Explores Recommendation 3 implementation steps, with a focus on using real-world contexts with plausible numbers and the use of visual representations to improve students' understanding of procedures for multiplication and division with fractions. • Continues the focus on fraction division and Recommendation 3, with exploration of visual representations used for fraction division tasks and use of predicting and judging the reasonableness of answers for fraction division problems. • Reviews the implementation steps and potential roadblocks associated with Recommendation 3 and supports collaborative planning of a lesson that incorporates those implementation strategies to mitigate anticipated roadblocks. • Incorporates additional activities, including using an online app for dividing fractions, analyzing students' work and identifying strengths and roadblocks related to Recommendation 3, engaging with a sorting task while consolidating knowledge related to Recommendations 2 and 3, revisiting and updating the Teacher Reflection Tool, and completing a module reflection.	**Meeting 3A** (60 minutes) **Module 3 interim activities** (teacher self-paced; 120 minutes) **Meeting 3B** (60 minutes)
Module 4 Implementation of fraction computation	• Continues the focus on fraction computation and Recommendations 2 and 3, including activities focused on the relationship between fraction multiplication and fraction division. • Applies new classroom implementation strategies associated with implementation steps for Recommendations 2 and 3 and includes approaches to mitigate anticipated roadblocks. • Incorporates additional activities, including using an online app for multiplying fractions, reading about Recommendation 3 implementation steps, analyzing your students' work and identifying strengths and roadblocks related to Recommendations 2 and 3, and completing a module reflection.	**Meeting 4A** (60 minutes) **Module 4 interim activities** (teacher self-paced; 120 minutes) **Meeting 4B** (60 minutes)

Module and title	Description of module content	Module sections
Module 5 Understanding ratio, rate, and proportion	• Introduces Recommendation 4, with a focus on Implementation Steps 1 and 2, associated Roadblocks 4.1 and 4.2, and activities to support understanding of proportional relations and using visual representations to solve ratio, rate, and proportion problems. • Uses an online app to explore and use visual representations, including tape diagrams, to solve ratio and rate problems. • Incorporates additional activities, including using visual representations to explore the multiplicative relations within and between equivalent ratios; using double number lines to solve ratio, rate, and proportion problems; making connections between visual representations and the cross-multiplication strategy; and completing a module reflection.	**Meeting 5A** (60 minutes) **Module 5 interim activities** (teacher self-paced; 120 minutes) **Meeting 5B** (60 minutes)
Module 6 Implementation of ratio, rate, and proportion	• Explores Recommendation 4 further, with a particular focus on Implementation Step 3 and associated Roadblock 4.3. • Applies and supports reflection on classroom implementation based on Recommendation 4 implementation steps as well as approaches for mitigating anticipated roadblocks. • Incorporates additional activities, including using the Teacher Reflection Tool, to support reflection on understanding and use of the practice guide, and on learning completed across the professional learning modules.	**Meeting 6A** (60 minutes) **Module 6 interim activities** (teacher self-paced; 120 minutes) **Meeting 6B** (60 minutes)

Note: The information about how to carry out the recommendation is in the form of steps for implementing the recommendation, which are referred to as “implementation steps” throughout this professional development.

Module 5 Introduction

Module 5 explores Recommendation 4, with a focus on developing students' understanding of proportional relations and using visual representations to solve ratio, rate, and proportion problems. The module reviews Recommendation 4, Implementation Steps 1 and 2 and potential Roadblocks 4.1 and 4.2.

Additional activities include using an online app to explore and use visual representations, including tape diagrams, to solve ratio problems in the context of missing paint, reading about using double number lines for ratio tasks, revisiting, and updating the Teacher Reflection Tool, and completing a module reflection.

Objectives for Module 5

By the end of Module 5, teachers will be able to do the following:

- Explain the importance of developing students' conceptual understanding of strategies, including the buildup strategy and the unit ratio strategy, for solving ratio, rate, and proportion problems before using the cross-multiplication strategy.
- Use visual representations such as ratio tables, double number lines, and tape diagrams to solve ratio, rate, and proportion problems.
- Support student understanding of and ability to accurately apply the cross-multiplication strategy for solving ratio, rate, and proportion problems.
- Understand common roadblocks to solving ratio, rate, and proportion problems.

Recommendation focus for Module 5

The Teaching Fractions Toolkit professional development focuses on practice guide Recommendations 2-4 in the context of grade 6 math content. The design of the professional development also is in line with Recommendation 5 and its implementation steps, which relate to professional development programs for teachers of students learning fractions content.

Module 5 specifically focuses on building knowledge related to Recommendation 4 in the context of grade 6 standards and extends to knowledge and skills often taught in grades 7-8.

Recommendation 4: "Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems."

Sections of Module 5

This module includes an initial facilitated meeting (Meeting 5A), a set of self-paced activities for teachers to do between meetings (interim activities) that is estimated to take 2 hours, and a second facilitated meeting (Meeting 5B).

Module 5

Understanding Ratio, Rate, and Proportion focuses on Recommendation 4 with an emphasis on conceptual understanding of strategies for solving ratio, rate, and proportion problems.

Meeting 5A ➔ **Module 5 interim activities** ➔ **Meeting 5B**

Preparation and resources for Module 5

Before leading Module 5, be sure to review all resources and facilitator instructions in this guide and in the slide decks, review pages 35-41 of the practice guide, explore the module activities yourself if you have not done so before, and anticipate what questions your participants might have. Make sure that a room is reserved with a workspace for teachers to explore tasks and discuss ideas if meeting in person or ensure a videoconference setup is available if meeting online. Ensure that you will have internet access so you can access and demonstrate the online apps. Based on the size of your group, consider when you will use breakout groups to increase opportunities for engagement and discussion. Also consider how you will gauge participants' developing understanding. In larger groups you might consider using informal checks for understanding or exit tickets.

To facilitate this module, in addition to this facilitator guide, you will need the materials listed below that are also posted on the Teaching Fractions Toolkit website. It may be helpful to print copies of these to refer to when preparing for and facilitating the meetings so you do not have to view everything electronically when leading the meetings:

- **Practice guide:** The full practice guide that serves as the foundation for this professional development toolkit.
- **Slide deck:** The Meeting 5A slide deck and the Meeting 5B slide deck.
- **Participant workbook:** The Module 5 workbook, that includes all handouts used in this module. Each participant will need access to an electronic copy or a print copy of the workbook. Participants may prefer a print copy so they can write on it.
- **Additional resources:** Module activities refer to online apps, readings, videos, and other resources. Links to these resources are summarized in the following table for your quick reference also embedded in the participant workbooks and on the Teaching Fractions Toolkit website.

Type	Resource	Link and description
Online app	Comparing Paint Mixtures	www.geogebra.org/m/rdgswrwq The Mixing Paint Mixtures app supports users in representing ratios of paint in a mixture. The app shows the color and volume of paint associated with the number of cups of paint selected. It also provides three different visual representations and the ratio as fractions and percentages.
Teacher Reflection Tool		Review and update or complete the Recommendations 3 and 4 checklists in the Teacher Reflection Tool. Then complete examples 4a-4c of the Recommendation 4 notes and reflections.
Practice guide reading	Recommendation 4	Read pages 35-41 to learn about Recommendation 4.

