IES Institute of Education Sciences

Regional Educational Laboratory Northeast & Islands

at Education Development Center

Mathematics Intervention Toolkit: Mathematical Language Module

Participant Workbook

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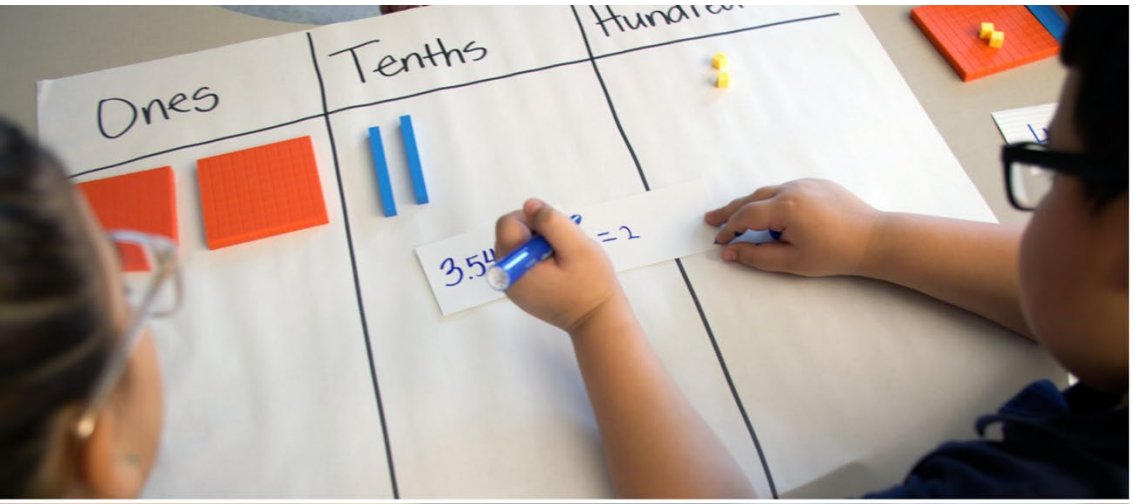
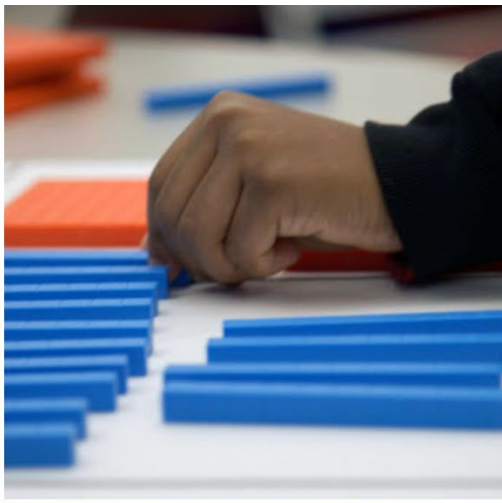
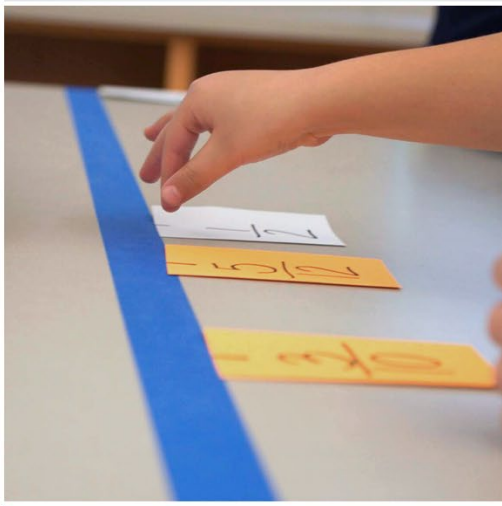
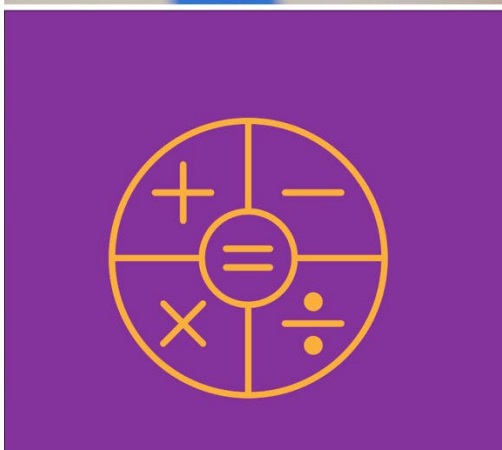








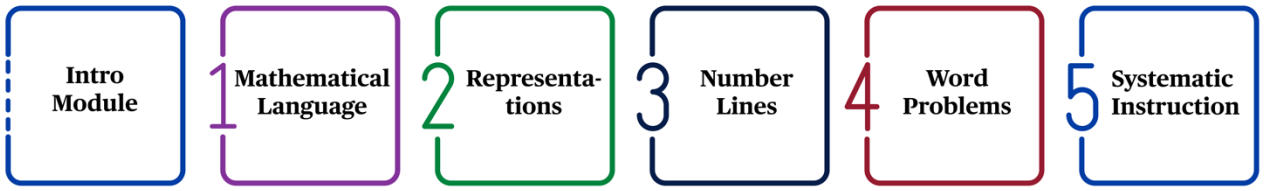
Contents

Introduction to the Course

Welcome! This professional development (PD) course is designed to build participants' knowledge and practices for supporting students struggling with mathematics. It focuses on the evidence-based recommendations of the What Works Clearinghouse Practice Guide *Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades* [1] (WWC Guide). These recommendations are based on a rigorous review and synthesis of research studies of effective intervention practices. The course is designed to connect this important research to participants' classroom practice.

The course has a series of **modules** to support in-depth professional learning. It starts with an Introductory Module and continues with five modules, each focusing on one recommendation (figure 1). The current module, Module 1, provides a deep dive into the recommendation for **mathematical language**.

Figure 1. Course Sequence



The course is specifically designed for **teachers of mathematics intervention in grades 3–6**. This includes teachers with different roles, such as interventionists, Title I teachers, math specialists, general educators, and special educators. Participants will be able to apply the strategies in a variety of intervention settings, including separate intervention classes, intervention/enrichment blocks, and designated times for intervention during core mathematics classes. Similarly, the course will support participants who use a variety of intervention programs/curricula or who do not have a program.

The full course is intended to provide about **28 hours** of professional learning during one or two school years. It uses a **hybrid format** that combines online learning, Professional Learning Community (PLC) sessions, and opportunities for classroom implementation. The course focuses on key **Number and Operations** topics, such as **fractions**, that are a high priority for mathematics intervention.

[1] Fuchs, L. S., Newman-Gonchar, R., Schumacher, R., Dougherty, B., Bucka, N., Karp, K. S., Woodward, J., Clarke, B., Jordan, N. C., Gersten, R., Jayanthi, M., Keating, B., and Morgan, S. (2021). *Assisting students struggling with mathematics: Intervention in the elementary grades* (WWC 2021006). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. Retrieved from http://whatworks.ed.gov/.

Module Overview

The Mathematical Language Module is part of a PD course on the recommendations of the What Works Clearinghouse (WWC) Practice Guide *Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades.*[2] These recommendations are based on a rigorous review and synthesis of research studies of effective intervention practices. In this module, you will learn about the recommendation and evidence-based strategies for building students' understanding and use of mathematical language.

Recommendation for Mathematical Language

Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts.

Implementation Steps

1. Routinely teach mathematical vocabulary to build students' understanding of the mathematics they are learning.
2. Use clear, concise, and correct mathematical language throughout lessons to reinforce students' understanding of important mathematical vocabulary words.
3. Support students in using mathematically precise language during their verbal and written explanations of their problem solving.

Source: WWC Guide, pp. 11–20

Professional Learning Goals

Participants will:

- Build knowledge of the WWC Guide's recommendation for teaching mathematical language and supporting student communication.
- Build knowledge of evidence-based strategies and how to implement them with students struggling with mathematics.
- Strengthen their ability to plan for and implement the strategies with students.

[2] Fuchs, L. S., Newman-Gonchar, R., Schumacher, R., Dougherty, B., Bucka, N., Karp, K. S., Woodward, J., Clarke, B., Jordan, N. C., Gersten, R., Jayanthi, M., Keating, B., & Morgan, S. (2021). *Assisting students struggling with mathematics: Intervention in the elementary grades* (WWC 2021006). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. Retrieved September 23, 2023, from http://whatworks.ed.gov/.

Key Questions

Participants will explore these questions:

1. **What** is the WWC Guide's recommendation for mathematical language?
2. **Why** is understanding mathematical language important for student learning?
3. What are **strategies** for **how to** implement the recommendation?
4. What are **ways to apply** the recommended strategies with your students?
5. What are **potential challenges** and ways to address them?

Math Content Focus

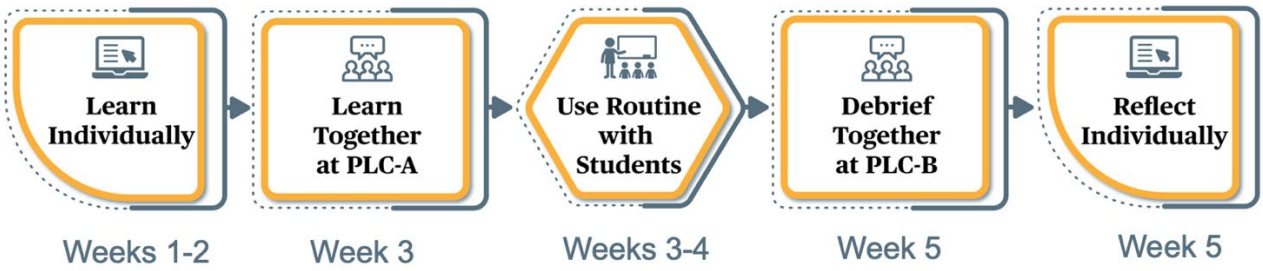
The module focuses on evidence-based strategies for building students' understanding and use of mathematical language. These strategies are applicable across mathematics content areas. The module provides examples and activities for a variety of mathematics topics, including:

- A card sorting activity to build understanding of the mathematical term, *unit fraction*.
- Graphic organizers to define and represent a variety of mathematical terms.
- Vocabulary charts with words for describing whole numbers, fractions, and decimals.
- An instructional routine to support students' use of math language for place value, whole numbers, fractions, and decimals.

Module Sequence

This 5-week module provides about 6 hours of professional learning, including an Online Session, two PLC sessions, and opportunities to apply strategies with students (figure 2).

Figure 2. Sequence of activities in the module.



The sequence begins with learning individually about the recommendation in the Online Session by doing self-paced, asynchronous activities, such as videos and readings. This session will prepare you for participating in the PLC sessions. At PLC Session-A, you and your colleagues will discuss the recommendation for mathematical language, try strategies, and prepare to use an instructional routine. After the session, you will use the routine with students one or more times. At PLC Session-B, you will share experiences with the routine and discuss ways to strengthen strategies. The module closes with the opportunity to reflect individually.

Resources

The Mathematical Language Module includes the following resources for participants:

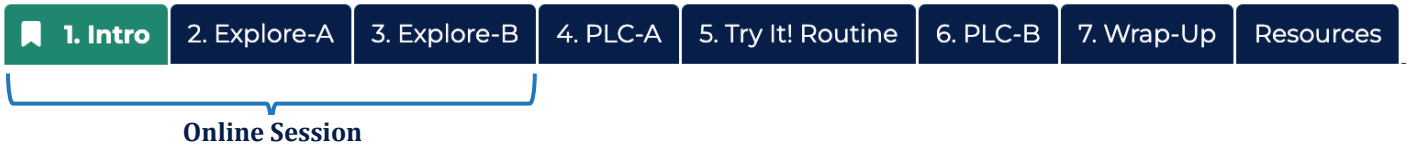
- **Online Component**: Provides activities and resources (see next section).
- **Participant Workbook** (current document): Provides all the handouts, a Routine Teaching Guide, and appendices with reproducible student handouts and other resources.
- **Classroom Video Example**: The video titled *Instructional Routine: Explaining Reasons Using Mathematical Language* shows an intervention teacher and students using the routine.
- **Debriefing Slides Template**: Participants will use this file to create several slides about their experiences teaching the routine. The same template is used for Modules 1–4.
- **Optional: Slides for Teaching the Routine**: These slides are an optional resource that participants can choose to use when they teach the routine to students.
- **Glossary**: Provides relevant mathematical terms with definitions for the course.

The **Mathematics Intervention Toolkit,** which includes all the course resources, is available for free at https://ies.ed.gov/ncee/rel/math-support-grades-3-6.

Online Component

This component provides resources for all the sessions in the module. It is organized by tabs (figure 3), which are numbered to show the sequence.

Figure 3: Tab Menu of the Online Component for All Sessions in the Module



Description of tabs

- **Tabs 1–3, Online Session**: Provide self-paced, asynchronous activities for participants to do independently. The Intro tab provides an overview of the module's goals and key questions. On the Explore-A and Explore-B tabs, participants learn about the recommendation.
- **Tab 4. PLC Session-A**: Provides resources to use during or after the facilitated session.
- **Tab 5. Try It! Routine**: Provides information and resources to help participants prepare for and use the routine with students.
- **Tab 6. PLC Session-B**: Provides resources to use during or after the facilitated session.
- **Tab 7. Wrap-Up**: Provides short reflection activities.
- **Resources:** Provides a hyperlinked list of module resources and additional resources.

Course Checklist

Use this checklist to keep track of your progress in the course.

	Introductory Module	Dates
☐	Kick-Off Session	
☐	Wrap-Up (Complete tab 4 of Online Component)	
	Module 1. Mathematical Language	
☐	Online Session (Complete tabs 1–3 of Online Component)	
☐	PLC Session-A	
☐	Try It!: Use Routine with Students	
☐	PLC Session-B	
☐	Wrap-Up (Complete tab 7 of Online Component)	
	Module 2. Representations	
☐	Online Session (Complete tabs 1–3 of Online Component)	
☐	PLC Session-A	
☐	Try It!: Use Routine with Students	
☐	PLC Session-B	
☐	Wrap-Up (Complete tab 7 of Online Component)	
	Module 3. Number Lines	
☐	Online Session (Complete tabs 1–3 of Online Component)	
☐	PLC Session-A	
☐	Try It!: Use Routine with Students	
☐	PLC Session-B	
☐	Wrap-Up (Complete tab 7 of Online Component)	
	Module 4. Word Problems	
☐	Online Session (Complete tabs 1–3 of Online Component)	
☐	PLC Session-A	
☐	Try It!: Implement Routine with Students	
☐	PLC Session-B	
☐	Wrap-Up (Complete tab 7 of Online Component)	
	Module 5. Systematic Instruction	
☐	PLC Session	
☐	Course Wrap-Up (Complete tab 3 of Online Component)	

Website URL: https://ies.ed.gov/ncee/rel/math-support-grades-3-6.

Module Checklist: Mathematical Language

Use this checklist to keep track of the module dates and your progress on the tasks.

Module Dates: _____ to _____

- **Online Session:** Complete during this time span: _____ to _____
- **PLC Session-A:** Attend session on date: _____ time: _____
- **Try It! Routine:** Implement during this time span: _____ to _____
- **PLC Session-B:** Attend session on date: _____ time: _____
- **Wrap-Up:** Complete by date: _____

Tasks

1–3. Online Session.

Complete the activities *before* PLC Session-A.

- ☐ Tab 1, Introduction. Read about the module's goals, key questions, and sequence.
- ☐ Tab 2, Explore-A tab. Do online activities to learn about the recommendation.
- ☐ Tab 3, Explore-B tab. Do more online activities to continue learning.

4. PLC Session-A.

- ☐ Participate in the PLC session: Discuss the recommendation, try strategies, and prepare to use an instructional routine.

5. Try It! Routine.

Implement the routine *before* PLC Session-B.

- ☐ Use the routine one or more times with students.
- ☐ Prepare slides for sharing experiences. Use the Debriefing Slides Template.

6. PLC Session-B.

- ☐ Participate in the session: Share experiences using the routine by showing slides and focusing on the debriefing questions. Discuss ways to strengthen strategies.

7. Wrap-Up.

Complete the activities *after* PLC Session-B.

- ☐ Do the closing activities on tab 7, Wrap-Up, to reflect on your learning in the module.

Website URL: https://ies.ed.gov/ncee/rel/math-support-grades-3-6.

Handouts

H1. Reference Sheet: Mathematical Language

Directions: During the module, record notes below to create a helpful resource for future use.

1. What is the WWC Guide's recommendation?[3]

Recommendation for Mathematical Language

Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts.

2. Why is understanding mathematical language important for student learning?

Add your ideas to the list.

Understanding and using mathematical language helps:

- Students build understanding of mathematical concepts and processes.
- Teachers and students communicate more clearly with each other.
- Students access language used in instruction during intervention and core mathematics class.
- Prepare students for the language they will be using in future mathematics classes.

3. How do you carry out the recommendation? What are the implementation steps?

The WWC Guide provides three main implementation steps (below) that involve the use of evidence-based strategies (see next page).

Main Implementation Steps

1. Routinely teach mathematical vocabulary to build students' understanding of the mathematics they are learning.
2. Use clear, concise, and correct mathematical language throughout lessons to reinforce students' understanding of important mathematical vocabulary words.
3. Support students in using mathematically precise language during their verbal and written explanations of their problem solving.

[3] The mathematical language recommendation and implementation steps are from the WWC Guide, pages 11–20.

4. What are strategies for how to implement the recommendation with students struggling with mathematics?

As you go through the module, use this chart to list strategies you want to remember and apply.

Recommended Strategies for Mathematical Language
Teach mathematical vocabulary. • Build understanding of new vocabulary by connecting terms to concrete representations, such as manipulatives, and semi-concrete representations, such as drawings.
Use clear mathematical language in instruction. • Consistently use correct mathematical language in your daily teaching to help students build familiarity and understanding of mathematical terms.
Support student communication. • Use sentence starters and frames to provide students with a starting point and structure for expressing their ideas.

H2. How to Carry Out the Recommendation

Read this excerpt from the WWC Guide[4] to learn about implementing the recommendation.

Recommendation for Mathematical Language

Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts.

The WWC Guide describes three main implementation steps; each involves the use of evidence-based strategies to support students struggling with mathematics.

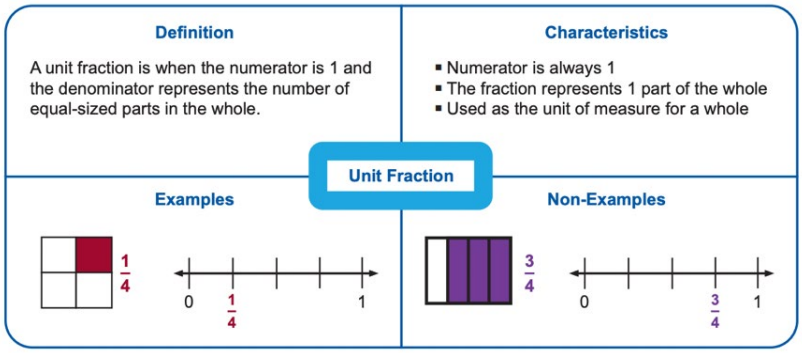
Implementation Step 1. Teach Mathematical Vocabulary

Routinely teach mathematical vocabulary to build students' understanding of the mathematics they are learning.

Introduce new mathematical vocabulary during instruction to provide context and meaning to the words. Use student-friendly definitions with simple and familiar mathematical terms. Link new vocabulary to a variety of examples when possible, including concrete or semi-concrete representations. Here are descriptions and examples of recommended strategies:

- **Use graphic organizers.** In Example 1, a graphic organizer pairs a new mathematical term, *unit fraction*, with its student-friendly definition, characteristics, examples, and non-examples. Use this type of graphic organizer with any mathematics terminology to visually and symbolically depict the meaning.

Example 1. Graphic organizer to depict the meaning of a mathematical term



[4] Excerpt from WWC Guide, pages 11-20, with adaptations by the Toolkit authors. The WWC Guide has three additional examples for the lower elementary grades.

- **Use concrete and semi-concrete representations, hand gestures, and role playing.** Simply providing a definition of a term is not sufficient for developing students' understanding of mathematical vocabulary and concepts. Deepen students' understanding of the terms by connecting them to concrete and semi-concrete representations. Hand gestures and role playing can also provide context and meaning to mathematical vocabulary. The context provided by representations, hand gestures, and role playing will help students better understand what they are learning. Example 2 demonstrates a role-playing activity to help students learn and understand the meaning of *divide*.

Example 2. Role-playing with hand gestures that teach the meaning of mathematical ideas or vocabulary.

Teacher: Look at this problem: 12 ÷ 3 = ___.

This problem asks us to **divide** 12 into three equal groups. To help us understand what it means to **divide** an amount of something into equal groups, let's pretend we are dividing a group of 12 apples among three families. We want to find out how many apples each family will receive.

If we pretend that each paper plate is a basket for each family and each counter is an apple, how many plates will we need? How many counters do we need?

Student: There are three families, so we need three plates to show how many shares we need. There are 12 apples, so we need 12 counters to give out. (*The teacher asks guiding questions if students need help.*)

Teacher: Now I'd like you to divide the counters among the three families.

> ***Students role play*** *the situation of dividing up the apples by putting counters on the plates to give each family an equal share.*

Teacher: How many apples does each family get? How did you find out? Did one family get more than another?

Student: There were four counters on each plate. I put one counter on each plate until I ran out of counters. So, each family gets four apples. I checked to make sure that each family would get the same number of apples so that it was fair.

Teacher: That's correct. When we **divided** 12 apples among three families, we gave each family four apples.

> *The teacher shows the action of divvying up the counters onto the plates. The teacher's* ***gestures*** *look like a dealer dealing out cards or more generally a hand motion that gives one item per plate until the apples are all distributed equally.*

Teacher: Let's write this as an equation: 12 ÷ 3 = 4.

Teacher: Let's act out a new problem. We need to **divide** 20 apples equally among three families. How many apples will each family get? Use counters and plates to solve this problem.

Create a shared list of mathematical terms to use across grades. To support learning across grades and settings, schools should consider creating a shared list of mathematical terminology that strategically

becomes more sophisticated with each grade. A shared list can ensure that teachers and students use consistent vocabulary and language across intervention and core classrooms, further supporting the learning of students who receive instruction in both settings. Example 3 shows several precise mathematical terms that teachers may use across grades and settings in the school.

Example 3. Shared word list for all teachers (grades K–6) in the school

Informal Terms *Rather than using this term…*	Correct Mathematical Terms *Consider using this term…*
Reduce	Simplify
Borrowing or Carrying	Regrouping
Flat Shape or Fat Shape	Two-Dimensional or Three-Dimensional Shapes
Bigger, Smaller	Greater Than, Less Than
Flip-Flop Property	Commutative Property

Note: This list is not comprehensive. It contains only a sample of words that might appear on a more comprehensive shared list used in a school.

Implementation Step 2. Use Clear Mathematical Language in Instruction

Use clear, concise, and correct mathematical language throughout lessons to reinforce students' understanding of important mathematical vocabulary words.

Use and emphasize clear, concise, and correct mathematical language throughout instruction: when referring to a new or previously learned topic, when discussing homework, and when responding to questions. It may take several lessons for students to understand new mathematical vocabulary and develop a deep understanding of the mathematics connected to the words. Consistent use of mathematical language helps students learn how the terms should be used and develop a deeper understanding of the terms.

- **Use correct terminology to prevent confusion.** When teachers use conversational or informal language instead of mathematical language, students may get confused. For example, some teachers may refer to the commutative property (a + b = b + a) as the *flip flop property*. Although this creative name for the property may be viewed as a fun memory device, replacing accurate terms with informal language such as this can cause serious confusion later in the students' schooling when other teachers do not use the *flip flop property* or when learners don't know the connection to the correct formal term *commutative property*. Using and practicing correct terminology from the start can eliminate later challenges.
- **Model precise mathematical language** when explaining your thought process and demonstrating how to solve a problem. In Example 4, the teacher uses the term *ratio* while reasoning out loud for solving a word problem. The teacher's ongoing and repetitive use of the term helps reinforce the

meaning of a ratio and how the ratio in the problem provides critical information for solving the problem.

Example 4. Teacher uses mathematical vocabulary (ratio) when thinking aloud[5]

First, the teacher reads the problem out loud. During the think-aloud below, the teacher points to the work while talking so that students attend to the correct parts.

> **Vocabulary: ratio**
>
> **Problem:** Kesha likes to exercise. For every 8 minutes that she uses the exercise bike, she does push-ups for 2 minutes. If she exercises for 30 minutes, for how many minutes does she do push-ups?

Teacher: I start by asking myself: What is the problem asking me to figure out? I see that the problem wants me to figure out how many minutes Kesha spends doing push-ups in 30 minutes of exercise. I use the **ratio** for minutes spent using the bike and minutes spent doing push-ups. I know that together those amounts will add to thirty minutes.

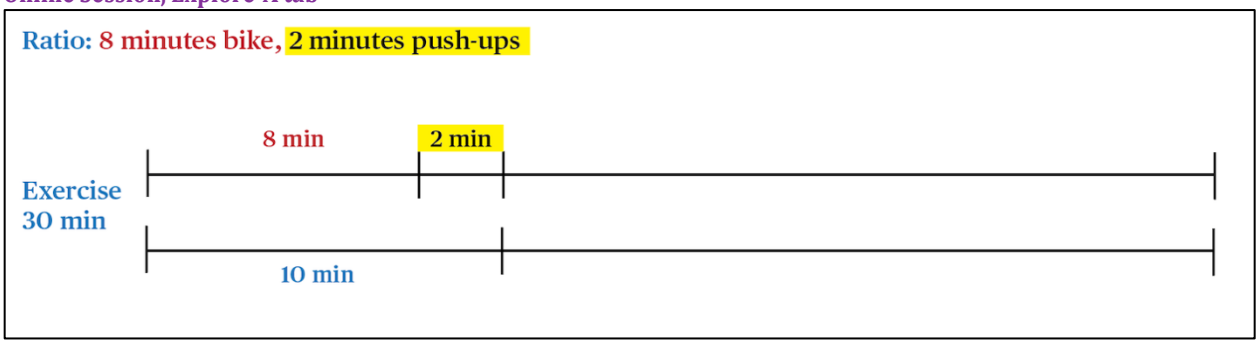
Remember, a **ratio** is a statement of how two numbers compare or relate to one another. The problem compares two numbers when it gives a **ratio** for minutes spent using the bike and minutes spent doing push-ups. I am going to write down the **ratio** for minutes spent exercising on the bike and minutes spent doing push-ups here.

> **Exercise time:** 30 minutes
> **Ratio:** 8 minutes to bike
> 2 minutes to do push-ups

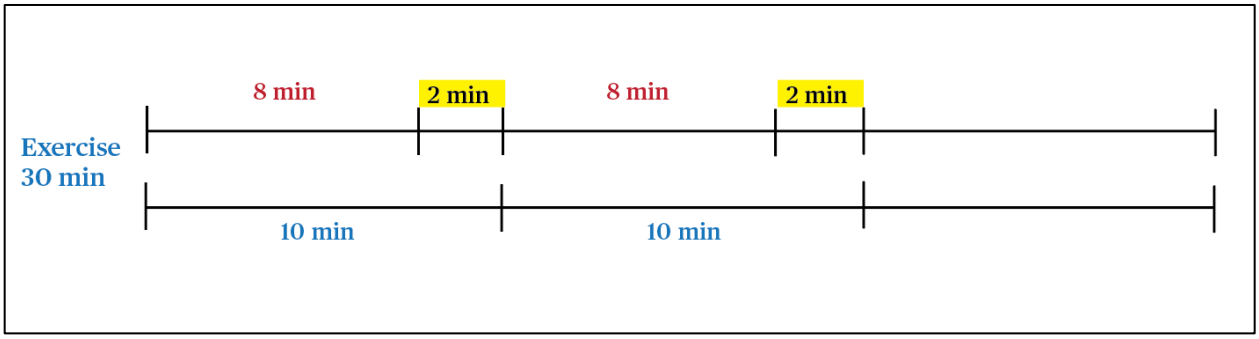
I create a strip diagram that will include all 30 minutes of exercise.

Now I use the **ratio**. Because I am using a **ratio**, every time Kesha spends 8 minutes exercising on the bike, she spends 2 minutes doing push-ups. I write the **ratio** once on the strip to see how many minutes she spent doing both activities. The first time she is on the bike for 8 minutes, she then does 2 minutes of push-ups. Together, that equals 10 minutes of exercise.

[5] Graphics for this example were adapted from WWC Guide, pages 15-16, by the Toolkit authors.

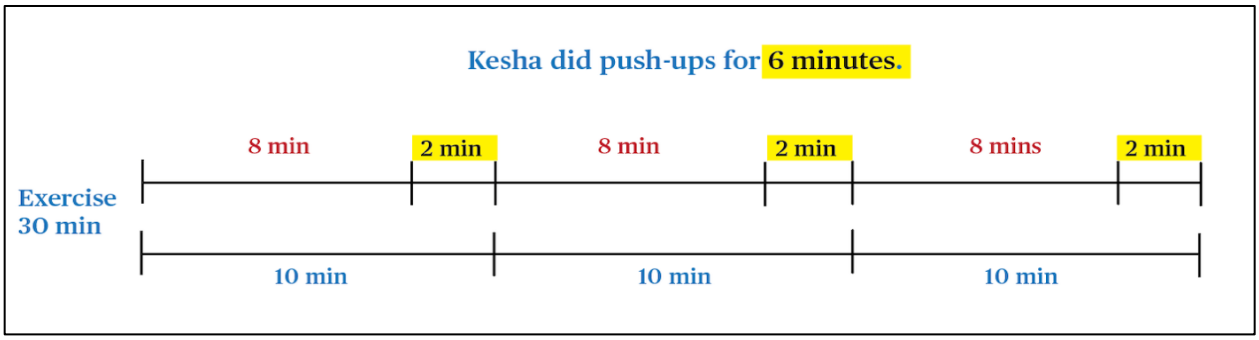


Now I keep using the same **ratio** until I get to 30 minutes of exercise. I add another 8 minutes of biking and 2 minutes of push-ups. I check: Have I reached 30 minutes yet? I have not.



I use the **ratio** again, and Kesha exercises for 10 more minutes. Now I have 10 + 10 + 10. That is 30 minutes. Now I can use my recordings on the strip diagram to figure out the answer to the question: How many minutes did Kesha spend doing push-ups during 30 minutes of exercise?

I can skip-count by 2s: 2, 4, 6. Now I write the answer: Kesha did push-ups for **6 minutes.**



Some words may have more than one meaning or are used in more than one context in mathematics. Use examples to provide instruction on the various ways words are used. In Example 5**,** the teacher leads an activity to expand students' understanding of the terms *factor* and *product*. The students have already learned that factors are the two numbers multiplied to get a product in a multiplication equation. In this lesson, students learn that a product can have multiple factors, not just two factors. Understanding that a number can have multiple factors helps students in being able to find a common denominator for adding or subtracting fractions. This new understanding of *factor* also prepares students for finding the greatest common factor between two numbers.

Example 5. Teacher leads an instructional activity to broaden students' understanding of the terms *factor* and *product*

The lesson below focuses on the mathematical terms ***factor*** and ***product***. The teacher also uses other correct mathematical language like *multiplication* and *equal.*

Teacher: Some multiplication problems ask us to multiply two numbers to find the **product**. The two numbers we multiply are called **factors**. Today, I am going to give you a product and you need to find the missing **factor**.

First, let's try finding a missing **factor** with 24 as the **product**. I will give you the first factor. I know 24 is an even number, so I know that 2 can be multiplied by another number to get 24. That means 2 is a **factor** of 24. Does anyone know what number times 2 would equal 24?

*The teacher gives students time to respond and arrive at 12 as another **factor** of 24. If students are struggling, the teacher may have students use 24 counters and group them into equal groups of 2 to show that 2 times 12 equals 24. Teacher writes on the board as each **factor** is given by students.*

Product: 24 Factors: 2×12

Teacher: Now I want you to work with a partner to find other **factor** pairs that can be multiplied to get a **product** of 24. *Allow students time to work with partners*.

Teacher: I'm going to ask each set of partners to give me **two factors** that when multiplied equal the **product** of 24. We will continue until we have listed all the possible **factors**.

*Students share their responses. As students respond, the teacher records the **factors** on the whiteboard with 24 at the top, labeled **product**. If students need to use the counters in arrays to find factors for 24, give them access to this support. Showing the different arrays on a document camera or in a small group can help students see the patterns of the **factors**. If using counters to create these arrays, emphasize that there are always 24 counters, and counters are not being added or taken away when constructing the different arrays for the **product** 24. Written on the whiteboard is:*

Product: 24 Factors: 2×12 6×4 8×3 12×2 4×6 24×1 3×8 1×24

Teacher: Remember, the two numbers that are multiplied to get a **product** are called ***factors***. Let's list each factor for 24 one at a time, in numerical order, so we know which numbers are **factors** of 24.

The factors for 24 are: 1, 2, 3, 4, 6, 8, 12, and 24.

Support students in using mathematically precise language during their verbal and written explanations of their problem solving.

Have students provide verbal and written explanations of mathematics concepts during intervention to help them develop key mathematical insights. Explaining their work provides students with opportunities to communicate their mathematical understanding using newly learned vocabulary and allows teachers to check for understanding and provide feedback.

- **Use sentence starters and guiding questions.** Students will likely need support to explain their thinking using mathematical language. Offer students a framework for providing explanations, such as a set of guiding questions or sentence starters (Example 6). It is also helpful for teachers to restate the students' explanations using correct language if students do not.

Example 6. Sentence starters and sentence frames[6]

Sentence starters give students a starting point for expressing their ideas. Teachers can provide a consistent set of sentence starters for students to use in discussions and activities. *Examples:*

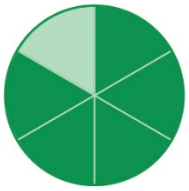
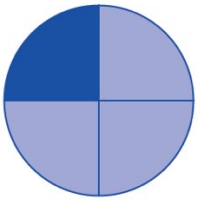
- **The fractions are equivalent because…**
- **My drawing of the fraction shows…**
- **My answer makes sense because…**
- **I agree/disagree because…**

Sentence frames have one or more blanks for students to complete in their responses. *Examples:*

- **I know that ___ is a unit fraction because…**
- **The fraction ___ is _____ $\frac{1}{2}$ because…**
  [greater than, less than]
- **Use vocabulary charts and classroom word walls.** Remind students to include the mathematical language modeled and taught during instruction by displaying mathematical vocabulary on the classroom wall. It's helpful to include definitions, examples, and visual representations for each term, as shown in Example 7. These kinds of supports can be useful for both verbal and written explanations.

[6] Example 6 was added by the Toolkit authors.

Example 7. A mathematical language chart that supports upper elementary school students as they use mathematical language to present their thinking

Term	Definition	Example/Representation
Numerator	The number of equal-sized parts being considered or used. It is the number of times the unit fraction is repeated. In this example, 5 is the numerator.	$\frac{5}{6}$ 
Denominator	The number of equal-sized parts that make up the whole. In this example, 4 is the denominator.	$\frac{1}{4}$ 
Unit Fraction[a]	Used as the unit of measure for a whole (e.g., is copied 8 times to create a whole, or 1). A unit fraction is a fraction with 1 in the numerator.	$\frac{1}{8}, \frac{1}{5}, \frac{1}{4}, \frac{1}{2}$ $\frac{1}{8}$ is shown by the shaded part. 

a. Picture of one-eighth added by Toolkit authors.

H3. Video Observations

Background Information: The video, *Instructional Routine: Explaining Reasons Using Mathematical Language*, shows an intervention teacher using the routine with grade 4 students.

Directions: In the module, you will watch the video two times:

- In the Online Session, watch individually to take a close look at strategies in action. Write notes on this handout below.
- At PLC Session-A, you and your colleagues will rewatch the video so that it's fresh in your minds for the discussion and for preparing to use the routine.

Video Watching Norms

- Observe, without judging, the teacher and students.
- Look for ideas to apply in your practice.

Focus Question

As you watch, focus on this question and write notes:

1. What **strategies** for mathematical language do you notice in the video?

Puzzle Used in Video

A 303	B 660
C 1,044	D 389

Ideas to Share at PLC Session-A

2. What are **one or two ideas** from the video that stood out for you?

H4. Vocabulary Strategy: Use Graphic Organizers

Graphic organizers are a helpful strategy for reinforcing the meaning of vocabulary terms. Include a student-friendly definition, characteristics, examples, and non-examples.

1. Complete two graphic organizers.

a. Add more characteristics, examples, and non-examples for the term ***polygon***.

Polygon

Definition

A polygon is a 2-dimensional (2D) shape that is closed and has only straight sides.

Characteristics

- Sides are straight.

Examples


Non-Examples


b. Complete this graphic organizer for the term ***factor*.**

Factor

Definition

Two numbers that are multiplied to get a product are called **factors**.

Characteristics

Examples

2 x 6 = 12

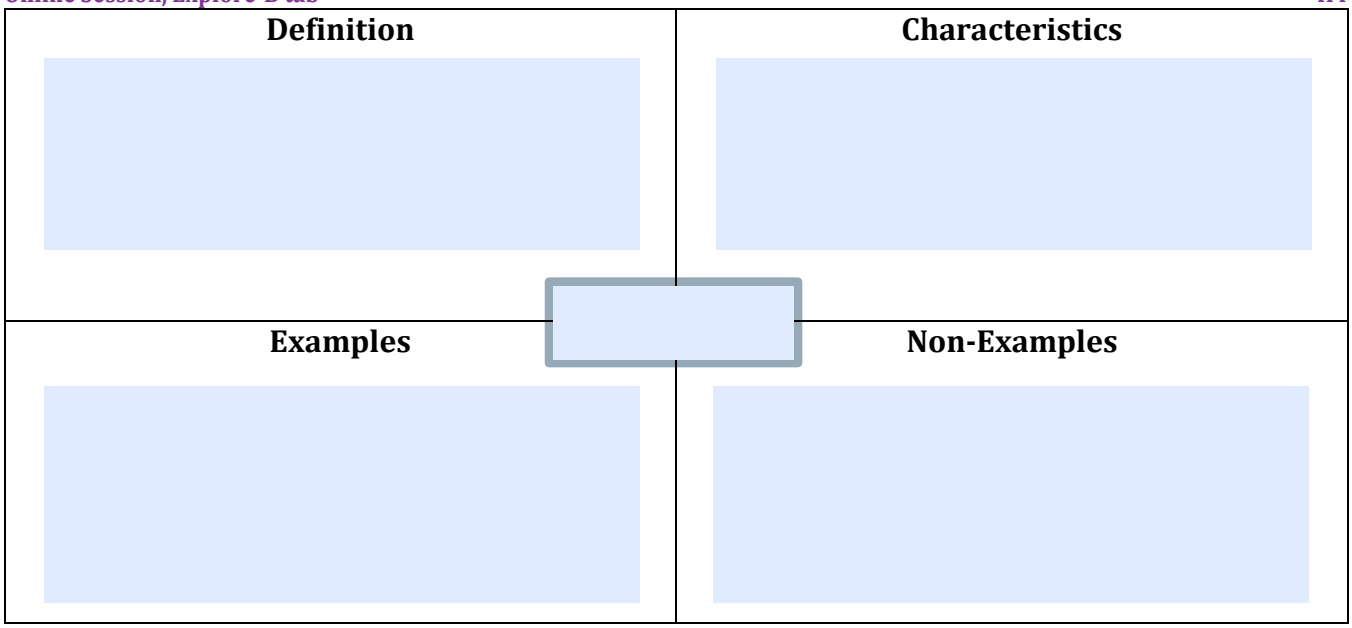
2 and 6 are factors **of 12.**

- The factors of 12 are 1, 2, 3, 4, 6, and 12.
- The factors of 20 are 1, 2, 4, 5, 10, and 20.