Meeting 5A

During Meeting 5A, you will introduce participants to Recommendation 4 and support them in exploring how to carry out that recommendation's implementation steps by engaging with sample ratio, rate, and proportion problems. Participants will continue their learning through use of visual representations while solving additional ratio, rate, and proportion problems.

The agenda for Meeting 5A is as follows.

Agenda and resources for Meeting 5A

Minutes	Topic and resources
5	**Introduction** (slides 1-3)
10	**Recommendation 4 exploration** (slides 4-9)
15	**Strategies to solve ratio, rate, and proportion problems** (slides 10-15) • Module 5, Handout 1: Exploring strategies to solve ratio, rate, and proportion problems
15	**Visual representations to solve ratio, rate, and proportion problems** (slides 16-22) • Module 5, Handout 2: Exploring visual representations
5	**Teacher Reflection Tool: Recommendations 3 and 4** (slides 23-25) • Update the Recommendation 3 checklist and notes. • Complete the Recommendation 4 checklist, notes, and reflection questions
5	**Wrap-up** (slides 26-30) • Instructions for Module 5 interim activities

Facilitator directions for Meeting 5A

The directions for how to facilitate each Meeting 5A agenda topic are provided below. The left side includes the main instructions, and the right side includes background information and tips.

Introduction (5 minutes, slides 1-3)

1. **Review the agenda** (slide 2).
2. **Review the module objectives** (slide 3).
   - Module 5 introduces Recommendation 4 and supports the exploration of this recommendation. The primary focus of Module 5 is to support teachers in understanding and explaining the importance of developing students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before using cross-multiplication.
   - Multiple strategies and visual representations will be introduced, and participants will have an opportunity to apply and discuss these strategies.
   - Participants will have an opportunity to reflect on their understanding and use of Recommendations 3 and 4 in practice.

Recommendation 4 exploration (10 minutes, slides 4-9)

1. **Introduce Recommendation 4** by briefly reviewing both the recommendation itself and its three implementation steps (slides 5-7).
   - Reinforce evidence that suggests the importance of proportional reasoning. Proportional reasoning is a critical skill for students to develop in preparation for more advanced topics in mathematics.
   - Evidence also suggests that students often learn a strategy to solve a problem in one context but cannot apply the same strategy in other contexts.
   - Stress that during Module 5, participants will learn how to make connections among problems that are the same mathematically but differ in context.
2. **Highlight key vocabulary and areas of emphasis** needed when solving ratio, rate, and proportion problems (slides 8-9).

Recommendation 4: "Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems."

Recommendation 4 implementation steps:

- "Develop students' understanding of proportional relations before teaching computational procedures that are conceptually difficult to understand (e.g., cross-multiplication). Build on

- Thinking proportionally includes understanding the multiplicative relationship between two quantities.
- Contexts that require proportional reasoning include problems that involve the following:
  - Ratios (the relation between two quantities, such as the ratio of green to red buttons).
  - Rates (the relation between two quantities measured in different units, such as distance per unit of time).
  - Proportions (two equivalent ratios).
- Proportional reasoning often is needed to solve problems in everyday contexts.
- Develop reasoning strategies prior to introducing cross-multiplication.
- Use a progression of problems to build informal reasoning strategies.
- Use visual representations for teaching ratio, rate, and proportion problems.
- Return to informal strategies and visual representations after cross-multiplication to demonstrate the connection between the algorithm and the informal strategies.

students' developing strategies for solving ratio, rate, and proportion problems."

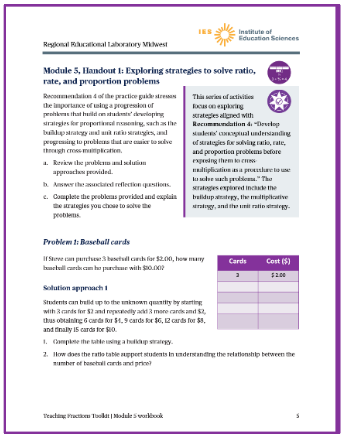
- "Encourage students to use visual representations to solve ratio, rate, and proportion problems.
- "Provide opportunities for students to use and discuss alternative strategies for solving ratio, rate, and proportion problems."

Tip: Teachers can use a progression of problems that build on students' developing strategies for proportional reasoning.

Tip: It is advisable to encourage students to apply their own strategies, discuss with students varied strategies as well as the strengths and weaknesses of those strategies, and help students understand why a problem's solution is correct. If students do not generate these strategies on their own, teachers should introduce the strategies as ways of solving ratio, rate, and proportion problems (practice guide, p. 37).

Strategies to solve ratio, rate, and proportion problems (15 minutes, slides 10-15)

1. **Refer participants to Module 5, Handout 1** in the participant workbook (slides 11-15).
   - Click once and show problem 1 (slide 11).
   - Ask participants to review problem 1 and fill in the table on the handout.
   - Ask participants how the ratio table supports students in understanding the relationship between the number of baseball cards and the price.
2. **Describe the buildup strategy** using the baseball problem to illustrate the solution approach (slide 11).
   - The buildup strategy is presented in the ratio table. Point out how students can use repeated addition to find other ratios and solve this problem.
   - Explain that problems that facilitate the use of the buildup strategy should have an integral relation between the component numbers in the two ratios—in which the numbers in one ratio can be generated by repeatedly adding numbers in the other ratio, allowing students to build up to the unknown number (practice guide, p. 37).
   - The practice guide also recommends using visual representations to solve ratio, rate, and proportion problems (Recommendation 4, Implementation Step 2). In this example, the ratio table makes the repeated addition in the buildup strategy visible.
3. **Point out the multiplicative relationship** in the ratio table (slide 12).
   - Point out how some students can use their knowledge of repeated addition being associated with multiplication to solve this problem.
   - Students may recognize that they can multiply to solve a problem. In this case, "$2 times 5 is $10, and "$3 times 5 is $15."
4. **Stress the importance of providing opportunities for students to choose and apply their own strategies** (slide 13).
   - Ask participants to review and solve Problem 2 about purchasing notebooks. Prompt them to answer how many notebooks Carlos can purchase with $24 and how much 60 notebooks will cost him.


Regional Educational Laboratory Midwest

IES Institute of Education Sciences

Module 5, Handout 1: Exploring strategies to solve ratio, rate, and proportion problems

Recommendation 4 of the practice guide stresses the importance of using a progression of problems that build on students' developing strategies for proportional reasoning, such as the buildup strategy and unit ratio strategies, and progressing to problems that are easier to solve through cross-multiplication.

a. Review the problems and solution approaches provided.

b. Answer the associated reflection questions.

c. Complete the problems provided and explain the strategies you chose to solve the problems.

This series of activities focus on exploring strategies aligned with **Recommendation 4:** "Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems." The strategies explored include the buildup strategy, the multiplicative strategy, and the unit ratio strategy.

Problem 1: Baseball cards

If Steve can purchase 3 baseball cards for $2.00, how many baseball cards can he purchase with $10.00?