Non-examples

2. Create your own graphic organizer by using this template.

Focus on a mathematical term that your students find difficult.



3. Write down ideas for using these graphic organizers with students.

a. How might you use these graphic organizers with your students?

b. How might you integrate a graphic organizer into a math lesson?

c. What mathematical terms would you choose?

d. What questions would you ask students if they do not know what to put in a category?

H5. Challenges and Suggestions

Directions: Read about two potential challenges and the expert panel's advice.[7] Then, choose **one challenge** (A or B) to focus on and add your responses to the questions below.

Challenge A

"I don't know what words I'm supposed to use. Everyone seems to use different terminology."

Panel's Advice

Review your state's mathematics standards to identify the important language for students to learn. Also consider state assessment guidelines and the curriculum materials used in the school. Consult with your colleagues to draft a list of accurate and precise vocabulary that the school can agree to use in mathematics classes across grade levels and settings. This could be a shared list of mathematical language on which teachers across the school agree.

Challenge B

"Teaching vocabulary takes time that we don't have."

Panel's Advice

Integrate language instruction throughout mathematics. Introduce and use mathematical words intentionally and throughout lessons, to reinforce their meaning. Taking this approach does not require adding an activity that might take up additional instructional time. There does not need to be an entire mathematics intervention lesson focused on vocabulary.

Add Your Ideas for Challenge ___A ___B

1. In what ways does this challenge resonate with your experiences?

2. How might you apply one or more of the panel's suggestions?

3. What are ways to collaborate with colleagues to help address this challenge?

[7] Excerpt from WWC Guide, page 39; questions from the Toolkit authors.

H6. Reflection for Online Session

Directions: Reflect on your learning by writing responses to these questions. Bring your completed handout to PLC Session-A so you can refer to the ideas you recorded.

1. What are **two important ideas** that you learned about the recommendation for mathematical language?

2. What is **one instructional strategy** (from this session) that you want to try with your students or that you currently use? How is this strategy helpful for students?

3. What is **one idea or strategy** that you want to discuss with colleagues at PLC Session-A?

H7. Card Sorting Routine: Unit Fractions

Step 1. Sort Cards and Explain Reasons.

a. **Take turns.** Take a card and look at the fraction. The shaded part shows the fraction. Decide which category to place it in:

Unit Fractions **Non-Unit Fractions**

b. When you place a card in a category, explain your reasons by using **one** sentence frame:

The fraction ___ is a unit fraction because…

The fraction ___ is a non-unit fraction because…

c. Then your partner will respond by saying:

I agree because… **I disagree because…**

Step 2. Discuss and Generalize.

a. Look at a chart with the correct cards in each category. Check your work and ask questions.

b. Discuss the **unit fractions category**:

- What do the examples of unit fractions have in common?
- What are the characteristics of unit fractions? Make a class list.

c. Discuss the **non-unit fractions category**:

- What do the non-unit fractions have in common?
- What are the characteristics of non-unit fractions?

Step 3. Wrap up and Reflect.

a. Ask students to complete the exit task (shown below).

Exit Task

Write a fraction in each blank. Explain your reasons by completing these sentence frames:

1. **The fraction ___ is a unit fraction because…**
2. **The fraction ___ is a non-unit fraction because…**

b. Then, have students share in pairs or with the whole group.

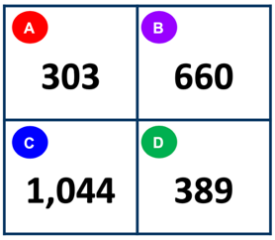
H8. Video Discussion

Directions: We will rewatch the video, *Instructional Routine: Explaining Reasons Using Mathematical Language*, together so that it is fresh in our minds for the discussion and for preparing to use the routine ourselves. This handout has new focus questions.

Video Watching Norms

- Observe, without judging, the teacher and students.
- Look for ideas to apply in your practice.

Puzzle Used in Video



Focus Questions

As you watch, focus on these **new** questions and write notes:

1. How does the teacher engage and support students in talking about their mathematical ideas and using mathematical vocabulary?

2. What do you notice about the students' explanations?

Ideas to Apply

3. What are one or two ideas from the video that you would like to try with students?

H9. Walk-Through of Routine: Script

Instructional Routine: Explaining Reasons Using Mathematical Language

Steps:

1. Introduce learning goals.
2. Choose and reason.
3. Share ideas using sentence frames.
4. Connect to mathematical vocabulary.
5. Apply vocabulary to explain reasons.
6. Wrap up.

Purpose: To get familiar with the routine by going through some of the steps in the roles of teachers and students. We will use a script to get a sense of what teachers and students might say and do. The script is designed for this PD activity and is **not** intended for teaching students. After we finish the walk-through, we will discuss ways to use the routine with students.

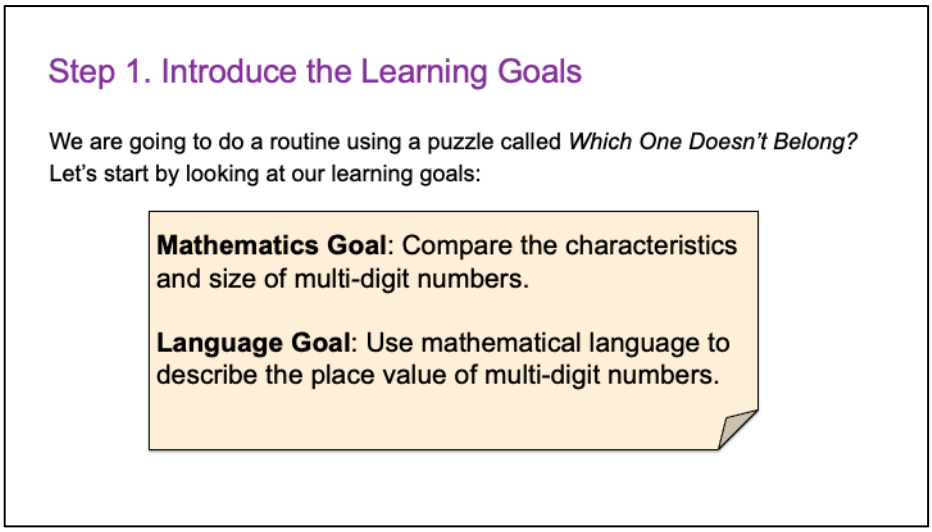
The script is set up as follows:

- The script is labeled with the roles of "Teacher" and "Student" to provide examples of what the teacher or students might **say**. You can read the text aloud or say it in your own words.
- *Text in italics* describes actions that the teacher and students will **do**.
- [Brackets] provide information for doing the walk-through.
- The script includes **slide images** to orient you to what is happening in each step. These slides were created for the walk-through.

Roles:

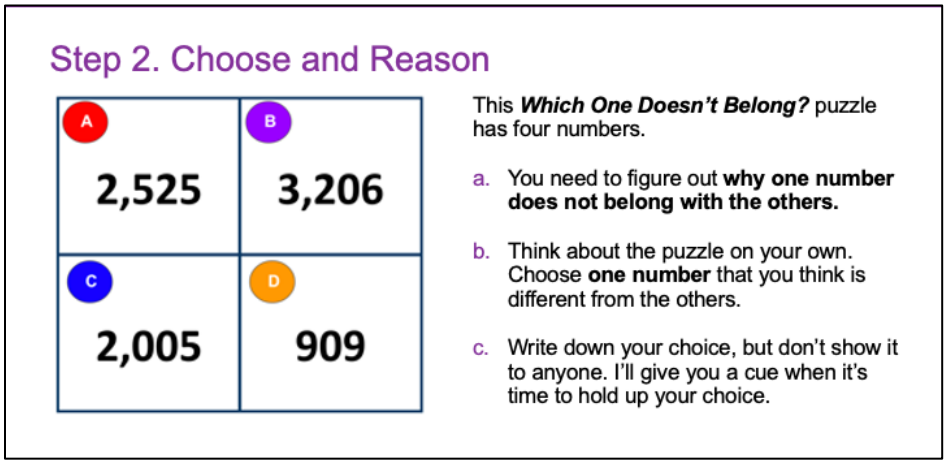
- **Teacher:** We will take turns in the role of teacher. The script provides suggestions about when to switch the teacher role.
- **Students:** Everyone will take on the role of students when they are not the teacher.

Step 1. Introduce the Learning Goals



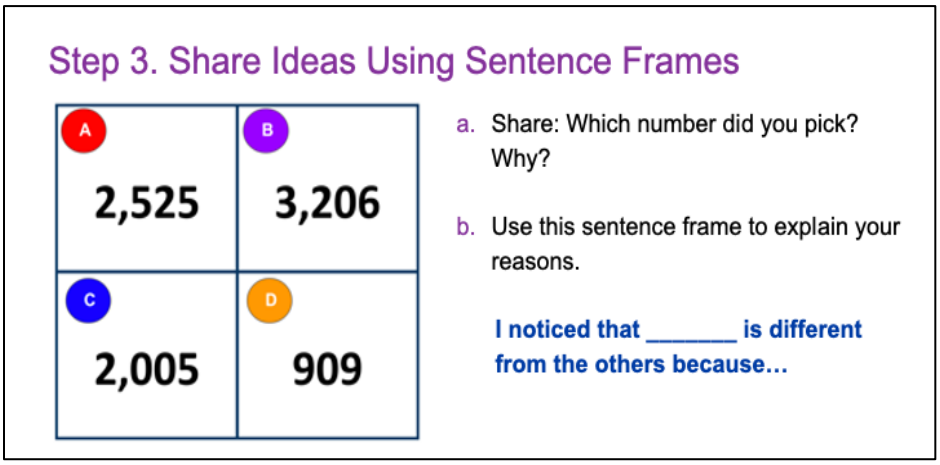
Teacher: Today, we are going to do an interesting puzzle called *Which One Doesn't Belong?* Our two learning goals build on the work we have been doing on place value. The mathematical goal is to compare the characteristics of multi-digit numbers. For our language goal, we will work on clearly describing those characteristics and place values of the numbers. At the end of the routine, we will revisit the goals to reflect on our learning.

Step 2. Choose and Reason



Teacher: Here's the *Which One Doesn't Belong?* puzzle. It has four different numbers. You need to figure out why one of the numbers doesn't belong with the others. You will have about 1 minute of individual think time to choose a number that you think is different from the others. Write down your choice on your whiteboard, but don't show it to anyone yet. When you are ready, we will share which numbers we picked and why.

Step 3. Share Ideas Using Sentence Frames



[Switch to a new teacher.]

Teacher: Please hold up your whiteboards so I can see which numbers were picked. *Students hold up the whiteboards.*

Teacher looks at students' choices to see what was selected and to think about which choice to focus on first.

Teacher: On your turn, please explain why the number you chose doesn't belong. When you share your reasoning, use the sentence frame, **I noticed that _____ is different from the others because…**

Student 1: I noticed that D is different because it starts and ends with the same number.

Teacher: Oh, I see what you mean. What does D start and end with?

Student 1: Nine.

Teacher: Great. And how would you read the whole number?

Student 1: Nine hundred nine.

Teacher: Dana, you also chose 909. What was your reason?

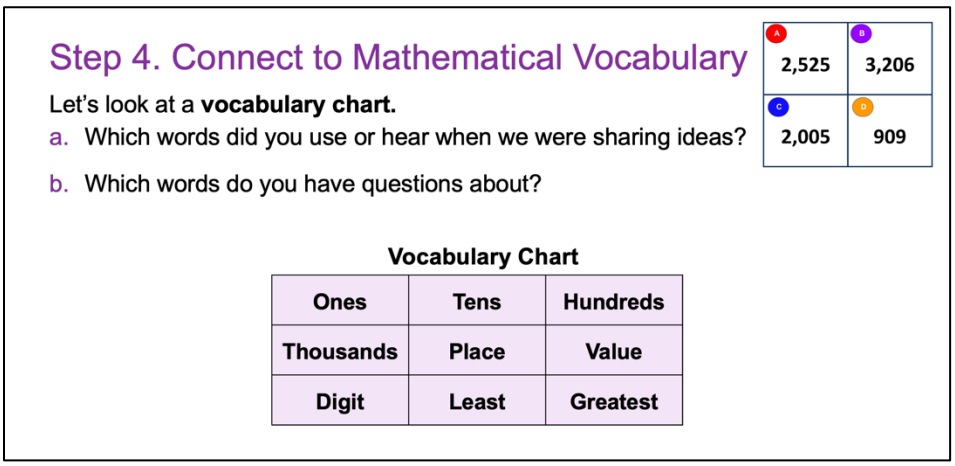
Student 2: It is the only one with three numbers. The others have four.

Teacher: Okay, I'm hearing you say that they have three and four numbers. Bari said that D starts and ends with the same number. We can use the word *digit* to describe the numerals in each of the place value locations of a number. So, 909 is different from the others because it has how many digits?

Student 2: Three digits.

Teacher: That's right. Nine hundred nine has three digits. The digit 9 is in the ones place and in the hundreds place, and the digit 0 is in the tens place.

Step 4. Connect to Mathematical Vocabulary



[Switch to a new teacher]

Teacher: When we share our explanations in mathematics class, it's helpful for us to use mathematical terms so others can clearly follow our thinking. I created a vocabulary chart of some mathematical terms that are important to use when describing and comparing numbers. Let's read the words on the chart.

Teacher points to words as the class says them aloud.

Teacher: Which words did you use or hear another student use when we were sharing ideas?

Student 1: I heard *hundreds*.

Student 2: I used the word *digit*.

Teacher notes their responses on the chart.

Teacher: Which words on the chart do you have questions about?

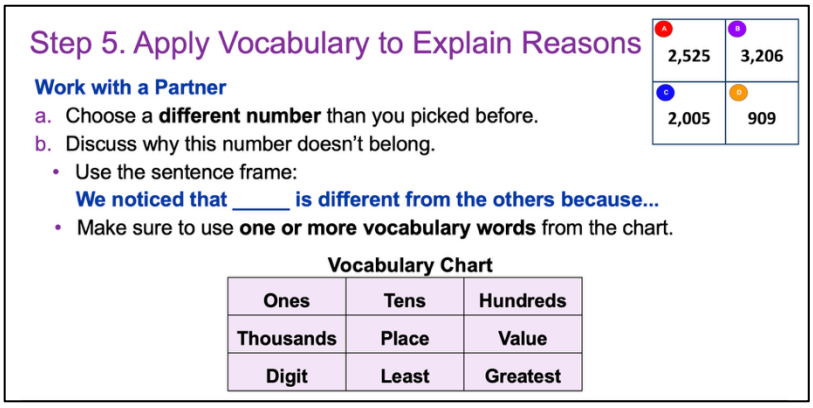
Student 1: I'm not sure what *place* means.

Teacher: *Place* means the location of digits in the number and gives you information about the value. So, let's look at 909 again. *Pointing to the number*. This is the ones place. This is the tens place, and this is the hundreds place. *Point to the 2 in choice A.* Can you tell me what this place is called?

Student 1: That's the thousands place.

Teacher: That's right. Any other questions about the vocabulary?

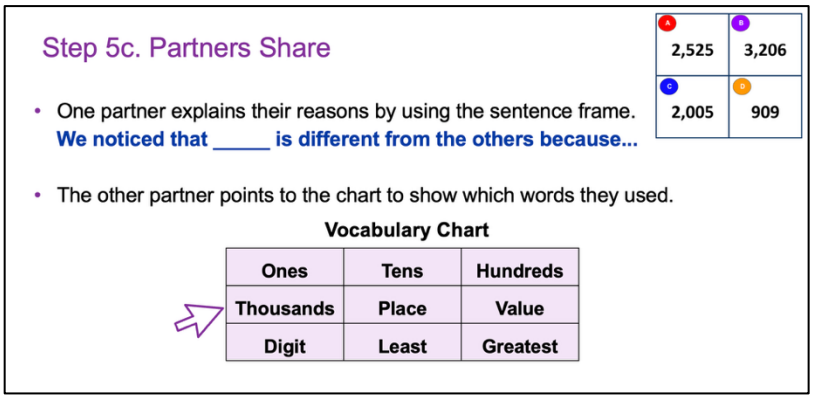
Step 5. Apply Vocabulary to Explain Reasons



Teacher: For the next step, you will work with a partner using the vocabulary chart and sentence frame. Together, you and your partner will pick a number that neither of you picked and talk about reasons why you think it doesn't belong. Use your sentence frame to explain your reasoning. Make sure to use at least one word from the vocabulary chart in your explanation. After you've had about 3 minutes to work together, we will come back together to share explanations with the group.

Discuss a new number, and complete a sentence frame with a partner.

Note about Step 5: Teachers can choose whether to have students complete the sentence frame in writing or just prepare to share it verbally.



[Switch to a new teacher]

Teacher: We'll take turns having partners share their reasons by using the sentence frame:

We noticed that ___ is different from the others because...

One partner will explain your reasons to the group. Then, the other partner will point to the mathematical term or terms on the vocabulary chart that they used.

Let's start with this pair.

Student 1: We noticed that C is different from the others because it has two zeroes.

Student 2 (Partner): *Point to the zeroes in 2,005 on the puzzle card.*

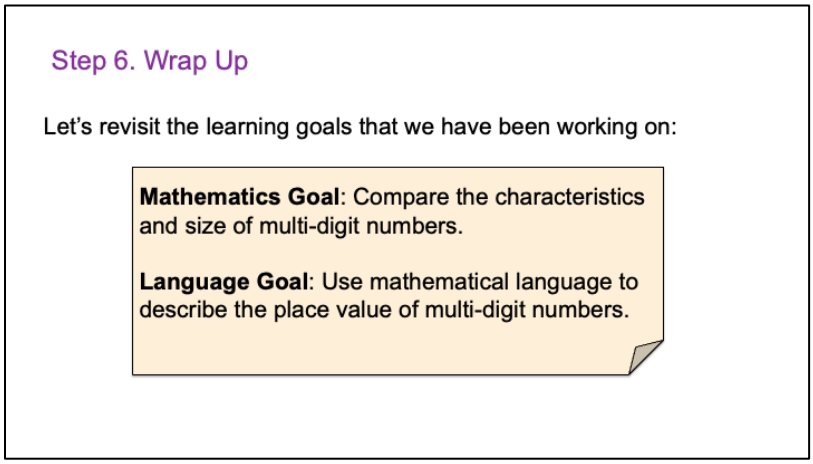
Teacher: Okay, you are noticing two zeroes in this number, and the other choices either have one zero or no zeroes. Can one of you describe the location of the zeroes in this number? G*esture to 2,005.*

Student 2: The number has a 0 in the tens place and in the hundreds place. And these two only have one 0 and it's in the tens place.

Teacher: That's right. And how do you read this number? *Point to 2,005.*

Student 1: Two thousand five.

Step 6. Wrap Up



Teacher: Here are the goals we have worked on together today. You compared the characteristics of multi-digit numbers to decide which ones didn't belong. You used place-value language and other comparison language to explain your reasons. As the last step of the routine, I want to wrap up by giving you a chance to reflect on your learning. First, think for a moment about these goals and what you learned.

Next, turn and talk to a partner about what you learned today. Use the sentence starter,

One thing that I learned today about place value was…

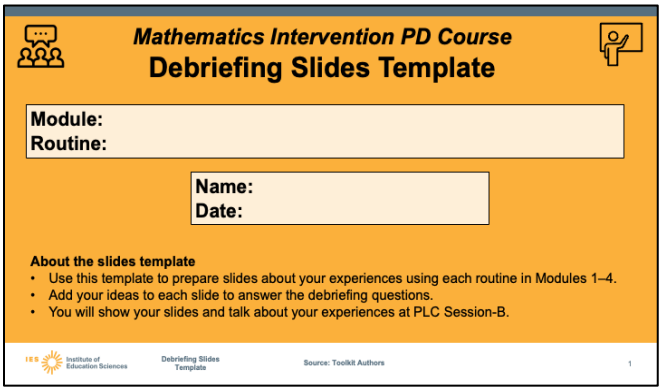
Students: *Participate in a pair-share to reflect on their learning.*

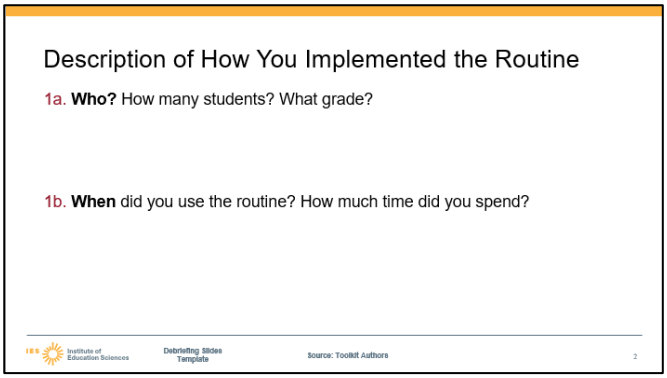
Teacher: *After the pair-share, connect this activity to what comes next in your lessons.*

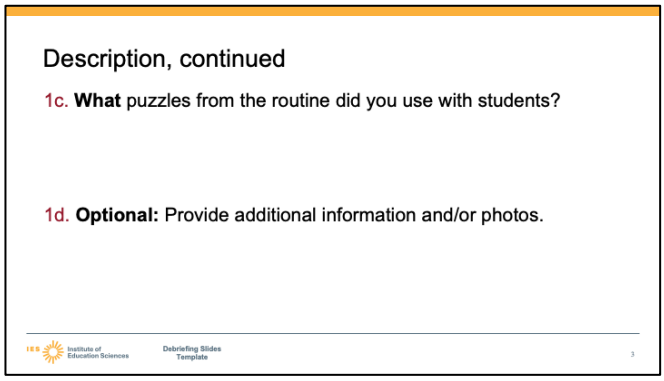
The end of the walk-through!

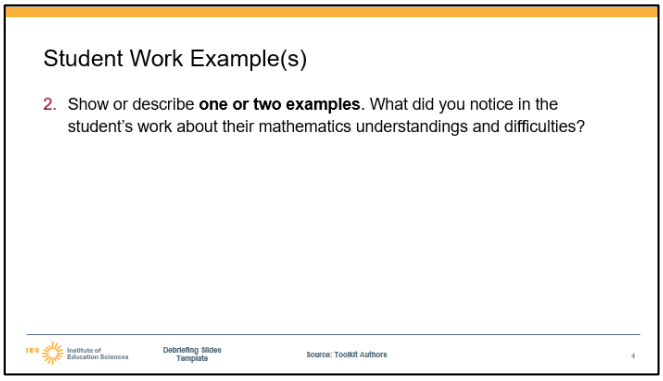
H10. How to Use the Debriefing Slides Template

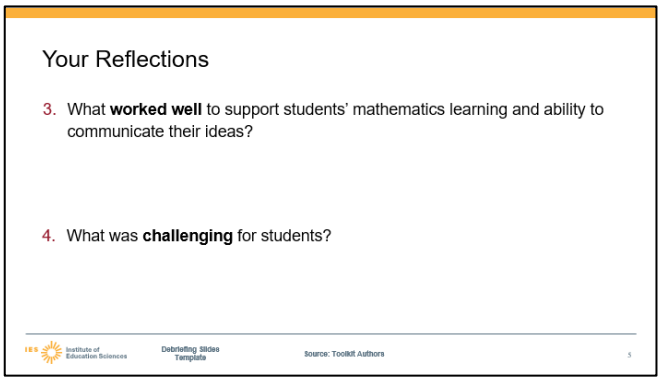
After you use the routine with students, reflect on your teaching experiences. Use the Debriefing Slides Template to prepare slides to share at PLC Session-B. The template has a set of five debriefing questions for you to answer. PLC group members will use the same questions to help ensure that the debriefing process is focused and productive. Add your ideas to each slide.

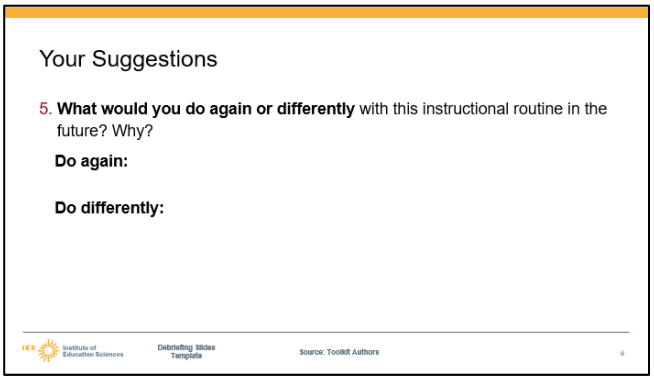












H11. Debriefing Questions and Protocol

Debriefing Questions

1. Description of how you implemented the routine. Who? When? What?
2. Show or describe one or two student work examples. What did you notice in students' work about their mathematics understandings and difficulties?
3. What worked well to support students' mathematics learning and ability to communicate their ideas?
4. What was challenging for students?
5. What would you do again or do differently in the future? Why?

Debriefing Protocol

Part 1: Sharing Experiences

1a. Each person takes a turn presenting their experiences using the routine.

- *Presenter* describes experiences and answers the debriefing questions (~6 min.).
- *Group members* are active listeners during the presentation (avoid interruptions).
- *Timekeeper* gives a 1-minute warning and says when time is up.

1b. After each presenter finishes, group members can:

- Ask clarifying questions and note ideas they would like to discuss in Part 2. (~1–2 min.)

***Time**:* About 7–8 minutes per presenter; total time varies by number of presenters.

Part 2: Group Discussion of Common Themes and Suggestions

The group discusses the following questions.

1. Think about the ideas shared by your colleagues. What's one idea that you want to use with this routine or apply in other mathematics activities?
2. Based on what you learned about students' understanding and challenges with using mathematical language, what next steps will you take?
3. What other strategies, challenges, or questions related to the routine would you like to discuss further?

***Time**:* About 6–10 minutes for group discussion.

H12. Recap Strategies: Mathematical Language

Directions: Review this list of strategies,[8] and add more ideas. Star * strategies that you want to try or to strengthen in your practice.

1. What are recommended strategies for mathematical language?

- **a.** Provide a **variety of examples** of new mathematical terms.
- **b.** Use **concrete and semi-concrete representations** to build understanding of mathematical vocabulary and to help students explain their ideas.
- **c.** **Connect** mathematical vocabulary to concrete, semi-concrete, and abstract representations.
- **d.** Use **graphic organizers** for mathematical vocabulary (such as to link definitions, characteristics, examples, and non-examples).
- **e.** Use **clear, correct, and consistent mathematical language** during instruction.
- **f.** Use **sentence starters** to support students in explaining their ideas orally and in writing.
- **g.** Use mathematical **vocabulary charts** and **word walls** to support students' use of mathematical language.
- **h.** Use **questioning strategies** to support students in explaining their ideas.
- **i.** Provide **consistent opportunities for students to discuss and explain** mathematics concepts and strategies.
- **j.** Use **partner discussions** to support students in sharing ideas and listening to others' ideas.

2. What things should you avoid? What should you do instead?

- Avoid teaching mathematical terms in isolation.
  - Instead, integrate vocabulary into mathematical instruction to help students build meaning through context.
- Avoid using informal, conversational, or catchy terms in place of formal mathematical vocabulary.
  - Instead, consistently use precise, formal mathematical vocabulary and support students in building understanding of this terminology. The use of imprecise, informal vocabulary can cause students confusion when they encounter the precise language in other classes or contexts.

[8] Strategies adapted from WWC Guide by Toolkit authors. These lists are not exhaustive.

H13. Strengthen Strategies

Directions: Use the prompts to reflect on current practices, brainstorm ideas, and plan actions.

1. Brainstorm Together

How will you strengthen your strategies for mathematical language? Choose prompts and list ideas.

a. Try the strategy of . . .

b. Increase . . .

c. Decrease . . .

d. Make sure to . . .

e. Find out about students' understanding of ___ by . . .

f. Build on students' strengths with ___ by . . .

g. Address students' challenges with ___ by . . .

2. Set an Implementation Intention

Look over your ideas above and choose one or two to focus on. Work individually to write an implementation intention and plan specific actions to take.

Implementation Intention
Set a goal to strengthen your instructional strategies for mathematical language. What will you focus on?
Plan **two actions** to take. What will you do? When?

H14. Self-Reflection Form: Mathematical Language

Introduction

This form provides an opportunity to reflect on your learning and current understanding of the recommendation for mathematical language. This self-reflection can help you celebrate progress and guide you in identifying areas for continued growth in your professional learning. It is not intended to be evaluative and will not be submitted or shared with others unless you choose to do so.

The form has two parts. In **Part 1**, you are asked to self-assess your understanding of specific strategies from the module on a scale of 1–3 (1: A little understanding; 2: Some understanding; and 3: A strong understanding). It is not expected that participants would have a "strong understanding" of every strategy at the end of a module. Please be assured that it's fine to select "some" or "a little" understanding. Your learning is evolving, and you will have opportunities to strengthen your understanding of the strategies in the subsequent modules and in your classroom. Continuing to use strategies with your students is a critical step in deepening your understanding and skills.

In **Part 2**, you are asked to reflect on the strategies and select one that you would like to improve. Focusing on one strategy is helpful for planning concrete next steps that are manageable to carry out. If you would like support, reach out to your facilitator and colleagues in the course. You may also want to revisit resources on specific strategies in the Participant Workbook and the online component.

This self-reflection form captures your thinking at one point in time. It's helpful to revisit the form later in the course to consider how your understanding has changed and to plan ways to continue your professional learning of the recommended strategies.

Part 1. Reflect on your understanding of strategies.

Directions: The table has a list of recommended strategies for teaching mathematical language. Read each strategy and self-assess your level of understanding by using this **rating scale**:

1. **A Little Understanding:** Have a vague sense of the strategy.
2. **Some Understanding:** Able to explain the strategy in general terms.
3. **A Strong Understanding:** Able to explain the strategy in detail and give examples.

NA: Have not learned about the strategy yet.

Strategies What is your current level of understanding for each strategy?	**Select Your Rating**
a. How to use **concrete representations** (such as manipulatives) to build understanding of mathematical vocabulary.	○ 1 ○ 2 ○ 3 ○ NA
b. How to use **semi-concrete representations** (such as drawings or diagrams) to build understanding of mathematical vocabulary.	○ 1 ○ 2 ○ 3 ○ NA
c. How to connect mathematical vocabulary to **abstract representations** (such as numbers or equations).	○ 1 ○ 2 ○ 3 ○ NA
d. How to use **graphic organizers** for mathematical vocabulary (such as to link definitions, characteristics, examples, and non-examples).	○ 1 ○ 2 ○ 3 ○ NA
e. How to use **sorting activities** (such as sorting examples and non-examples) to reinforce the meaning of mathematical terms.	○ 1 ○ 2 ○ 3 ○ NA
f. How to use **clear, correct, and consistent mathematical language** during instruction.	○ 1 ○ 2 ○ 3 ○ NA
g. How to use **sentence starters** and **sentence frames** to support students in explaining their ideas.	○ 1 ○ 2 ○ 3 ○ NA
h. How to use mathematical **vocabulary charts** and **word walls**.	○ 1 ○ 2 ○ 3 ○ NA
i. How to use **questioning strategies** to support students in explaining their ideas.	○ 1 ○ 2 ○ 3 ○ NA
j. How to provide **consistent opportunities for students to discuss and explain** mathematical concepts and strategies.	○ 1 ○ 2 ○ 3 ○ NA

Part 2. Reflect on your professional learning and use of strategies.

Directions: Look over the strategies on the prior page and then answer these questions.

1. What is one strategy that you find particularly helpful for building students' understanding and use of mathematical language? Why?

2. What is one strategy that you would like to improve in your practice? What are two actions you will take to strengthen your use of this strategy?

3. Reflect on your learning in this module on the mathematical language recommendation. What are one or two ways that your knowledge and practices have changed?

Routine Teaching Guide

This section has information and resources for teaching the *Instructional Routine: Explaining Reasons Using Mathematics Language.* Routine handouts are labeled with the letter **R** and a number.

Contents

Appendix A has reproducible handouts and other resources for the routine, such as large versions of the puzzles.

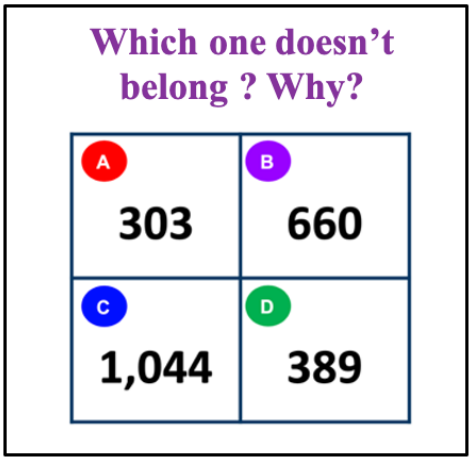
Slide Decks for Routine:

- Debriefing Slides Template for Sharing Experiences
- Optional Slides for Teaching the Routine

Introduction to the Routine

This Routine Teaching Guide provides information, suggestions, and resources to support teachers in using an instructional routine with students and then sharing their experiences. The routine incorporates multiple strategies from the WWC Guide's recommendation for mathematical language. It can be used with different mathematics topics and has a consistent sequence of steps to allow for ease of implementation by teachers and students.

In this routine, the teacher uses a *Which One Doesn't Belong (WODB)*[9] puzzle to engage students in communicating mathematical ideas. The WODB puzzles show four related but different choices of numbers, shapes, graphs, equations, or other representations. Students are asked to consider the four choices, select one that doesn't belong with the others, and explain their reasons.



The puzzles are designed so that there are reasons why each choice doesn't belong, which allows students to identify different reasons and add to the discussion. Because there is no incorrect choice, the focus is on the reasoning shared via the explanations. Having multiple solutions supports student participation and communication. In addition, the puzzles have accessible entry points that allow students to come up with reasons at their level of mathematical understanding.

The instructional routine adds a consistent structure and scaffolds to the WODB puzzles, providing students with increased support for explaining their reasons using mathematical language. The routine incorporates the use of sentence starters/frames, vocabulary charts, and questioning strategies to bolster student communication.

At PLC Session-A, you and your colleagues will learn how to use the instructional routine by watching and discussing a classroom video example, doing a walk-through, and discussing ways to use the routine with students. Everyone is expected to use the routine with students one or more times and then to share experiences at PLC Session-B.

[9] The *Which One Doesn't Belong?* activity format is from the website: https://talkingmathwithkids.com/wodb/. The Toolkit authors designed a set of puzzles specifically for the module's instructional routine.

Frequently Asked Questions (FAQs) about the Routine

A. What recommended strategies does the instructional routine incorporate?

The routine focuses on the WWC Guide recommendation for mathematical language, particularly implementation Step 3: "Support students in using mathematically precise language during their verbal and written explanations of their problem solving."

The routine incorporates these strategies:[10]

- **Puzzles that have multiple responses** so students can share different ideas.
- **Response cards** for students to choose an answer (A, B, C, or D) and hold it up.
- **Sentence starters/frames** provide a starting point for students to explain their ideas.
- **Questioning and rephrasing** to elicit and reinforce students' use of mathematical language.
- **Mathematical vocabulary chart** supports students' use of relevant mathematical terms.
- **Partner work and different discussion formats** provide opportunities and support for students to share ideas.

B. What mathematics content does the routine focus on?

This versatile routine can be used with a variety of mathematics topics. You have a choice of seven puzzles on the topics of whole number place value, fractions, and decimals (see handout **R5**). Alternatively, you can focus on different mathematics topics by using puzzles from other sources or by creating your own.

C. How do you integrate the routine with your intervention program?

The routine is designed to reinforce the use of mathematical language and to support students in strengthening their communication skills. Opportunities for students to discuss their ideas in pairs, small groups, and with the whole group are important and powerful for mathematics intervention class.

The routine is about 30 minutes long and can be used in one lesson. It works well to integrate the routine into a lesson by choosing a puzzle on a relevant math topic. The puzzles can be used to reinforce and review content from prior lessons. This guide provides puzzles on the topics of whole number place value, fractions, and decimals (**R5**). If your curriculum includes WODB puzzles, you can use those puzzles with the routine.

[10] Strategies adapted from WWC Guide by Toolkit authors.

Using the Routine with Students

Review these expectations and suggestions for course participants.

Expectations

- Use the routine with students **at least one time** *before* PLC Session-B.
- It's okay to **adjust the routine** to fit your teaching situation. The six-step routine takes about 30 minutes. If you have shorter intervention classes, one option is to use Steps 2–5 and skip Steps 1 and 6. Another option is to shorten the whole group share-out in Step 5.
- After you use the routine, prepare to share experiences at PLC Session-B by using the **Debriefing Slides Template** to create six slides. Add your ideas to the slides to answer the debriefing questions.

Suggestions

- The routine is likely to take more time the first time you use it. That's okay!
- Use the Two-Page Overview **(R1)** when you teach. It outlines each step. Feel free to put the routine into your own words.
- It's helpful to use a routine more than once. Some teachers said they felt unsure the first time and that they felt more comfortable when they used the routine again. Consider using the routine again with different puzzles or intervention classes.
- Keep in mind that the routine is also new for your students. Encourage and support their participation.

R1. Two-Page Overview of Routine

Instructional Routine: Explaining Reasons Using Mathematical Language
Total time: About 30 min.

Step 1. Introduce learning goals. (Whole Group) ~2 min.

a. **Display the learning goals and introduce them to students.**

Step 2. Choose and reason. (Whole Group and Individual) ~3 min.

a. Show the puzzle and provide directions.
b. Give students time to work independently to choose a number that does not belong and think about their reasons.
c. Ask students to write their letter choice on a sticky note or whiteboard, but not to show their answers until you give them a cue to hold them up.

Step 3. Share ideas using sentence frames. (Whole Group) ~6 min.

a. Ask students to hold up their response so you can see which numbers they chose. Use that information to sequence the order for sharing.
   - Affirm that all their choices are correct because there are reasons why each number doesn't belong. Providing this affirmation can help students feel more comfortable sharing.
b. Introduce the sentence frame and ask students to use it to explain their reasoning:

I noticed that_____ is different from the others because…

c. Have students share their reasons* using the sentence frame. Give students who chose the same number an opportunity to share their reasons as well.

Tip: If one number was not selected, do not discuss it yet. Use that number for Step 5.

***Sample responses** for each puzzle are provided on the routine handout **R5**.

Step 4. Connect to mathematical vocabulary. (Whole Group) ~5 min.

a. Show the vocabulary chart and ask: Which words on this chart did you say or hear when we shared ideas?
   - Students share words that they heard or used.
b. Then ask students: Which words on the vocabulary chart do you have questions about?
   - Students may ask about the meaning of one or more words. Respond by explaining the mathematical words and asking students to give examples.