Cards	Cost ($)
3	$ 2.00

Solution approach 1

Students can build up to the unknown quantity by starting with 3 cards for $2 and repeatedly add 3 more cards and $2, thus obtaining 6 cards for $4, 9 cards for $6, 12 cards for $8, and finally 15 cards for $10.

1. Complete the table using a buildup strategy.
2. How does the ratio table support students in understanding the relationship between the number of baseball cards and price?

Teaching Fractions Toolkit | Module 5 workbook 5

(Module 5, Handout 1, Workbook)

Tip: Click through the animation in the slides to show the solutions building in the ratio table.

Tip: Showcasing multiple approaches can reinforce the multiple ways for solving the problem and arrive at the same solution. Reinforce that some strategies are more efficient than others.

- Prompt the participants to describe possible ways that students might solve this problem. Potential answers could include the following:
  - Students could use a buildup strategy to fill in the ratio table (24 notebooks for $16.00, 36 notebooks for $24.00, 48 notebooks for $32.00, 60 notebooks for $40.00).
  - Students could use a multiplicative strategy to find their answers and fill in the ratio table. Because $8.00 x 3 = $24.00, students should see that they need to multiply 12 notebooks by 3 to get 36 notebooks.
  - Students could use a multiplicative strategy to find how much 60 notebooks will cost. Divide 60 notebooks by 12 to find out how many times $8.00 needs to be charged: 60 ÷ 12 = 5, 5 x $8.00 = $40.00.

5. **Present an example problem that encourages students to use a more sophisticated approach**, the multiplicative approach (slide 14).
   - Introduce problem 3, which is similar to prior examples but has larger numbers and ask participants to solve this problem.
   - Share that teachers can present similar problems, but with larger numbers, which demonstrates to students how time-consuming it can be to add up repeatedly to the unknown value. Students will see the advantage of multiplying and dividing rather than depending on repeated addition (54 ÷ 3 = 18, 18 x 2 = 36).
   - Point out that problems 1-3 share a similar structure and can be solved in similar ways.

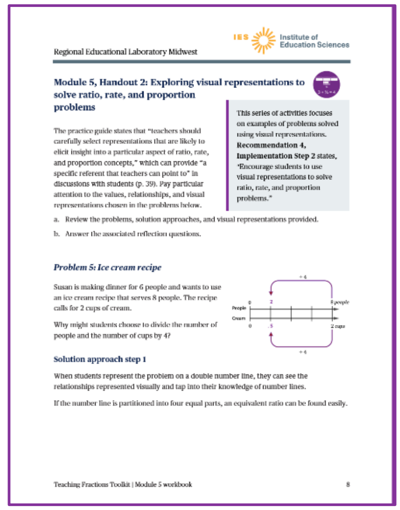
Tip: Although the intent is for students to use a multiplicative strategy, some participants may find a unit rate and multiply it by the number of students. (The unit rate is $\frac{2}{3}$ cup of flour per student. $\frac{2}{3}$ cup x 54 students = 36 cups.) This is a valid strategy even though it is not the most efficient strategy for the values in this problem. If this solution comes up, it is a good opportunity to stress that the numbers in a problem may suggest using one strategy over another.

6. **Explain that the unit ratio strategy is another approach** to solve ratio, rate, and proportion problems (slide 15).
   - Share that teachers can present problems that cannot be solved immediately through either repeated addition or multiplying or dividing a given number by a single integer.
   - The balloon problem is an example of a problem that involves ratios without an integral relation. In other words, when trying to solve $\frac{6}{24} = \frac{5}{x}$, no whole number factor can be multiplied with 5 to get 24.

- If you divide 6 by 6 to get 1 and divide $24 by 6 to get $4, you know that one balloon costs $4. If you want five balloons, multiply $4 by 5 and get $20 for the five balloons.

Visual representations to solve ratio, rate, and proportion problems (15 minutes, slides 16-22)

1. **Refer participants to Module 5, Handout 2** in the participant workbook (slides 17-22).
2. **Review key ideas related to Recommendation 4, Implementation Step 2:** "Encourage students to use visual representations to solve ratio, rate, and proportion problems" (slide 17).
   - Ratio, rate, and proportion problems can be represented and solved using several different visual representations:
     - Ratio tables (horizontal or vertical).
     - Double number lines.
     - Tape diagrams.
   - Teachers should carefully select visual representations to elicit particular insights and relationships, including multiplicative relationships within and between ratios.
   - Teachers should sometimes encourage students to create their own representations.
3. **Explain that the double number line can be used to represent and solve** ratio, rate, and proportion problems (slides 18-19).
   - Click once and show the participants the problem and ask them to think about how the problem can be shown on a double number line.
   - Point out that the problem asks "How much cream is needed to make the recipe for 6 people?
   - Share that the double number line makes the multiplicative relationships visible and assists students when setting up and solving problems.
   - Share that the double number line naturally builds on the work students have done previously with partitioning and showing relationships with fractions on a number line.
   - Ask participants why students might divide the people and cups by 4 in the problem.


Regional Educational Laboratory Midwest

IES Institute of Education Sciences

Module 5, Handout 2: Exploring visual representations to solve ratio, rate, and proportion problems

This series of activities focuses on examples of problems solved using visual representations. **Recommendation 4, Implementation Step 2** states, "Encourage students to use visual representations to solve ratio, rate, and proportion problems."

The practice guide states that "teachers should carefully select representations that are likely to elicit insight into a particular aspect of ratio, rate, and proportion concepts," which can provide "a specific referent that teachers can point to" in discussions with students (p. 39). Pay particular attention to the values, relationships, and visual representations chosen in the problems below.

a. Review the problems, solution approaches, and visual representations provided.

b. Answer the associated reflection questions.

Problem 5: Ice cream recipe

Susan is making dinner for 6 people and wants to use an ice cream recipe that serves 8 people. The recipe calls for 2 cups of cream.

Why might students choose to divide the number of people and the number of cups by 4?

Solution approach step 1

When students represent the problem on a double number line, they can see the relationships represented visually and tap into their knowledge of number lines.

If the number line is partitioned into four equal parts, an equivalent ratio can be found easily.

Teaching Fractions Toolkit | Module 5 workbook 8

(Module 5, Handout 2, Workbook)

Tip: Slides 18 and 19 are animated to show the multiplicative relationships and solution process. Pause after each step to provide time for the participants to notice the relationships on the double number line.

Tip: On the double number line, you can see multiple equivalent ratios simultaneously. In the ice cream recipe problem, you can point out how easy it is to find the unit ratio by finding the numbers halfway between zero and the first partition. Half of 2 people is 1 person, and half of 0.5 cups is 0.25 cups. It is 0.25 cups per person.