Step 5. Apply vocabulary to explain reasons. (Pairs and whole group) 8–10 min.

a. Provide directions to students for the next task. Select one option:
   - If a number was *not* chosen by any student, ask everyone to focus on it.
   - If all numbers were already chosen, ask pairs to choose a different number than the one that they chose the first time.

b. Have students work in pairs to talk about why the new number does not belong. Ask them to use **one or more mathematical terms** from the vocabulary chart and this sentence frame:

We noticed that _____ is different from the others because…

Invite the pairs to share their reasons with the whole group. Ask questions and make comments to reinforce their use of mathematical terms.

Step 6. Wrap up. (Whole Group). ~3 min.

a. Revisit the goals and ask students to reflect on their learning.

Tips for Using the Routine

- Ask **follow-up questions** to support students in elaborating on ideas. Examples:
  - Can you tell us more about your reason for choosing that number?
  - How is that number different from the other three numbers?
  - How would you explain [student name's] reason in your own words?
  - How is your reason similar to or different from [student's name]?
- *If students are new to WODB puzzles,* you may want to start with a non-mathematics puzzle, such as one with animals (**Appendix A**). Another option is to choose a puzzle on a familiar mathematics topic that students are likely to feel comfortable talking about.
- *If students are already familiar with WODB puzzles*, explain that you will use some new steps for doing the activity to focus on building mathematical language.

Suggestions Related to Time

- *If your intervention time is less than 30 minutes,* it's okay to shorten the routine. One option is to use Steps 2–5 and skip Steps 1 and 6. Another option is to shorten the whole group share-out in Step 5.
- *If you have additional time,* consider doing a second puzzle on a similar mathematics topic and using the same vocabulary chart. This gives students another opportunity to use the mathematical terms. Do Steps 1–3 with the addition of asking students to use one or more terms from the vocabulary chart when they explain their reasons.

R2. Planner for Routine

1. List Ideas to Apply for Teaching the Routine

a. What ideas from the video and walk-through do you want to apply with your students? What do you want to make sure to do when you implement this routine?

2. Make Specific Plans

Suggested Prerequistes for the Routine

Students should have prior experiences with the mathematics topic that is the focus of the puzzle. The routine helps to reinforce previously learned mathematical vocabulary and concepts. You can use the routine as a review, a content-building activity, or for formative assessment.

a. **When** will you use the routine with your students before PLC Session-B?

The routine will take about 30 minutes.

b. **Which puzzle(s)** do you want to use with students?

Look over the choice of puzzles on whole numbers, fractions, and decimals on handout **R5**. You can also use a puzzle from your curriculum or other sources.

3. Consider Potential Challenges and Ways to Address Them

a. How will you provide additional support if students have difficulty explaining their reasons or doing other parts of the routine? (See handout **R3** for suggestions.)

R3. Suggestions for Addressing Potential Challenges

Directions: Look over the suggestions and add your ideas. Star * ideas to try.

1. **If students are hesitant to talk…**

- **a.** Start with a non-mathematics puzzle to give students an opportunity to share their reasons without needing to use mathematical language. This will help students get familiar with the activity.
- **b.** Start with a mathematics puzzle on a familiar topic (from a prior grade) to help students feel more comfortable talking about their reasons why one choice doesn't belong.
- **c.** In Step 3, have students first talk with a partner before sharing with the whole group.

2. **If students all choose the same number…**

- **a.** After students share about that number, have students work in pairs to choose a different number and talk about their reasons. Remind students that all four choices have reasons why they do not belong.
- **b.** Select a different number and ask students questions, such as:
  - A student in another class selected [choice]. What reasons do you think the student gave for why this number is different from the other numbers?
  - I think that [choice] doesn't belong. What do you notice about the [place value/size] of this number? How is it different from the other numbers?

3. **If students have difficulty explaining their reasoning**:

- **a.** Do a sample explanation together to model using the sentence frame.
- **b.** Ask students to *show and explain* by pointing to the numbers as they talk about reasons.
- **c.** In Step 5, ask students to explain their reasons in their own words using the sentence frame. Then, ask them to restate what they said by using one mathematical term from the vocabulary chart.

4. Suggestions for selecting puzzles:

- **a.** Choose a puzzle on a mathematics topic for which students have prior knowledge. This helps students feel more comfortable explaining their reasons by using mathematical language.
- **b.** Select a back-up puzzle or two in case your first choice is too easy or too difficult. It's okay to switch puzzles.

R4. Preparation and Materials Checklist

Use this list to prepare for using the routine with students.

☐ **1. Choose which puzzle(s) you will use**. The handout **R5. Puzzles for the Routine**, provides a choice of puzzles on the topics of whole number place value, fractions, and decimals. Also, there are two introductory puzzles with non-mathematics topics in **Appendix A**.

☐ **2. Prepare the puzzle(s) to display.** Print the puzzle or write it on the board or chart paper. If you write the puzzle, make sure to label the choices (A, B, C, and D) to support students in talking about the choices. Large versions are provided in **Appendix A**.

☐ **3. Prepare learning goals to display.** The handout **R5. Puzzles for the Routine**, provides example goals for each puzzle. You can use them as is, adapt them, or write your own. Decide how you will display the goals for students, such as on chart paper or a whiteboard.

☐ **4. Prepare sentence frames.** Students will use a sentence frame to explain their choice and why it is different. Print and cut the larger sentence frames provided in **Appendix A** or write them on a whiteboard or chart paper for students to reference.

☐ **5. Prepare the vocabulary chart.** The handout **R5. Puzzles for the Routine**, provides an example vocabulary chart for each puzzle. Large versions of the charts are in **Appendix A**.

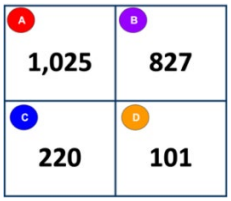
- You can use the vocabulary chart as is or adapt it by adding, changing, or cutting words.
- Plan how you will display the vocabulary chart for students to see in Steps 4 and 5. You may want to print a copy for each pair of students or write the mathematical words on the board or chart paper.
- One option is to write the words on index cards, one per card. Have students choose one or two cards with words they want to use in their explanations.

R5. Puzzles for the Routine

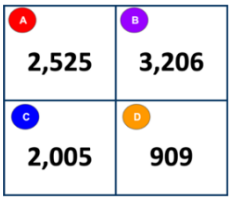
There are seven puzzles on the topics of whole number place value, fractions, and decimals. Each puzzle heading hyperlinks to a page with more information about the puzzle, including sample responses, learning goals, and a vocabulary chart.

Whole Number Place Value

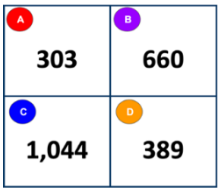
Puzzle 1. Place value for whole numbers in hundreds and thousands.



Puzzle 2. Place value for whole numbers in hundreds and thousands.

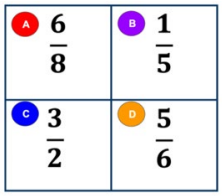


Puzzle 3. Place value for whole numbers in hundreds and thousands.

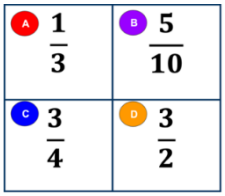


Fractions

Puzzle 4. Fractions less than and greater than 1.



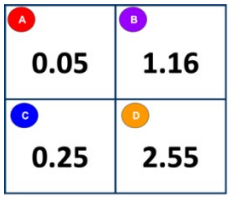
Puzzle 5. Fractions less than and greater than 1.



Decimals

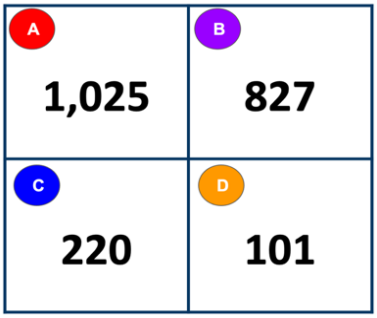
Puzzle 6. Decimals to tenths and hundredths.

Puzzle 7. Decimals to hundredths.



Puzzle 1

Mathematics Topic: Place value for whole numbers with hundreds and thousands.



Goals

- Mathematical Learning Goal: Compare the characteristics and sizes of multi-digit numbers.
- Language Goal: Use mathematical language to describe the place value of multi-digit numbers.

Vocabulary Chart

Ones	Tens	Hundreds
Thousands	Place	Value
Digit	Least	Greatest

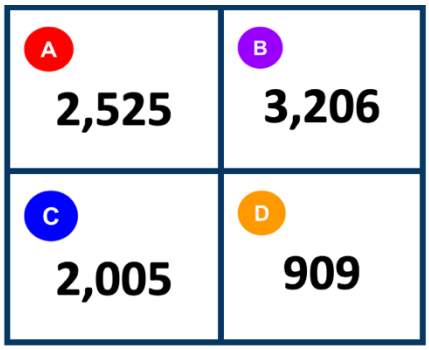
Sample Responses

This table has examples of possible responses. It is not an exhaustive list.

Choices	Which one doesn't belong? How is it different from the others?
A. 1,025	• The only number with thousands. • The number with the greatest value.
B. 827	• The only number with no zeros. • The only number with a 7 in the ones place. • The only number with an 8 in the hundreds place.
C. 220	• The only number with a 0 in the ones place. • The only even number. • The only number with two of the same digits (2) next to one another.
D. 101	• The only number with a 0 in the tens place. • The only number that doesn't have a 2 in the tens place. • The number with the least value. • The only number with the same digit (1) in the hundreds place and ones place.

Puzzle 2

Mathematics Topic: Place value for whole numbers with hundreds and thousands.



Goals

- Mathematical Learning Goal: Compare the characteristics and sizes of multi-digit numbers.
- Language Goal: Use mathematical language to describe the place value of multi-digit numbers.

Vocabulary Chart

Ones	Tens	Hundreds
Thousands	Place	Value
Digit	Least	Greatest

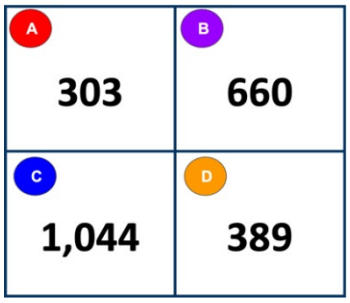
Sample Responses

This table has examples of possible responses. It is not an exhaustive list.

Choices	Which one doesn't belong? How is it different from the others?
A. 2,525	**a.** The only number with a 2 in the tens place. **b.** The only number with a 5 in the hundreds place. **c.** The only number that has a repeating pattern— 2525. **d.** The only number without a 0 in the tens place.
B. 3,206	**a.** The only number with a 3 in the thousands place. **b.** The only number with a 2 in the hundreds place. **c.** The only number with a 6 in the ones place. **d.** The number with the greatest value. **e.** The only even number.
C. 2,005	**a.** The only number with two zeros. **b.** The only number with a 0 in the hundreds place.
D. 909	**a.** The only number with three digits. **b.** The number with the least value. **c.** The only number that starts and ends with the same digit (9).

Puzzle 3

Mathematics Topic: Place value for whole numbers with hundreds and thousands.



Goals

- Mathematical Learning Goal: Compare the characteristics and sizes of multi-digit numbers.
- Language Goal: Use mathematical language to describe the place value of multi-digit numbers.

Vocabulary Chart

Ones	Tens	Hundreds
Thousands	Place	Value
Digit	Least	Greatest

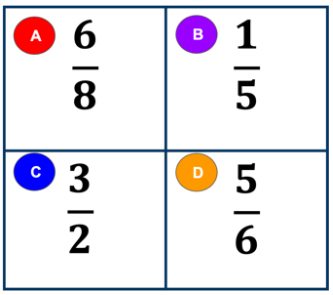
Sample Responses

This table has examples of possible responses. It is not an exhaustive list.

Choices	Which one doesn't belong? How is it different from the others?
A. 303	• The number with the least value. • The only number with 3 in the ones place and in the hundreds place. • The only number with a 0 in the tens place.
B. 660	• The only number that has the same digit (6) in the hundreds and tens place. • The only number with a 0 in the ones place. • The only number that has 5 as a factor.
C. 1,044	• The number with the greatest value. • The only number with thousands. • The only number without a 3 or a multiple of 3 as a digit. • The only number with 0 in the hundreds place.
D. 389	• The only number without a 0 in any place. • The only number that has three different digits (no duplicates of the same digit).

Puzzle 4

Mathematics Topic: Fractions less than and greater than 1.



Goals

- Mathematical Learning Goal: Compare the characteristics and sizes of different fractions.
- Language Goal: Be able to clearly describe fractions.

Vocabulary Chart

Wholes	Halves
Fifths	Sixths
Eighths	Unit Fraction
Numerator	Denominator
Less than	Greater than
Least	Greatest

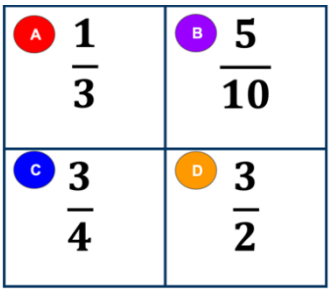
Sample Responses

This table has examples of possible responses. It is not an exhaustive list.

Choices	Which one doesn't belong? How is it different from the others?
A. $\frac{6}{8}$	• The only fraction with a 6 in the numerator. • The only fraction with an 8 in the denominator. • The only fraction that is equivalent to $\frac{3}{4}$. • The only fraction that is not in the simplest form. It can be simplified to $\frac{3}{4}$.
B. $\frac{1}{5}$	• The fraction with the least value. • The only unit fraction. • The only fraction with a 1 in the numerator. • The only fraction that is less than $\frac{1}{2}$. • The fraction that is closest to 0.
C. $\frac{3}{2}$	• The fraction with the greatest value. • The only fraction that is greater than 1. • The only fraction that is divided into halves. • The only fraction with a 2 in the denominator. • The only fraction that can be written as a mixed number: $1\frac{1}{2}$.
D. $\frac{5}{6}$	• The only fraction that is divided into sixths. • The only fraction that is $\frac{1}{6}$ away from a whole. • The fraction that is closest to one whole.

Puzzle 5

Mathematics Topic: Fractions less than and greater than 1.



Goals

- Mathematical Learning Goal: Compare the characteristics and sizes of different fractions.
- Language Goal: Be able to clearly describe fractions.

Vocabulary Chart

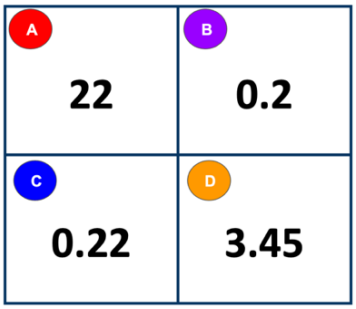
Wholes	Halves
Thirds	Fourths
Tenths	Unit Fraction
Numerator	Denominator
Less than	Greater than
Least	Greatest

Sample Responses

This table has examples of possible responses. It is not an exhaustive list.

Choices	Which one doesn't belong? How is it different from the others?
A. $\frac{1}{3}$	• The only unit fraction; the only fraction with a 1 in the numerator. • The only fraction that is less than $\frac{1}{2}$. • The fraction that is closest to 0. • The fraction with the least value.
B. $\frac{5}{10}$	• The only fraction that is *not* in the simplest form. It can be simplified to $\frac{1}{2}$. • The only fraction that does not have a 3 in the numerator or denominator. • The only fraction that is equivalent to $\frac{1}{2}$.
C. $\frac{3}{4}$	• The only fraction that is greater than $\frac{1}{2}$ but less than 1. • The fraction that is closest to 1. • The only fraction that is half of another fraction in the puzzle, $\frac{3}{2}$.
D. $\frac{3}{2}$	• The only fraction that can be written as a mixed number: $1\frac{1}{2}$. • The only fraction that is greater than 1. • The fraction with the greatest value.

Puzzle 6



Mathematics Topic: Comparing whole numbers and decimals with tenths and hundredths.

Goals

- Mathematical Learning Goal: Compare the characteristics and sizes of whole numbers and decimals.
- Language Goal: Use language to describe the place value of numbers.

Vocabulary Chart

Ones	Tens
Tenths	Hundredths
Decimal Point	Digit
Place	Value
Less than	Greater than
Least	Greatest

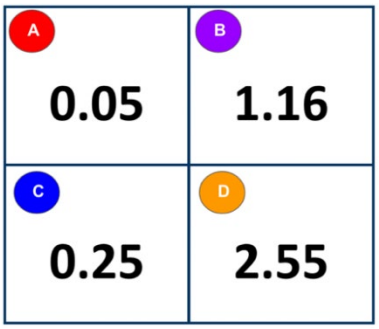
Sample Responses

This table has examples of possible responses. It is not an exhaustive list.

Choices	Which one doesn't belong? How is it different from the others?
A. 22	• The number that has the greatest value. • The only number that doesn't have a decimal point shown. • The only number that has a digit in the tens place.
B. 0.2	• The number with the least value. • The only number that has one digit after the decimal point.
C. 0.22	• The only number with a 2 in the hundredths place.
D. 3.45	• The only number that doesn't have a 2. • The only number with a 3 in the ones place. • The only number whose digits are all different.

Puzzle 7

Mathematics Topic: Comparing decimals with hundredths; understanding the value of zero in different places.



Goals

- Mathematical Learning Goal: Compare the characteristics and sizes of decimals to the hundredths place.
- Language Goal: Use language to describe the place value of decimals.

Vocabulary Chart

Ones	Tenths
Hundredths	Place
Decimal Point	Digit
Less than	Greater than
Least	Greatest

Sample Responses

This table has examples of possible responses. It is not an exhaustive list.

Choices	Which one doesn't belong? How is it different from the others?
A. 0.05	• The number with the least value. • The only number with a 0 in the tenths place. • The only number with two zeros.
B. 1.16	• The only number with a 1 in the ones place. • The only number with a 6 in the hundredths place. • The only number that doesn't have a 5 in the hundredths place.
C. 0.25	• The only number with a 2 in the tenths place. • The only number with a value of twenty-five hundredths, which is equivalent to $\frac{1}{4}$.
D. 2.55	• The number with the greatest value. • The only number larger than 2. • The only number with two digits that are 5s.

R6. Detailed Teaching Notes for the Routine

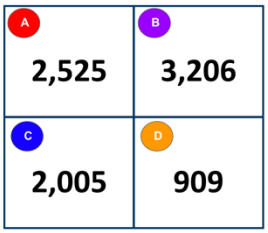
This section provides notes for teaching the routine. It includes pictures of the slides, but it is not necessary to use the slide deck, *Optional Teaching Slides for the Routine*. You can present the information to students by using chart paper, a whiteboard, or handouts to display the goals, sentence frames, and vocabulary chart. The notes are organized as follows:

A. **SAY:** Provides information about what to say and do. It is *not* meant to be a script. Use these examples to communicate the ideas in your own way.
   - Regular text indicates things to **say**.
   - *Italicized text* indicates things for the teacher or students to **do**.

B. **NOTES:** Provide directions and clarifying information for teachers. Some slides do not have any notes.

C. **TIPS:** Provides suggestions for implementing specific steps. Some slides do not have any tips.

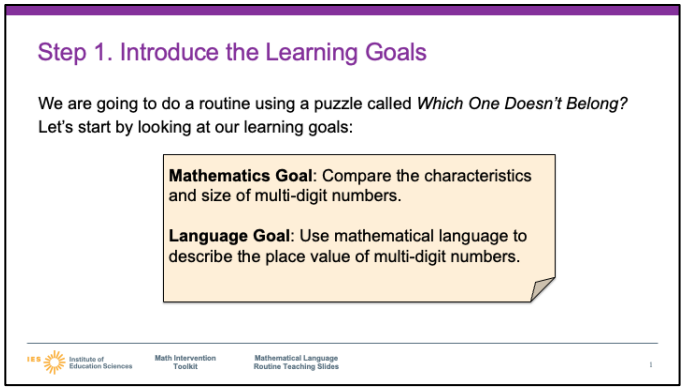
Example Puzzle: This deck shows Puzzle 2 (from the walk-through), but it can be used for any puzzle. To use a different puzzle, replace the goals, puzzle, and vocabulary chart on the slides.