- Stress in the ice cream recipe example that the number 8 is easily divided by 4, so the number line can be partitioned into four equal parts. Then 8÷4 is 2 and 2÷4 is 0.5.
- Click to move to the next slide and share that the values of 2 and 0.5 can be multiplied by the same value, 3, to find the amount of cream for 6 people.
- Explain that students often are able to recognize the similarities between prior work with ratio tables to work with double number lines.
- The double number lines provide the added benefit of being able to easily partition to find additional equivalent ratios.
- Ask the following questions:
    - How does the double number line reinforce multiplicative reasoning?
    - How would the visual representation change if Susan is making dinner for 7 people and wants to use an ice cream recipe that serves 9 people The recipe calls for 3 cups of cream. Create a double number line to represent this problem.

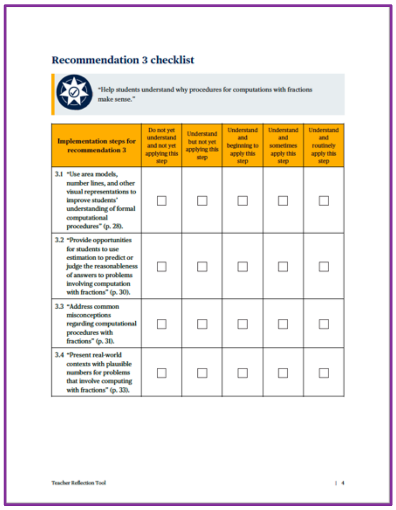
4. **Introduce the mixture problem that can be easily solved using tape diagrams** (slide 20).
    - The slide is animated.
    - Show the problem and then display the tape diagram that can be used to solve the problem.
    - Discuss how you might use the tape diagram to solve the problem and then step through the solution approach.
    - Explain that the total number of cups can be divided up and distributed equally across the diagram to maintain the 3-cup to 2-cup ratio.
    - This strategy can be applied when you are provided a ratio and a total amount of a mixture.
    - Give participants another example.
        - Sam wants to make 25 cups of fizzy juice with the same ratio of cranberry juice to bubbly water.
        - Show a tape diagram to find the amount of cranberry juice and bubbly water he will need. (Complete in Module 5, Handout 2).
        - The solution is 15 cups of cranberry juice and 10 cups of bubbly water.

Tip: If students choose to divide by the whole number 8, they will find the unit ratio as described above then they can multiply by 6 to get 1.5 cups of cream. If students choose to divide the number of people (8) by 2, they will find that 4 people will require 1 cup of cream. They will then need to determine how much cream will be needed for an additional 2 people. If 4 people require 1 cup of cream, 2 people will require .5 cups of cream. This will result in the same 1.5 cups of cream for 6 people. All of these approaches are valid. You can point out on the double number line that the ratios remain consistent (2:0.5, 4: 1, 6:1.5, 8:2). Students can check to see that these are equivalent ratios by dividing and each quotient is 4.

5. **Introduce a problem that can be solved by using two ratio tables** (slide 21).
   - Ask participants to use the ratio tables to help answer whether the two bead necklaces have equivalent ratios. Ask them to justify their answers.
   - This problem illustrates the use of two horizontal ratio tables and reinforces proportional thinking.
     - Some participants may justify their solution by stating that the ratio of 4 to 6 is equivalent to 2 to 3 and 8 to 12 is equivalent to 2 to 3.
     - Other participants may complete both ratio tables and state that they see 8 to 12 in both tables.
     - Other participants may set up a proportion and cross multiply and show $\frac{4}{6} = \frac{8}{12}$ because 4 x 12 = 6 x 8. They may state that since 48 = 48, the ratios are equivalent.
     - All these solutions are appropriate.
   - Ask participants to complete questions 2 and 3 on pages 10-11 of the participant workbook.
     - Do you need to completely fill out both ratio tables to find the solution to the question above? Why or why not?
     - How many square beads will Demetri need if he uses 40 round beads?
6. **Present Problem 8, "Which is the better deal?"** (slide 22)
   - Share that a goal is to help students recognize the advantages of one strategy over another based on the context and the numbers in the problem.
   - Different strategies can be used to solve the problem presented: ratio table, double number line, or unit price.
   - Ask the following questions of participants:
     - Which strategy would you use to solve this problem?
     - Why did you choose that strategy?
     - How did you represent the problem visually?
     - Where do you see the relationships from the problem in your visual representation?

Teacher Reflection Tool: Recommendations 3 and 4 (5 minutes, slides 23-25)

1. **Let participants know they will revisit the Teacher Reflection Tool** (pages 3-4) during the interim activities.
   - Participants will review their previously completed Recommendation 3 checklist, notes, and reflections during the interim activities and assess their growth.
   - Participants will be asked to identify and update their current levels on the scale for each implementation step and to reflect on the following questions:
     - Where have you deepened your understanding?
     - Which strategies have you applied in the classroom?
     - For which, if any, implementation steps could you use more support to understand or apply?
   - Participants will also complete the Recommendation 4 checklist and respond to the Recommendation 4 reflection questions.


Recommendation 3 checklist

"Help students understand why procedures for computations with fractions make sense."

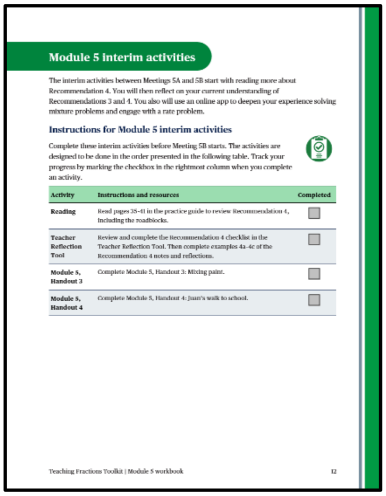
Implementation steps for recommendation 3	Do not yet understand and not yet applying this step	Understand but not yet applying this step	Understand and beginning to apply this step	Understand and sometimes apply this step	Understand and routinely apply this step
3.1 "Use area models, number lines, and other visual representations to improve students' understanding of formal computational procedures" (p. 28).	☐	☐	☐	☐	☐
3.2 "Provide opportunities for students to use estimation to predict or judge the reasonableness of answers to problems involving computation with fractions" (p. 30).	☐	☐	☐	☐	☐
3.3 "Address common misconceptions regarding computational procedures with fractions" (p. 31).	☐	☐	☐	☐	☐
3.4 "Present real-world contexts with plausible numbers for problems that involve computing with fractions" (p. 33).	☐	☐	☐	☐	☐

Teacher Reflection Tool | 4

(Module 5, Teacher Reflection Tool, Recommendation 3 checklist)

Wrap-up (5 minutes, slides 26-30)

1. **Review the list of Module 5 interim activities** that participants should complete before Meeting 5B and answer any questions from participants (slides 27-28). Explain the following:
   - For Module 5, Handout 3, participants will use an app to represent and solve mixture problems.
   - For Module 5, Handout 4, participants will explore a double number line to solve a rate problem.
2. **Verify that everyone knows the time and the location for Meeting 5B.**


Module 5 interim activities

The interim activities between Meetings 5A and 5B start with reading more about Recommendation 4. You will then reflect on your current understanding of Recommendations 3 and 4. You also will use an online app to deepen your experience solving mixture problems and engage with a rate problem.

Instructions for Module 5 interim activities

Complete these interim activities before Meeting 5B starts. The activities are designed to be done in the order presented in the following table. Track your progress by marking the checkbox in the rightmost column when you complete an activity.