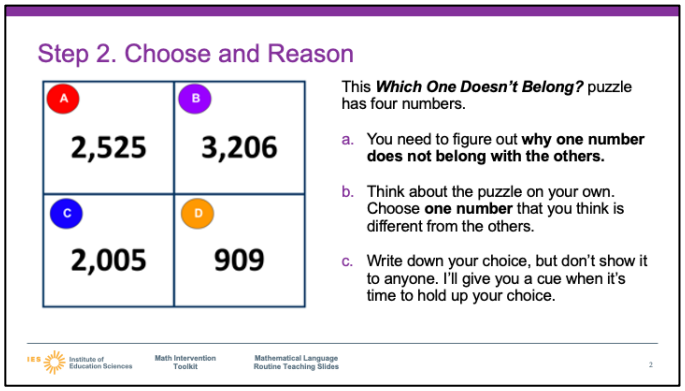
Step 1 (Slide 1)

NOTES: You may adapt these goals or use them as written.

SAY: Today, we will do an interesting puzzle called *Which One Doesn't Belong?* Our two learning goals build on the work we have been doing on place value. The mathematical goal is to compare the characteristics and sizes of multi-digit numbers. For our language goal, we will work on clearly describing the characteristics of the numbers. At the end of the routine, we will revisit the goals to reflect on our learning.



Step 2 (Slide 2)

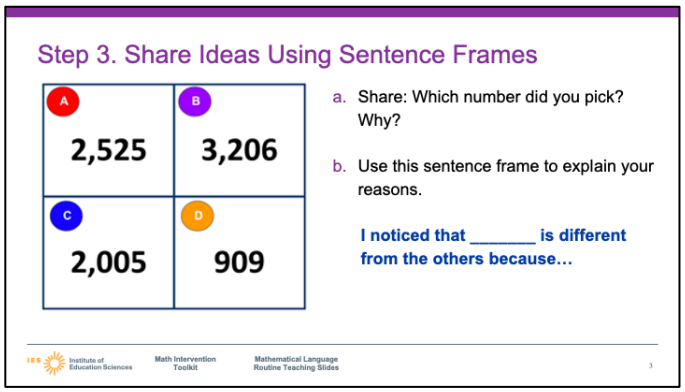


NOTES: Students will work individually to choose a number and plan what they will say. They will write their choice on a sticky note, a piece of paper, or a mini whiteboard. In Step 3, you will introduce students to the sentence frame to say their reason aloud; they do not need to write it down in Step 2.

SAY: Here's the *Which One Doesn't Belong?* puzzle. It has four different numbers. You need to figure out why one of the numbers doesn't belong with the others. You will have about 1 minute of individual think time to choose a number that you think is different from the others. Write down your choice on your whiteboard, but don't show it to anyone.

After the think time, we will share which numbers we picked and why.

Step 3 (Slide 3)



NOTES: Provide an opportunity for students to share their choices and reasons. Ask questions as needed to support students in communicating and elaborating on their ideas.

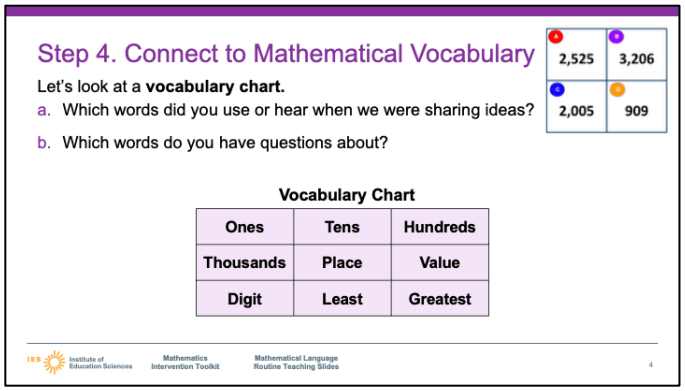
SAY: Please hold up your whiteboards so I can see which numbers were picked.

On your turn, please explain why the number you chose doesn't belong. When you share your reasoning, use the sentence frame, **I noticed that ___ is different from the others because…**

TIPS for Step 3:

- At the start of Step 3, ask students to hold up their choices. Use this information to plan which choice to start with and who to call on. It's helpful to start with a choice that was selected by two or more students. This helps get those students involved in the discussion from the start. It also gives students a chance to hear different reasons for the same choice.
- Support students in explaining and elaborating on ideas by asking questions, such as:
  - Can you tell us more about your reason for choosing that number?
  - How is that number different from the other three numbers?
  - How would you explain [student name's] reason in your own words?
  - How are your reasons similar to or different from [student's name]?

Step 4 (Slide 4)



SAY: When we share our explanations in math class, it's helpful for us to use mathematical terms so that others can clearly follow our thinking. I created a vocabulary chart of some mathematical terms that are important for describing and comparing numbers. I'll read the words on the chart.

a. Which words did you use or hear another student use when we were sharing ideas?

Students share words that they used or heard their classmates use.

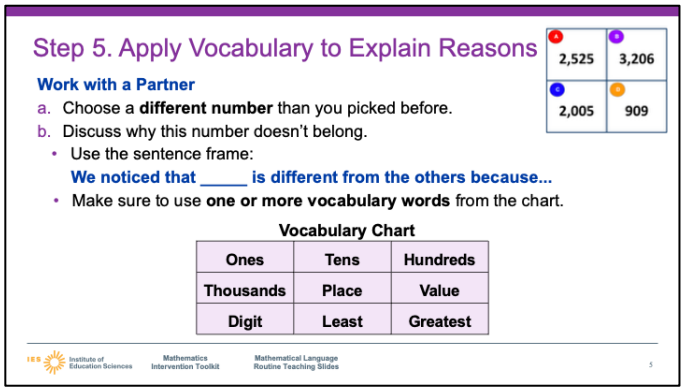
b. Which words on the chart do you have questions about?

Students can ask questions about the meaning of specific words. Provide explanations and show examples as needed. Keep this brief.

TIPS for Step 4:

- When students share which words on the chart they used or heard, point to those words on the vocabulary chart. You may want to star * or highlight them.
- When students ask questions about one or more mathematical terms, respond by providing an explanation to build understanding of the terms. Keep this brief to allow time for students to use the mathematical terms in Step 5.

Step 5a-b (Slide 5)

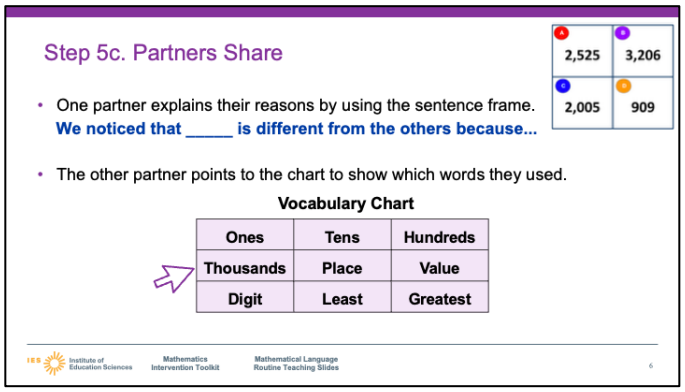


NOTES: In this step, students will discuss with a partner a different number than the one they used the first time. If one of the numbers was not selected in Step 3, you can ask all students to focus on that number. Or you can ask students to choose a different number than the one they selected before. The example below focuses on the latter approach.

SAY: For the next step, you will work with a partner. Together, pick a new number that neither of you picked and talk about reasons why you think it doesn't belong. Use your sentence frame to prepare an explanation of your choice. Try to use at least one word from the vocabulary chart in your explanation. After you've had about 3 minutes to work together, we will come back together to share explanations with the group.

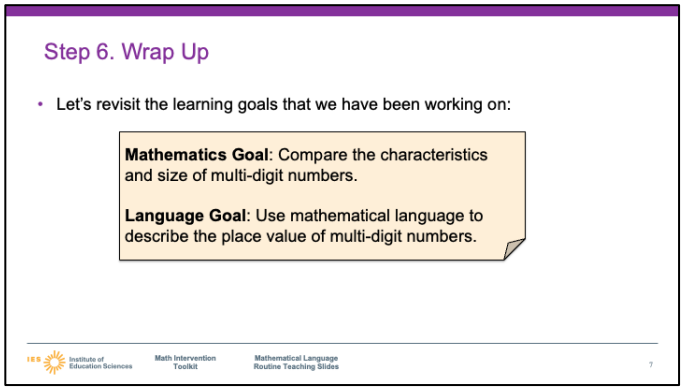
Step 5c (Slide 6)

SAY: We'll take turns having partners share their completed sentence frame. I want one partner to share your reasons. The other partner will point to the vocabulary words from the chart as their partner uses them. Those of you who are listening, be sure you understand the explanation and how they used the vocabulary. You can ask questions about their choice or reasoning.



Step 6 (Slide 7)

SAY: Here are the goals we have worked on together today. You compared the characteristics of multi-digit numbers to decide which ones didn't belong. You explained your reasons by using mathematical language to describe the place value of the numbers and make comparisons.



Slides for Other Puzzles

Slides 8-24 have large versions of puzzles 1–6, vocabulary charts, and two introductory puzzles.

Appendix A: Routine Resources for Use with Students

This appendix has reproducible handouts for the Instructional Routine: Explaining Reasons by Using Mathematical Language.

Sentence Frames

There are two copies of each sentence frame below. Provide one copy of each per pair of students.

For Step 3

I noticed that_____ is different from the others because . . .

For Step 3

I noticed that_____ is different from the others because . . .

For Step 5

We noticed that_____ is different from the others because . . .

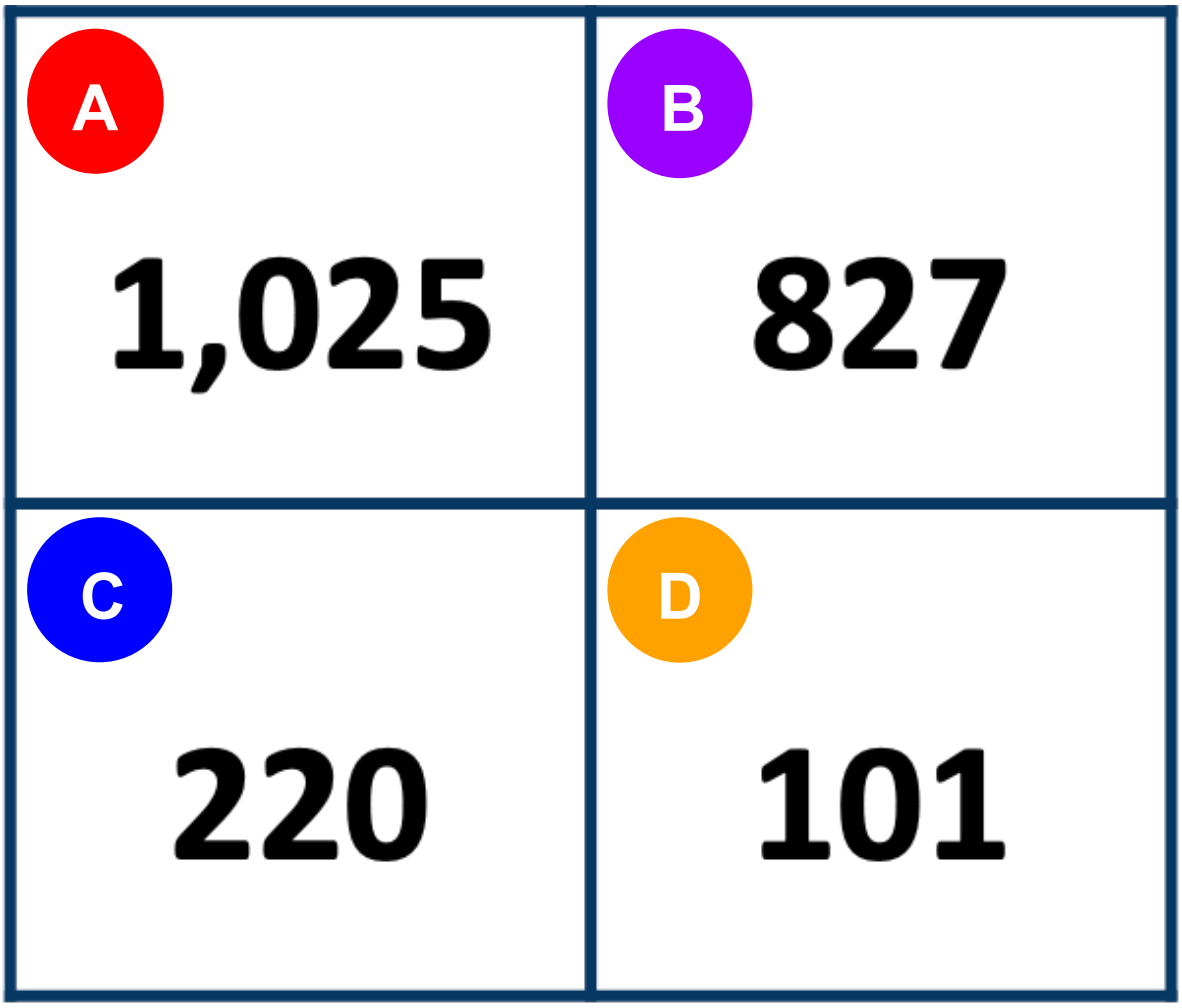
For Step 5

We noticed that_____ is different from the others because . . .

Puzzles: Large Versions

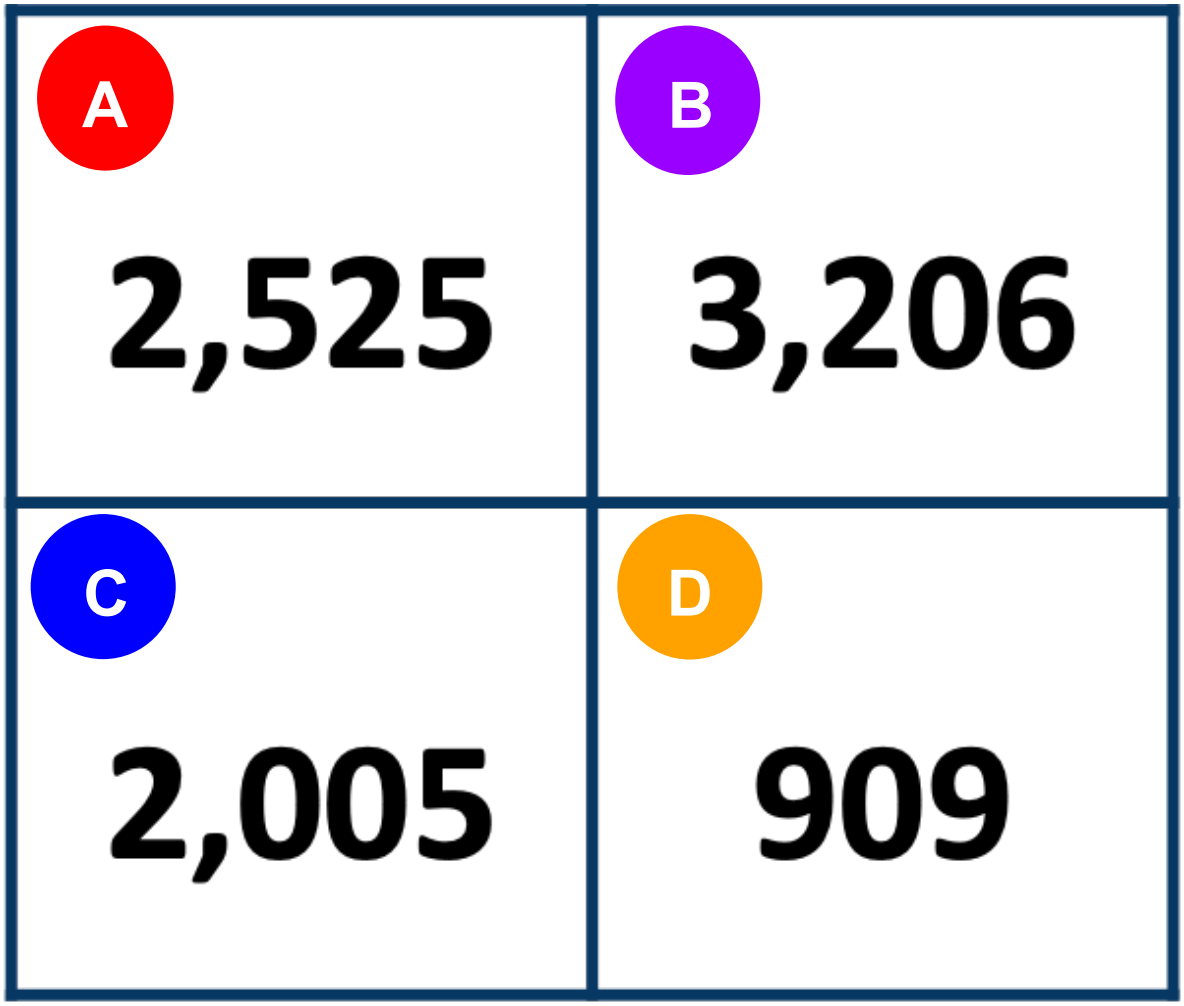
Puzzle 1

Which one doesn't belong? Why?



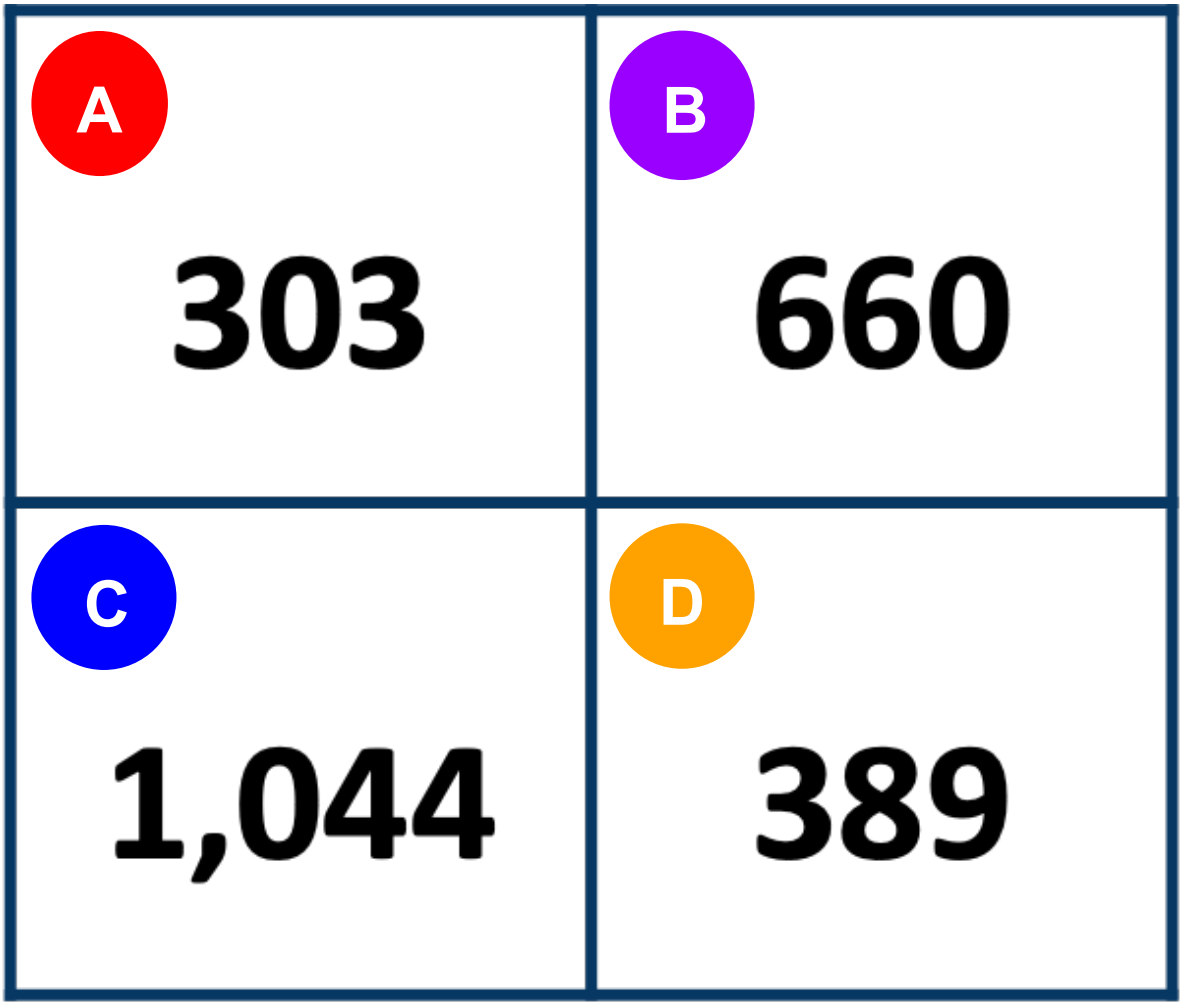
Puzzle 2

Which one doesn't belong? Why?



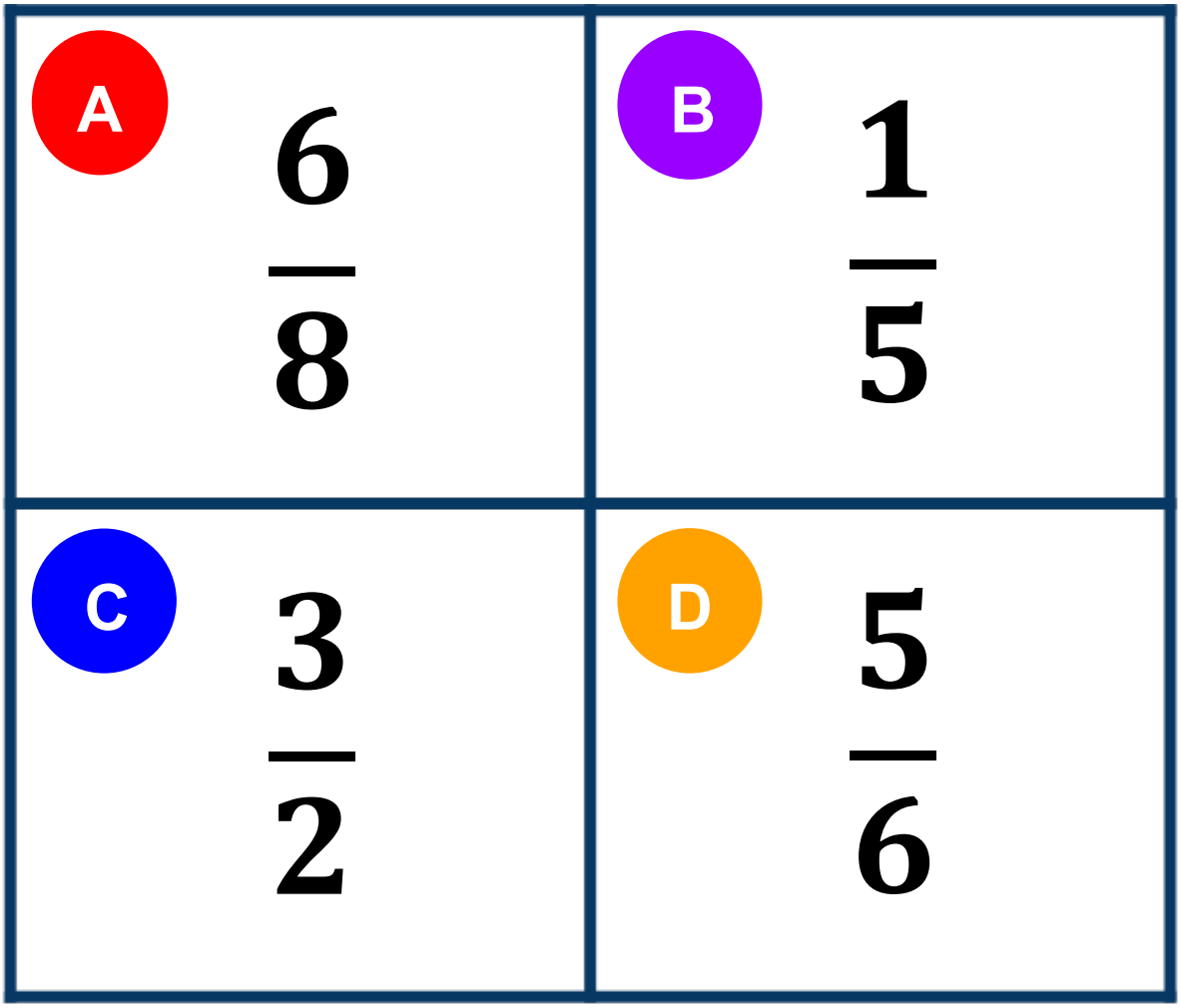
Puzzle 3

Which one doesn't belong? Why?



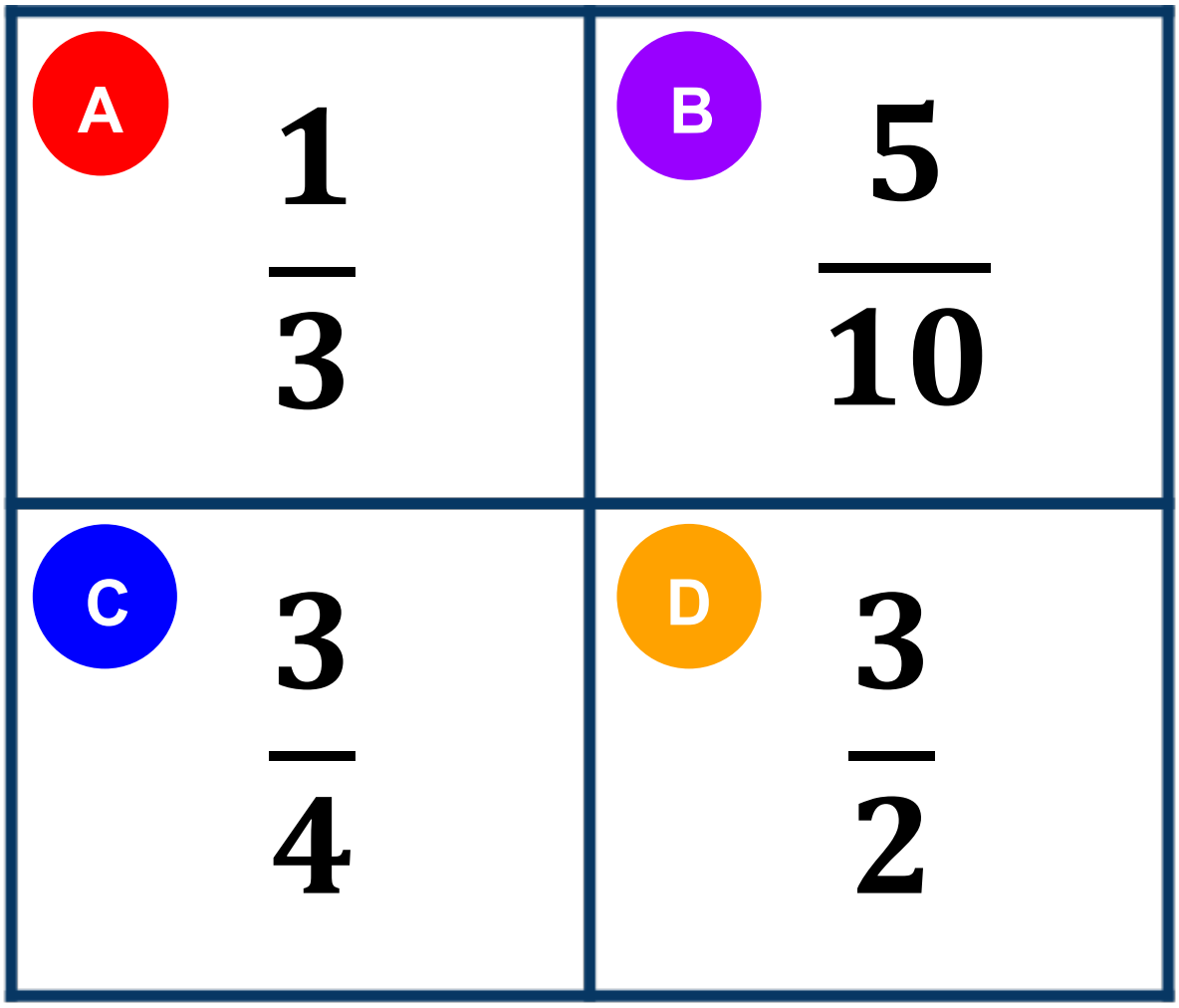
Puzzle 4

Which one doesn't belong? Why?



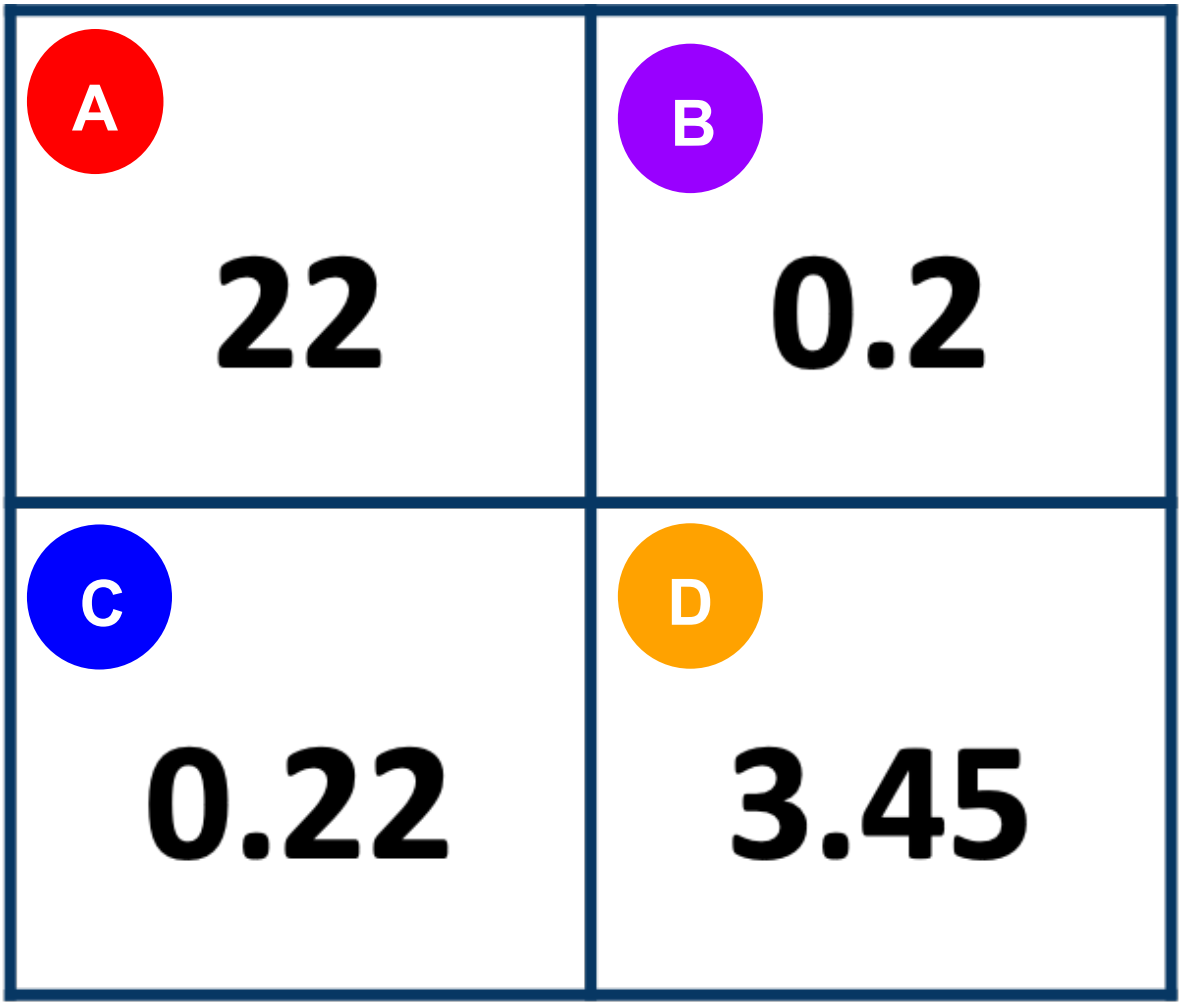
Puzzle 5

Which one doesn't belong? Why?



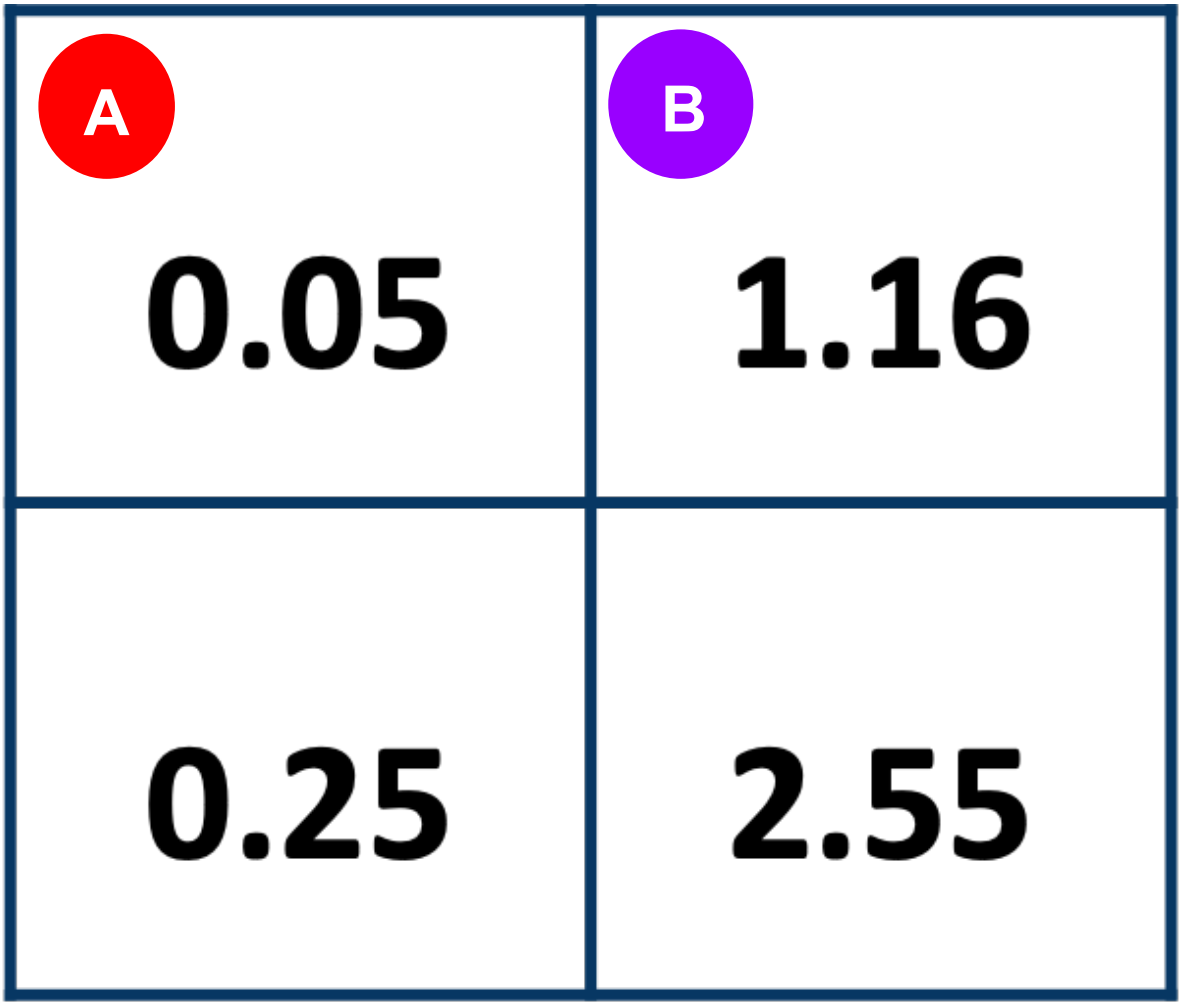
Puzzle 6

Which one doesn't belong? Why?



Puzzle 7

Which one doesn't belong? Why?



Vocabulary Charts: Large Versions

Problems 1–3. Vocabulary Chart

Ones	Tens
Hundreds	Thousands
Place	Value
Digit	Number
Least	Greatest

Problem 4. Vocabulary Chart

Wholes	Halves
Fifths	Sixths
Eighths	Unit Fraction
Numerator	Denominator
Less than	Greater than
Least	Greatest

Problem 5. Vocabulary Chart

Wholes	Halves
Thirds	Fourths
Tenths	Unit Fraction
Numerator	Denominator
Less than	Greater than
Least	Greatest

Problem 6. Vocabulary Chart

Ones	Tens
Tenths	Hundredths
Decimal Point	Digit
Place	Value
Less than	Greater than
Least	Greatest

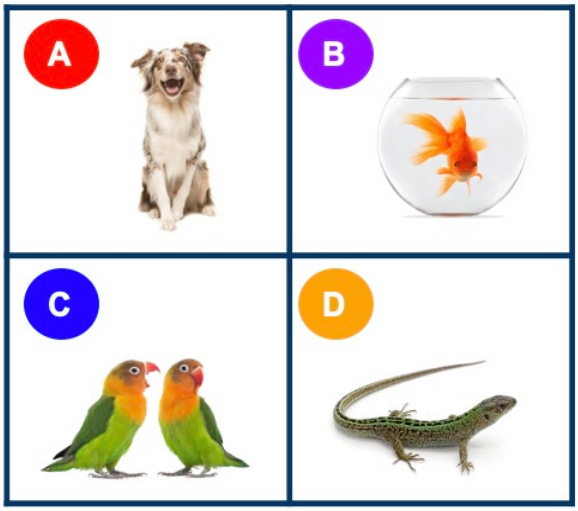
Problem 7. Vocabulary Chart

Ones	Tenths
Hundredths	Place
Decimal Point	Digit
Less than	Greater than
Least	Greatest

Optional: Introductory Puzzles with Non-Mathematics Topics

If students are new to *Which One Doesn't Belong?* puzzles, you may want to start with a non-mathematics puzzle. Use Steps 2 and 3 of the routine with the non-mathematics puzzle. Then, use a mathematics puzzle to do Steps 2–5. This handout has two introductory puzzles for the themes of pets and transportation.