Activity	Instructions and resources	Completed
Reading	Read pages 35-41 in the practice guide to review Recommendation 4, including the roadblocks.	☐
Teacher Reflection Tool	Review and complete the Recommendation 4 checklist in the Teacher Reflection Tool. Then complete examples 4a-4c of the Recommendation 4 notes and reflections.	☐
Module 5, Handout 3	Complete Module 5, Handout 3: Mixing paint.	☐
Module 5, Handout 4	Complete Module 5, Handout 4: Juan's walk to school.	☐

Teaching Fractions Toolkit | Module 5 workbook 12

(Module 5, interim activities, Workbook)

Module 5 interim activities

Between Meetings 5A and 5B, participants will complete interim activities, which should take approximately 2 hours. These activities include reading about Recommendation 4, reflecting on their current understanding of Recommendations 3 and 4, using an online app to deepen their experiences solving mixture problems, and engaging with a rate problem. Refer to the information for Meeting 5B for additional details about the interim activities.

Activity	Participants will …
Reading	Review [practice guide pages 35-41](). Pages 35-40 explain Recommendation 4 and its associated implementation steps, the first two of which were explored during Meeting 5A. Page 41 explains potential roadblocks and approaches related to Recommendation 4. Roadblocks 4.1 and 4.2 are discussed during Meeting 5B.
Teacher Reflection Tool	Review and update previous responses for the Recommendation 3 checklist in the Teacher Reflection Tool. Reflect on the following questions: • Where have you deepened your understanding? • Which strategies have you applied in the classroom? • For which, if any, implementation steps could you use more support to understand or apply? Then, complete the Recommendation 4 checklist in the Teacher Reflection Tool for the first time and respond to questions 4a-4c of the Recommendation 4 notes and reflections.
Module 5, Handout 3	Complete Module 5, Handout 3: Mixing paint. Participants will use an app to represent and solve mixture problems. Comparing Paint Mixtures online app: https://www.geogebra.org/m/rdgswrwq
Module 5, Handout 4	Complete Module 5, Handout 4: Juan's walk to school. Participants will explore a double number line to solve a rate problem.

Interim activities facilitator role

In the participant workbooks, participants have directions and all the resources they need to complete the interim activities between the two meetings. As a facilitator, you might wish to send an email midway between the two meetings to remind participants that they should be working on the interim activities and check in to see if any participants have questions or need support concerning the interim activities.

Meeting 5B

During Meeting 5B, you will have participants share and discuss their experience of working on ratio and rate problems that focus on Recommendation 4, Implementation Steps 1 and 2. Then you will facilitate an exploration of a process to build understanding of the cross-multiplication strategy by making connections between a multiplicative strategy between ratios on a double number line and the cross-multiplication strategy. Finally, you will have participants engage with proportion problems for which they use both a strategy with a visual representation and the cross-multiplication strategy. End the meeting by having participants complete a module reflection.

The agenda for Meeting 5B is as follows.

Agenda for Meeting 5B

Minutes	Topic and resources
5	**Introduction** (slides 1-2)
20	**Mixing paint and Juan's walk to school problems** (slides 3-16) • Module 5, Handout 3: Mixing paint • Module 5, Handout 4: Juan's walk to school
25	**A variety of strategies for solving ratio, rate, and proportion problems** (slides 17-24) • Module 5, Handout 5: Juan's weekend walks
10	**Wrap-up** (slides 25-30) • Module 5, Handout 6: Module 5 reflection

Facilitator directions for Meeting 5B

The directions for how to facilitate each section of the Meeting 5B agenda are in the following table.

Introduction (5 minutes, slides 1-2)

1. **Review the agenda** (slide 2).
2. **Let participants know that they will continue the focus on Recommendation 4 Implementation Steps 1 and 2, as well as Roadblocks 4.1 and 4.2.** The focus of Meeting 5B is exploring and making connections between a variety of strategies for solving ratio, rate, and proportion problems.
3. **Tell participants that Module 6 will focus on Recommendation 4, Implementation Step 3 and Roadblock 4.3**, as well as a final reflection activity for Modules 1-6.

Background: Participants engaged with ratio tables in Meeting 5A. Although there are no ratio tables in the handouts and examples in Meeting 5B, many of the strategies discussed during Meeting 5B also apply to working with ratio tables. Encourage participants to include ratio tables in their discussions, and they may choose to use a ratio table or other visual representation while working on part B of Module 5, Handout 5.

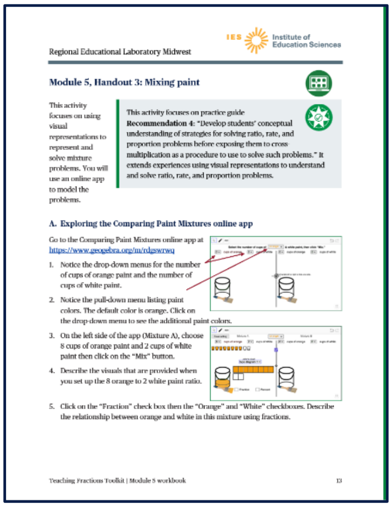
Background: Roadblocks 4.1 and 4.2 are explained on page 41 in the practice guide.

Mixing paint and Juan's walk to school problems (20 minutes, slides 3-16)

1. **Remind participants of the importance of providing opportunities to solve ratio, rate, and proportion problems using a variety of strategies**—both before and after learning the cross-multiplication strategy—and using a progression of problems that build on students' developing strategies. Teachers should explicitly discuss a variety of strategies and those strategies' strengths and weaknesses with students (slide 4).
2. **Debrief part A of Module 5, Handout 3: Mixing paint** (slide 5). Start by asking for insights or questions participants had while exploring the Comparing Paint Mixtures online app (https://www.geogebra.org/m/rdgswrwq).

   The app includes some features not highlighted in the handout that you may wish to point out to participants if they did not discover them during their explorations:

   - The diamond-shaped handle in the middle of the screen between mixtures, which is labeled "Pull



Regional Educational Laboratory Midwest

Module 5, Handout 3: Mixing paint

This activity focuses on using visual representations to represent and solve mixture problems. You will use an online app to model the problems.

This activity focuses on practice guide **Recommendation 4:** "Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems." It extends experiences using visual representations to understand and solve ratio, rate, and proportion problems.

A. Exploring the Comparing Paint Mixtures online app

Go to the Comparing Paint Mixtures online app at https://www.geogebra.org/m/rdgswrwq

1. Notice the drop-down menus for the number of cups of orange paint and the number of cups of white paint.
2. Notice the pull-down menu listing paint colors. The default color is orange. Click on the drop-down menu to see the additional paint colors.
3. On the left side of the app (Mixture A), choose 8 cups of orange paint and 2 cups of white paint then click on the "Mix" button.
4. Describe the visuals that are provided when you set up the 8 orange to 2 white paint ratio.
5. Click on the "Fraction" check box then the "Orange" and "White" checkboxes. Describe the relationship between orange and white in this mixture using fractions.

Teaching Fractions Toolkit | Module 5 workbook 13

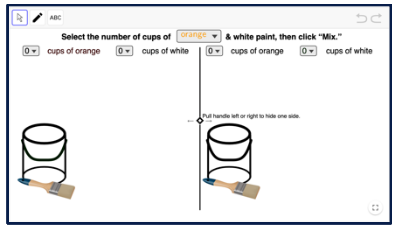
(Module 5, Handout 3, Workbook)

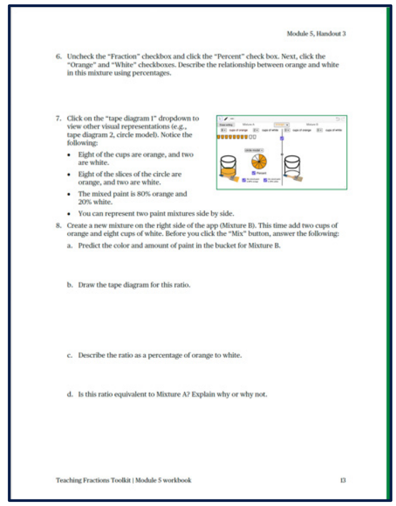
handle left or right to hide one side" before any quantities for cups of paint are selected, can be moved left or right.

- The Move, Pen, and Text tools in the upper left of the app allow writing on the app. After using the Pen or Text tool, select the Move tool to use the other features of the app.
- The upper right corner of the app has an Undo button and a Redo button.
- There is a Fullscreen button in the bottom right corner of the app.
- Clicking on a paint can outline or the paint color on a brush will hide the can and the brush. A button will appear that you can click to show the can and brush.

3. **Discuss the visuals and ratios for the orange paint mixtures in part A of Module 5, Handout 3, using the questions on slide 6** (slides 6-7). Use the Comparing Paint Mixtures online app or the image on slide 7 during the discussion. If you feel comfortable, you can go directly to the app and use the slides with app directions as reference. Share the following commonalities between the ratios and visual representations if participants do not mention them:
    - The total number of boxes or sectors, representing 10 cups total.
    - One group of boxes or sectors represents 2 cups, and a second group of boxes or sectors represents 8 cups.
    - Expressed as percentages, both ratios are 20% of one color and 80% of the other color.

    Share the following differences between the ratios and visual representations if participants do not mention them:
    - The groups of boxes are different colors and represent different colors of paint: for example, for Mixture A, the 8 orange boxes or sectors represent 8 cups, whereas for Mixture B the two white boxes or sectors represent two cups.
    - The values for the percentages are swapped: Mixture A is 80% orange, whereas Mixture B is 20% orange.
4. **Discuss the missing light blue paint problem on Module 5, Handout 3 part B** (slides 8-9). Ask participants to share how they used the tape diagram to



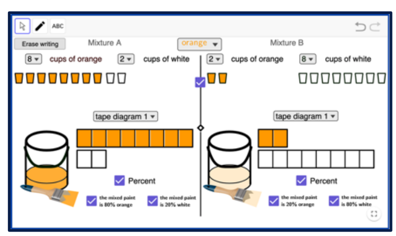

Module 5, Handout 3

6. Uncheck the "Fraction" checkbox and click the "Percent" check box. Next, click the "Orange" and "White" checkboxes. Describe the relationship between orange and white in this mixture using percentages.
7. Click on the "tape diagram 1" dropdown to view other visual representations (e.g., tape diagram 2, circle model). Notice the following:
    - Eight of the cups are orange, and two are white.
    - Eight of the slices of the circle are orange, and two are white.
    - The mixed paint is 80% orange and 20% white.
    - You can represent two paint mixtures side by side.
8. Create a new mixture on the right side of the app (Mixture B). This time add two cups of orange and eight cups of white. Before you click the "Mix" button, answer the following:
    a. Predict the color and amount of paint in the bucket for Mixture B.
    b. Draw the tape diagram for this ratio.
    c. Describe the ratio as a percentage of orange to white.
    d. Is this ratio equivalent to Mixture A? Explain why or why not.

Teaching Fractions Toolkit | Module 5 workbook 13

(Module 5, Handout 3, Workbook)

Tip: Noting differences between the visual representations and ratios for problems such as this one–that include the same values for different quantities–may help students use and attend to labels and how they set up visual representations.



Tip: The app can support understanding whether the ratios are equivalent by using the "Fraction" and/or "Percent" check boxes and comparing the corresponding values for the colors of paint in the mixtures. For example, in the image of the app find the number of cups of blue and white paint to make 30 cups of light blue paint. Slide 8 starts with the first question and the tape diagram as shown on workbook page 15, and animation on the slide shows the 6s in the tape diagram boxes and, after the next click, the bottom question. An example strategy follows:

- Because there are 5 boxes total in the tape diagram, the 30 cups of paint needed can be divided into 5 groups to determine how many cups are in each box.
- There are 6 cups in each group, and the number 6 can be written in each box of the tape diagram (click to show the animation when the 6s are added to the tape diagram).
- The sum of the tape diagram for each color is the number of cups needed for each color to make 30 cups of light blue paint: 6 cups of blue paint and 24 cups of white paint.

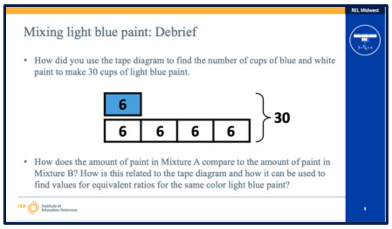
Ask participants to discuss how the amount of paint in Mixture A compares with the amount of paint in Mixture B. An example response follows:

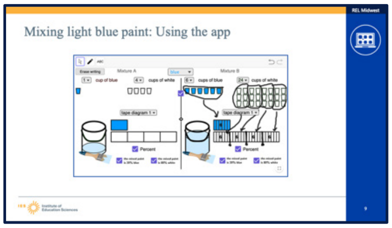
- The amount of paint in Mixture B is 6 times the amount of paint in Mixture A, which is the same as 30 cups of light blue paint divided equally into the 5 total number of boxes of the tape diagram (for both the blue paint and the white paint)–the number 6 in each box is the same as the number of times (6) the amount for Mixture B is greater than Mixture A.

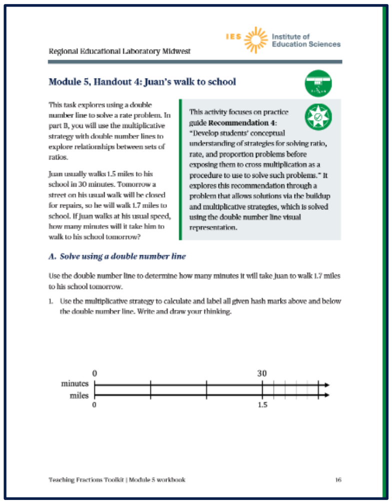
Make connections between the tape diagram visual representation on the handout and the tape diagram(s) on the app if participants do not mention them.