Introductory Puzzle 1. Pets

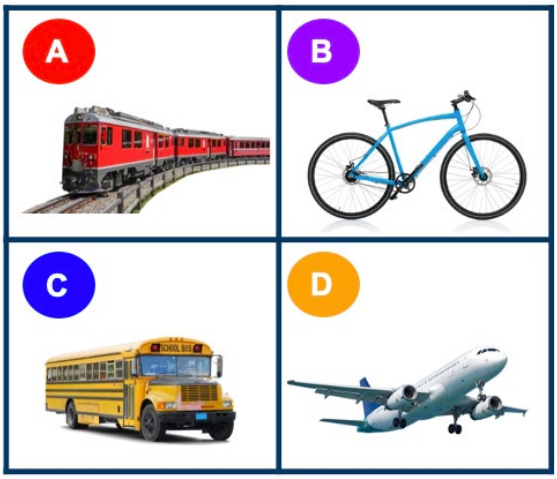


Sample Responses

This table has examples of possible responses. It is not an exhaustive list.

Choices	Which one doesn't belong? How is it different from the others?
A. Dog	• The only animal that has ears that you can see. • The only animal that barks. • The largest animal in size.
B. Goldfish	• The only animal that needs to live in water. • The only animal that has fins (no legs). • The only animal with gills.
C. Parrot	• The only image with two animals. • The only animal with feathers. • The only animal with two legs.
D. Lizard	• The only animal that is a reptile. • The only animal that can change the color of its skin. • The only animal that can shed its tail if it is in danger.

Introductory Puzzle 2. Transportation



Sample Responses

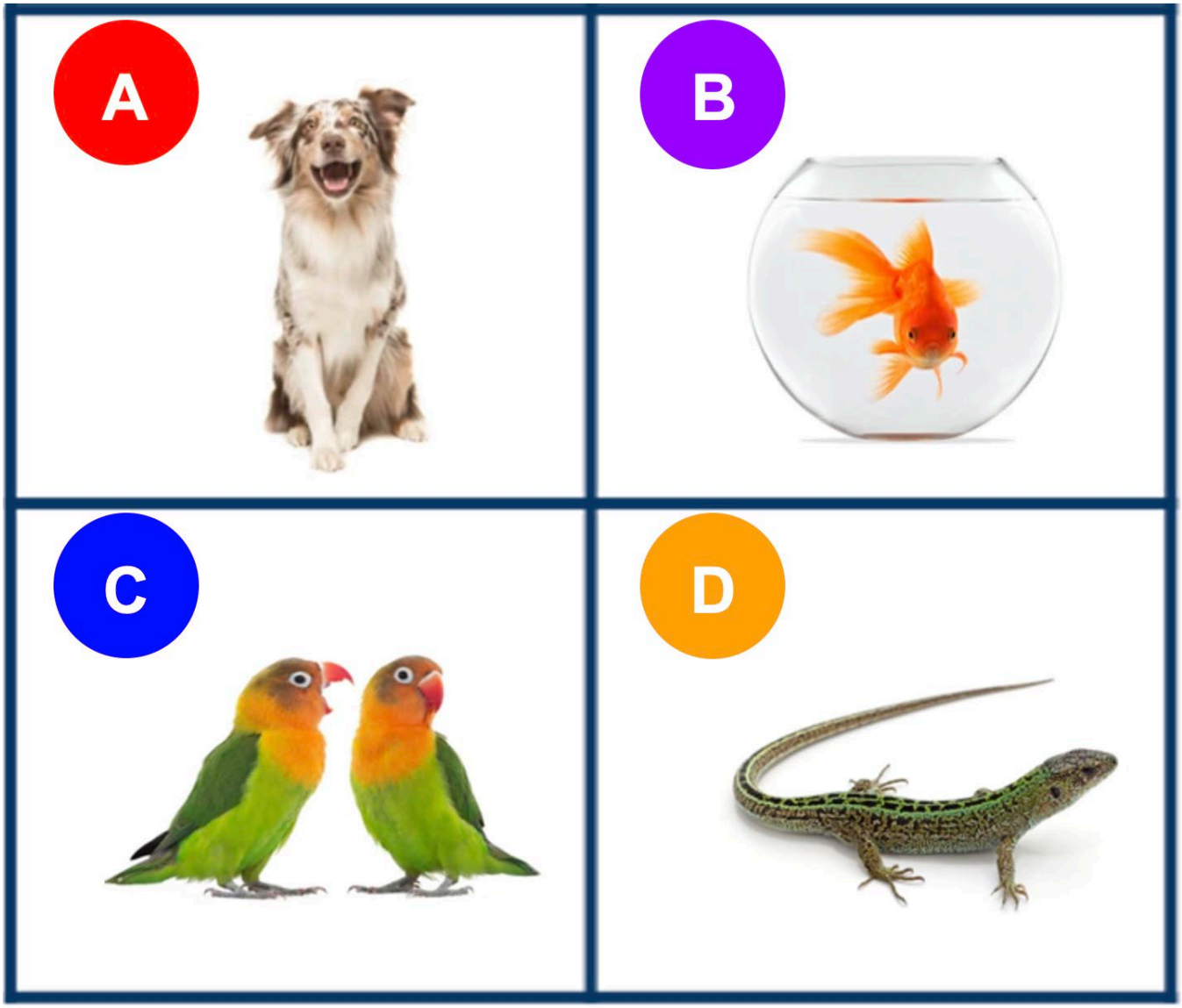
This table has examples of possible responses. It is not an exhaustive list.

Choices	Which one doesn't belong? How is it different from the others?
A. Train	• The only vehicle that is red. • The only vehicle that has cars that connect to each other. • The only vehicle that rides on tracks.
B. Bicycle	• The only vehicle with exactly two wheels. • The only vehicle with pedals. • The only vehicle designed for one person.
C. School Bus	• The only vehicle that has a stop sign attached to it. • The only vehicle that has the words *School Bus* on it.
D. Airplane	• The only vehicle that flies. • The only vehicle that has wings. • The only vehicle in which you can take off and land.

Large versions of the introductory puzzles are on the next two pages.

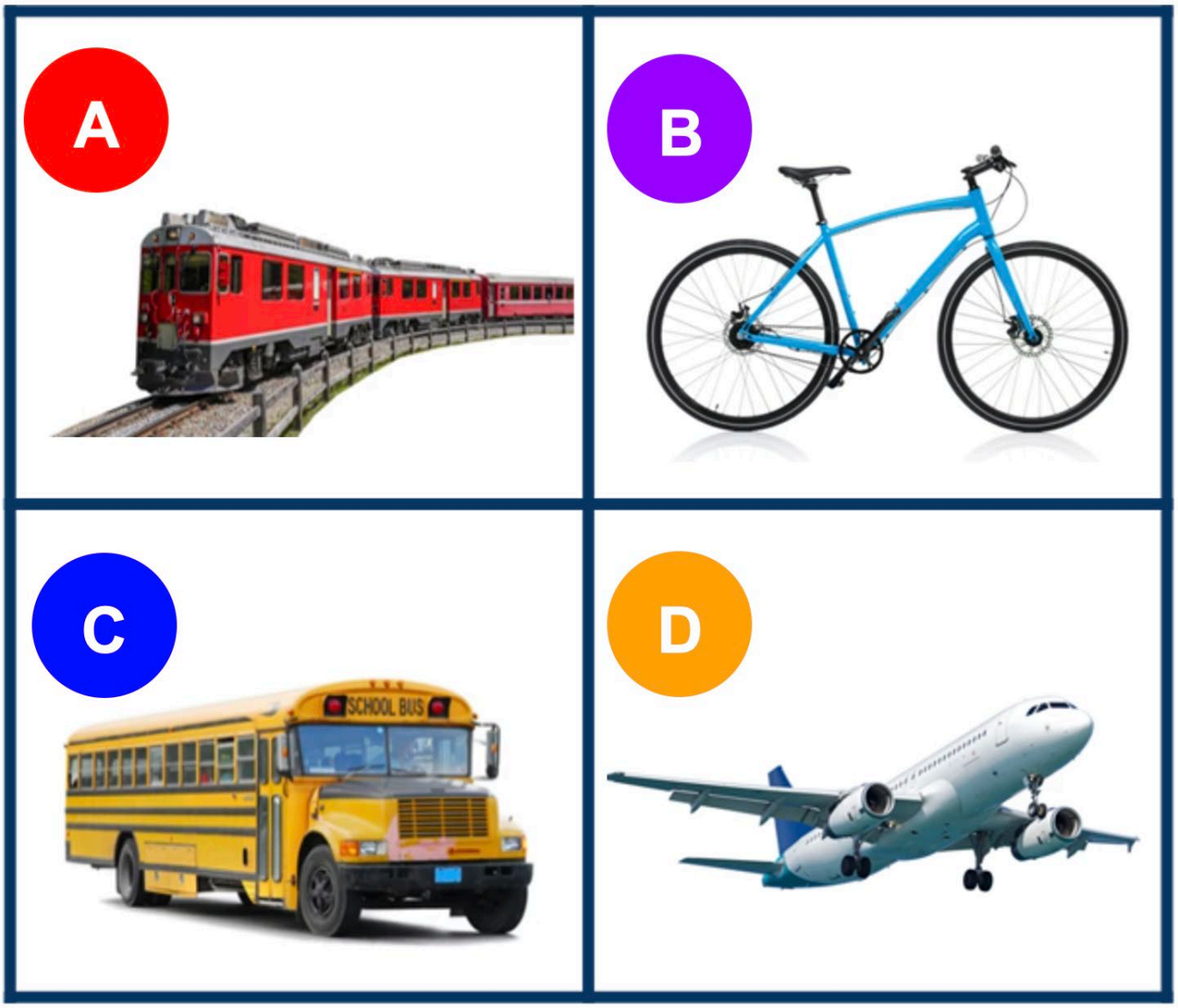
Introductory Puzzle 1

Which one doesn't belong? Why?



Introductory Puzzle 2

Which one doesn't belong? Why?



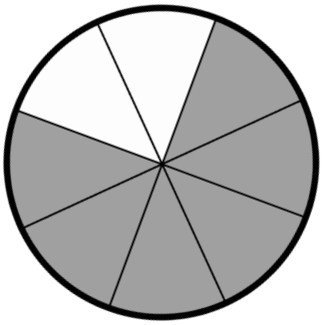
Appendix B: Resources for Card Sorting Activity

Use these resources for the card sorting activity. This activity focuses on the mathematical terms *unit fraction* and *non-unit fraction*.

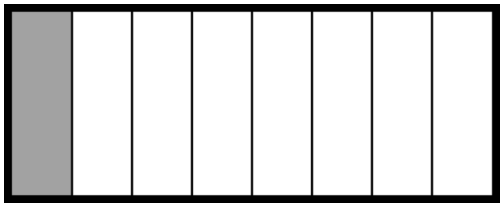
Fraction Cards: Set A

Card Set A has examples of unit fractions and non-unit fractions. It includes 12 cards with these representations: fraction circles, fraction rectangles/bars, number lines, and numbers. You can choose to focus on specific representations and select a subset of cards to use.

A.



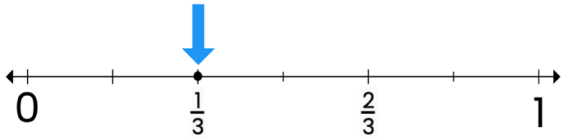
B.



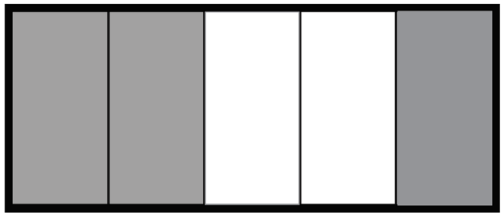
C.

$$\frac{1}{12}$$

D.



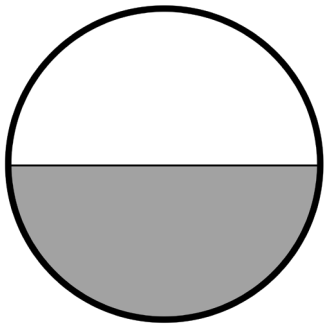
E.



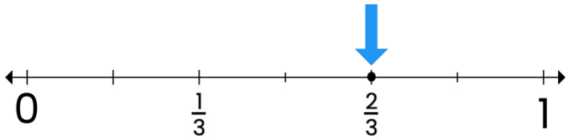
F.

$$\frac{11}{12}$$

G.

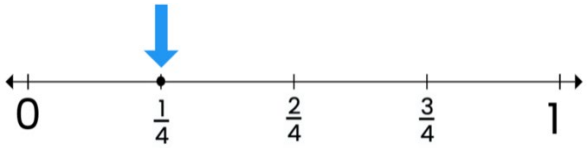


H.



I.

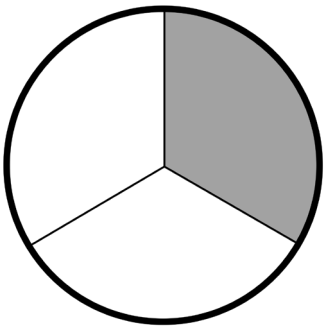
J.



K.

$$\frac{5}{4}$$

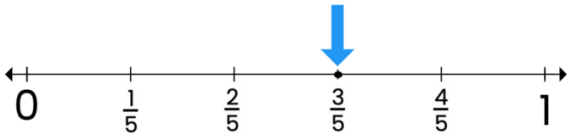
L.



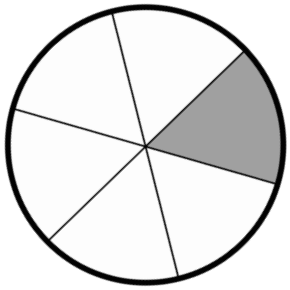
Fraction Cards: Set B

Card Set B also has examples of unit fractions and non-unit fractions. It includes 12 cards with these representations: fraction circles, fraction rectangles/bars, number lines, and numbers. You can use this card set on its own or combine it with Card Set A.

M.



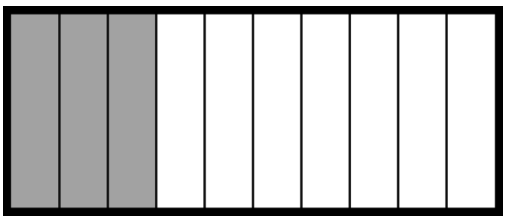
N.



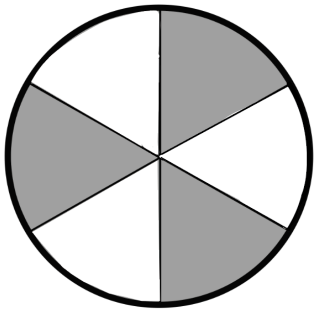
O.

$$\frac{1}{3}$$

P.



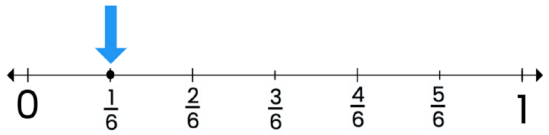
Q.



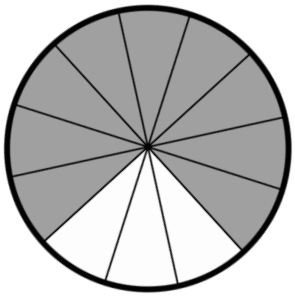
R.

$$\frac{2}{5}$$

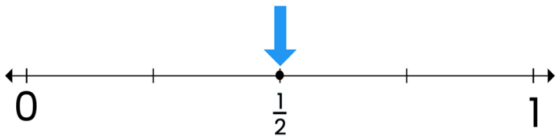
S.



T.



U.

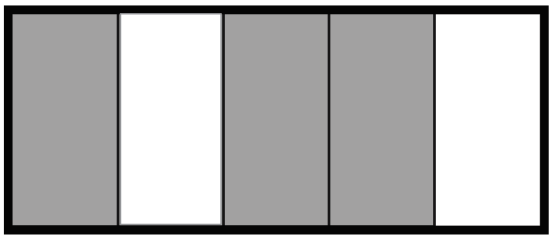


V.

W.

$$\frac{7}{6}$$

X.



Category Cards

Print and cut out one set of cards for each pair. There are five sets on the handout. Another option is to write the categories on index cards.

Unit Fractions	**Non-Unit Fractions**
Unit Fractions	**Non-Unit Fractions**
Unit Fractions	**Non-Unit Fractions**
Unit Fractions	**Non-Unit Fractions**
Unit Fractions	**Non-Unit Fractions**

Sentence Frames

The fraction ___ is a unit fraction because…

The fraction ___ is a non-unit fraction because…

I agree because...

I disagree because…

The fraction ___ is a unit fraction because…

The fraction ___ is a non-unit fraction because…

I agree because...

I disagree because…

The fraction ___ is a unit fraction because…

The fraction ___ is a non-unit fraction because…

I agree because...

I disagree because…

Exit Task

Name:_____ **Date:_____**

Write a fraction in each blank. Explain your reasons by completing these sentence frames.

1. **The fraction ___ is a unit fraction because…**

2. **The fraction ___ is a non-unit fraction because…**

Name:_____ **Date:_____**

Write a fraction in each blank. Explain your reasons by completing these sentence frames.

1. **The fraction ___ is a unit fraction because…**

2. **The fraction ___ is a non-unit fraction because…**

Answer Key: Card Sets A and B

Card Set A

Unit Fractions (7 cards)	Non-Unit Fractions (5 cards)
B	A
C	E
D	F
G	H
I	K
J	
L	

Card Set B

Unit Fractions (5 cards)	Non-Unit Fractions (7 cards)
N	M
O	P
S	Q
U	R
V	T
	W
	X

Appendix C: Answer Key for Handout H4

H4. Vocabulary Strategy: Use Graphic Organizers

1. Complete these graphic organizers

Answers will vary. Examples are shown in brackets and dark orange font.

a. Add more characteristics, examples, and non-examples for the term *polygon*.

Polygon

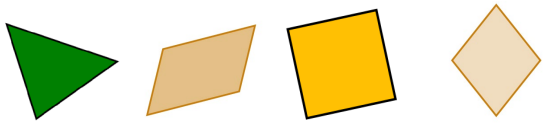
Definition

A polygon is a 2-Dimensional (2D) shape that is closed and has only straight sides.

Characteristics

[Example responses:
- Sides are straight.
- Sides do not cross.
- It is not round.
- It is a closed figure.
- It is 2-dimensional.]

Examples



Non-Examples

b. Complete this graphic organizer for the term *factor*.

Factor

Definition

Two numbers that are multiplied to get a product are called **factors**.

Characteristics

[Example responses:
- Divides a number evenly with no remainders.
- Every number, besides 0 and 1, has at least two factors—1 and the number itself (e.g., The factors of 7 are 1 and 7).]

Examples

2 x 6 = 12

2 and 6 are **factors** of 12.

- The factors of 12 are 1, 2, 3, 4, 6, and 12.
- The factors of 20 are 1, 2, 4, 5, 10, and 20.

Non-examples

[Example responses:
- 5 is **not** a factor of 12, because there will be a remainder.
- 24 is **not** a factor of 12. It is a multiple of 12.
- Fractions and decimals are not factors. Factors are whole numbers.]

2. Create your own graphic organizers by using this template.

Focus on a mathematical term that your students find confusing.

[Answers will vary.]

3. Write down ideas for ways to use these graphic organizers.

a. How might you use these graphic organizers with your students? How would you integrate a graphic organizer into a math lesson? What mathematical terms would you use?

[Answers will vary. Example responses provided below.

- It would be helpful to create a larger version of the graphic organizer template together as a class and have students take an active role in its creation by writing in the boxes.
- I also think it is useful for students to have their own copy of the completed graphic organizers stored in one place, like a notebook, where students can reference the graphic organizers that we make together.
- Mathematical terms that I am considering using focus on the hierarchy of 2-D shapes, such as *quadrilateral*, *parallelogram*, *rhombus*, and *trapezoid*. I am also thinking about using them for *factor* and *multiple*.]

b. What questions would you ask students if they don't know what to put in a category?

[Answers will vary. Example responses provided below.

- How can you explain _____ in your own words?
- What do you know about _____?
- Describe an example of what _____ is not.
- Draw or describe an example of _____.