5. **Discuss Module 5, Handout 4: Juan's walk to school part A, asking participants to explain their work and their thinking** for using the multiplicative strategy for the problem (slides 10-12). After participants have shared their thinking, emphasize how a multiplicative strategy can be used in at least two different ways in this example:
   - **The multiplicative relationship within a ratio–between the values for one hash mark** (similar to the numerator and the denominator within a ratio written as a common fraction). This relationship is the same for all ratios whose values can be represented on the double number line. For example, in this problem, the number of minutes is always 20 times the number of miles, and the

on slide 7, the 80% orange for Mixture A is different from the 20% orange for Mixture B, so the mixtures do not represent equivalent ratios, and the mixed paint colors are different.






Regional Educational Laboratory Midwest

Institute of Education Sciences

Module 5, Handout 4: Juan's walk to school

This task explores using a double number line to solve a rate problem. In part B, you will use the multiplicative strategy with double number lines to explore relationships between sets of ratios.

Juan usually walks 1.5 miles to his school in 30 minutes. Tomorrow a street on his usual walk will be closed for repairs, so he will walk 1.7 miles to school. If Juan walks at his usual speed, how many minutes will it take him to walk to his school tomorrow?

This activity focuses on practice guide **Recommendation 4:** "Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross multiplication as a procedure to use to solve such problems." It explores this recommendation through a problem that allows solutions via the buildup and multiplicative strategies, which is solved using the double number line visual representation.

A. Solve using a double number line

Use the double number line to determine how many minutes it will take Juan to walk 1.7 miles to his school tomorrow.

1. Use the multiplicative strategy to calculate and label all given hash marks above and below the double number line. Write and draw your thinking.

Teaching Fractions Toolkit | Module 5 workbook 16

(Module 5, Handout 4, Workbook)

number of miles is always 0.05 times the number of minutes.

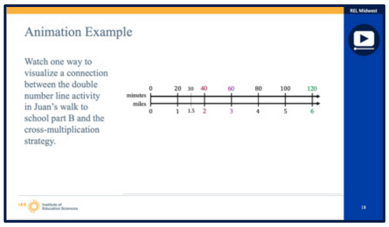
- **The multiplicative relationship between ratios**–between two values on the top number line and the corresponding two values on the bottom number line (similar to between the numerators and between the denominators when two ratios are written as a proportion). There is a common factor (or quotient) between the corresponding quantities in any two ratios, although this common factor may change if you use different ratios. For example, the factor 1.5 is the same between both 1 mile and 1.5 miles and between 20 minutes and 30 minutes; the factor of 2 is the same between 1 mile and 2 miles and between 20 minutes and 40 minutes.

6. **Discuss Module 5, Handout 4, part B, asking participants to describe what they noticed** while completing the two double number lines using the multiplicative strategy, and what they wondered while doing this activity (slides 13-16). Share some or all of the following key points if participants do not mention them:
   - The four ratios (30:1.5, 40:2, 60:3, and 80:4) are equivalent to each other.
   - The four ratios (30:1.5, 40:2, 60:3, and 80:4) are all equivalent to 120:6.
   - The double number lines emphasize the factors (the numbers in the circles) that can be multiplied by the values in each ratio (30:1.5, 40:2, 60:3, and 80:4) to create the equivalent ratio 120:6.
   - The factors (1.5, 2, 3, and 4) shown are all the values for miles in the second ratio in each example. For example, the factor 3 is used to multiply the values in the ratio 40:2 to create the ratio 120:6, and the number 3 is the same as the number of miles for the second ratio (60:3) in the example. In each case, the factor is the number of miles for the ratio in the double number line that is not being multiplied.
   - In each example, multiplying the number of miles together (3 x 2 and 4 x 1.5) and the factors together (3 x 2 and 4 x 1.5) results in 6, which is the number of miles in the ratio 120:6.

Tip: You do not need to respond to participant wonderings. If time allows, encourage participants to respond to each other's wonderings. Example wonderings are listed in the presenter notes for slide 13.

A variety of strategies for solving ratio, rate, and proportion problems (25 minutes, slides 17–24)

1. **Tell participants that you will show them one way to visualize a connection between the double number line activity in Juan's walk to school part B and the cross-multiplication strategy** (slide 18). Explain that you will show the animation once without stopping; then you will go back and show it again while pointing out important information and answering questions. Ask participants to react to the animation and ask questions. Share the following **key points in the process** if participants do not mention them:
   - The cross-multiplication strategy "is essentially a procedure to create equivalent ratios" (practice guide, p. 38), and this process shows one way to understand the cross-multiplication strategy by making connections to a multiplicative strategy shown on a double number line.
   - To find the equivalent ratio 120:6, the two ratios 40:2 and 60:3 are each multiplied by the value for miles of the other ratio; for example, the 40 and the 2 are multiplied by 3, which is the value for miles for the ratio 60:3.
   - The double number line is transformed into the ratios written as common fractions with equals signs between equivalent ratios.
   - The labels for the lines, minutes and miles, remain important when the ratios are written as equivalent fractions. Writing the ratios in the correct form means the quantities are in the same corresponding positions (numerator or denominator), similar to how a double number line is created.
   - When written as fractions, the pairs of factors (2 and 3) above and below the double number line can be rewritten as $\frac{2}{2}$ and $\frac{3}{3}$, highlighting how the ratios 40:2 and 60:3 can be changed to equivalent ratios (in this example, 120:6) by multiplying by forms of the number 1.
   - Transferring the ratio 120:6 also to the left side can emphasize how the ratios 40:2 and 60:3 are both equivalent to 120:6 and explicitly show 120:6 and 120:6 as equal and with common denominators (6). This also



Tip: If it seems helpful for participants, the facilitator may also make connections to example 5, "Why cross-multiplication works," in the practice guide on page 39.

allows a more explicit connection to the cross-multiplication algebraic algorithm shown afterward.

- The cross-multiplication strategy can be thought of as setting the resulting numerator products (120) equal to each other in an equation. The common denominators of the ratio 120:6 mean the numerators are equal. “If two equal fractions have the same denominator, then the numerators of the two equal fractions must be equal as well” (practice guide, p. 39).
- The algebraic form of the cross-multiplication strategy results in the multiplication of the numerator of the first ratio (a) with the denominator of the second ratio (d), which corresponds to the use of the multiplicative strategy with the double number line when 40 is multiplied by 3. Similarly, multiplying the numerator of the second ratio (c) with the denominator of the second ratio (b) corresponds to the use of the multiplicative strategy with the double number line when 60 is multiplied by 2.
- When all values in two ratios in proportion are known, the products of a x d and b x c are the same number, just as 120 = 120 in this example.
- If one value is not known, because the products a x d and b x c are the same number, the equation a x d = b x c can be solved to find the unknown value.

2. **Connect the discussion about the cross-multiplication strategy to the suggested approaches to Roadblock 4.1**, emphasizing that teachers may mitigate student misunderstanding by carefully presenting problems and strategies that can help students understand the logic behind the cross-multiplication strategy and write the ratios within the problem in the correct form (slide 19).
3. **Tell participants to also review example 5 on practice guide page 39** and suggest demonstrating with students why the cross-multiplication procedure works by starting with two equal fractions, such as $\frac{4}{6} = \frac{6}{9}$, and following the steps shown in example 5.

Tip: One way to check if two numeric ratios are equivalent is to use the cross-multiplication strategy: if the products of a x d and b x c are the same number, the ratios are equivalent.

Background: Roadblock 4.1: “Many students misapply the cross-multiplication strategy.” The suggested approach includes carefully presenting examples to “help students understand the logic behind the cross-multiplication procedure and why the ratios within the problem need to be in the correct form for the procedure to work” (practice guide, p. 41).

4. **Explain that participants will now work on the two problems on Module 5, Handout 5** individually and then they will discuss their thinking briefly as a group (slide 20). Participants will complete the handout in parts, first working on and discussing part A on workbook page 19, then working on and discussing part B on workbook page 20, and finally reflecting individually and then discussing part C on workbook page 21.
   - Emphasize that for both problems in parts A and B, participants will use at least two strategies: one that uses a visual representation and one that uses the cross-multiplication strategy. These strategies are partially completed on the handout for part A.
   - Point out the dashed portion of the given double number line for part A, meaning that the number lines are "open" and do not have a consistent scale with equivalent intervals along their entire length.
5. **Tell participants to work individually on part A** on workbook page 19 and tell them how much time they have (such as 2 minutes). Afterward, ask a small number of participants to share their work and discuss connections between the strategies (slide 21).
   - For the problem in part A, using the given double number line and the multiplicative strategy may be faster and more efficient for some participants than using the cross-multiplication strategy.
6. **Tell participants to work individually on part B** on workbook page 20 and tell them how much time they will have (such as 3 minutes; slide 22). Afterward, ask a small number of participants to share their work and discuss connections between the strategies (slide 23).
   - For the problem in part B, using the cross-multiplication strategy may be faster and more efficient for some participants than using a strategy with a visual representation.
7. **Tell participants to reflect individually on part C** on workbook page 21 and tell them how much time they will have (such as 2 minutes). Afterward, discuss the reflection questions (slide 24). Emphasize the following key points:
   - For some problems, such as Juan's Saturday walk (part A), a buildup or multiplicative strategy (with or without a visual representation) may be more efficient than using the cross-multiplication strategy.


Regional Educational Laboratory Midwest

IES Institute of Education Sciences

Module 5, Handout 5: Juan's weekend walks

This task explores using both visual representations and the cross-multiplication strategy to solve proportion problems.

A. Juan's Saturday walk

Juan walks with his friend Anita on Saturdays. Last Saturday, they walked 2 miles in 25 minutes. If Juan and Anita walk at the same speed next Saturday, how many minutes will it take them to walk 6 miles?

Use a double number line and the cross-multiplication strategy to solve the problem.

This activity focuses on practice guide **Recommendation 4:** "Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems." It explores this recommendation through problems that allows solutions via multiple strategies, which are solved using visual representations and the cross-multiplication strategy.

Teaching Fractions Toolkit | Module 5 workbook 19

(Module 5, Handout 5, Workbook)

Background: A multiplicative strategy can be used with an "open" double number line, which may increase the efficiency of using a double number line visual representation, as well as highlighting the multiplicative nature of proportional relationships.

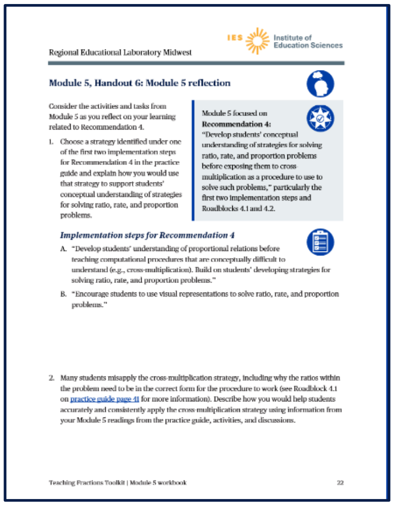
Tip: If there is time, you may ask participants to reflect on these additional questions:

- How may the Module 5, Handout 5 problems reinforce a conceptual understanding of the cross-multiplication strategy?
- How might your students engage with the problems?
- How might understanding and using a double number line for solving some ratio, rate, and proportion problems help students write the values in the ratios in the correct form—that is, in the correct corresponding positions?

- For some problems, such as Juan's Sunday walk (part B), a cross-multiplication strategy may be more efficient than other strategies.
- Using a variety of strategies, including for the same problem, and making explicit connections between them, may help reinforce the multiplicative nature of ratios and proportions and support students relying less exclusively on the cross-multiplication strategy (see Roadblock 4.2 on practice guide p. 41).

Wrap-up (10 minutes, slides 25-30)

1. **Have participants write reflections on Module 5, Handout 6** (slide 26) about a strategy identified under one of the first two implementation steps for Recommendation 4. How would they use that strategy to support students' conceptual understanding of strategies for solving ratio, rate, and proportion problems? How would they help students accurately and consistently apply the cross-multiplication strategy?
2. **Check-in with participants about any** questions and review the Module 6 schedule (slides 27-28).


Regional Educational Laboratory Midwest

IES Institute of Education Sciences

Module 5, Handout 6: Module 5 reflection

Consider the activities and tasks from Module 5 as you reflect on your learning related to Recommendation 4.

1. Choose a strategy identified under one of the first two implementation steps for Recommendation 4 in the practice guide and explain how you would use that strategy to support students' conceptual understanding of strategies for solving ratio, rate, and proportion problems.

Module 5 focused on **Recommendation 4:** "Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems," particularly the first two implementation steps and Roadblocks 4.1 and 4.2.

Implementation steps for Recommendation 4

A. "Develop students' understanding of proportional relations before teaching computational procedures that are conceptually difficult to understand (e.g., cross-multiplication). Build on students' developing strategies for solving ratio, rate, and proportion problems."

B. "Encourage students to use visual representations to solve ratio, rate, and proportion problems."

2. Many students misapply the cross-multiplication strategy, including why the ratios within the problem need to be in the correct form for the procedure to work (see Roadblock 4.1 on practice guide page 41 for more information). Describe how you would help students accurately and consistently apply the cross-multiplication strategy using information from your Module 5 readings from the practice guide, activities, and discussions.

Teaching Fractions Toolkit | Module 5 workbook 22

(Module 5, Handout 6, Workbook)

Credits

Information is provided about photos and other resources used in Module 5.

Photos

Photographic images used by the Regional Educational Laboratory Midwest at the American Institutes for Research on the indicated pages (front cover, back cover, page 1, and page 2) supplied by Getty Images in compliance with REL Midwest's annual license agreement.

Resources

The toolkit is based on the recommendations of the What Works Clearinghouse practice guide Developing Effective Fractions Instruction for Kindergarten Through 8th Grade.

The Comparing Paint Mixtures online app was adapted from the Comparing Mixtures (24 cups) resource by EDC in Maine using GeoGebra and licensed under a Creative Commons Attribution-ShareAlike 3.0 Unported International License (CC BY-SA 3.0).

IES Institute of Education Sciences

Regional Educational Laboratory Midwest