Institute of Education Sciences

Regional Educational Laboratory Northeast & Islands

at Education Development Center

Mathematics Intervention Toolkit: Representations Module

Participant Workbook

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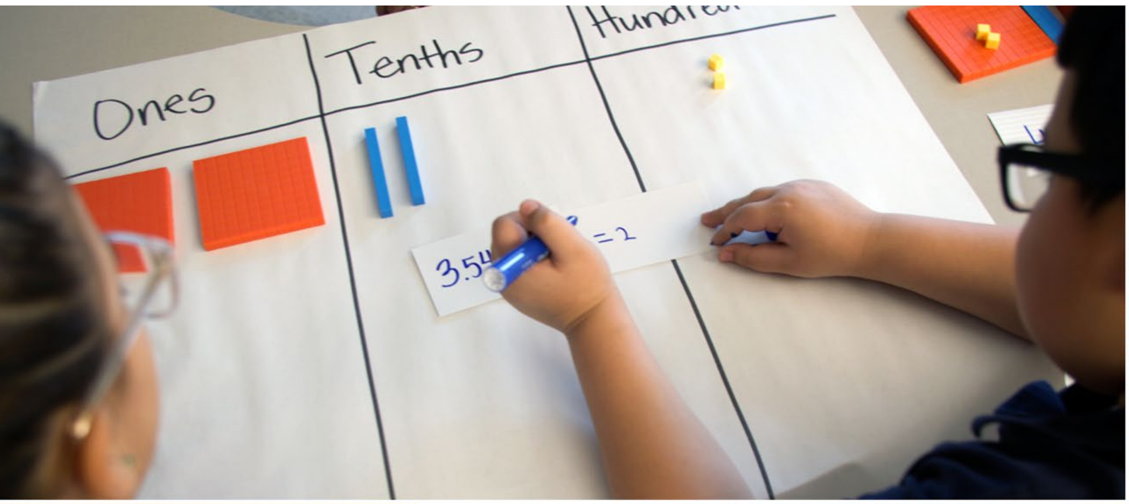
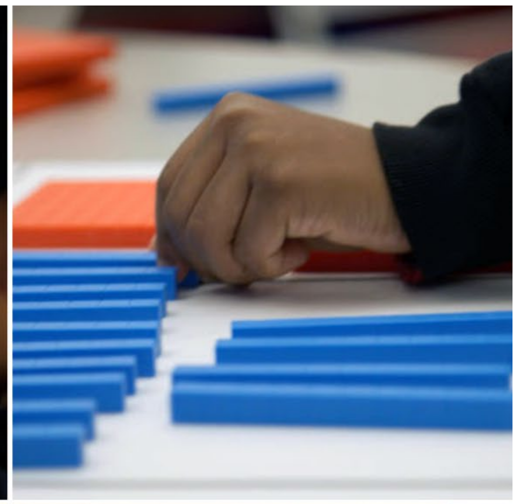
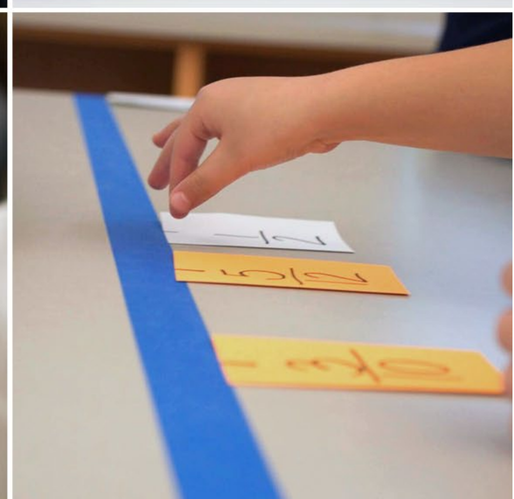






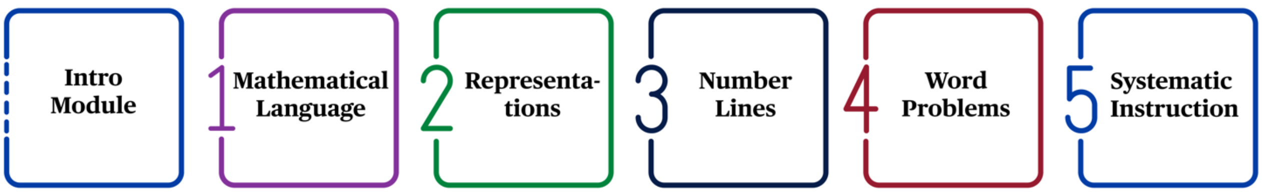
Contents

Introduction to the Course

Welcome! This professional development (PD) course is designed to build participants' knowledge and practices for supporting students struggling with mathematics. It focuses on the evidence-based recommendations of the What Works Clearinghouse Practice Guide *Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades* [1] (WWC Guide). These recommendations are based on a rigorous review and synthesis of research studies of effective intervention practices. The course is designed to connect this important research to participants' classroom practice.

The course has a series of **modules** to support in-depth professional learning. It starts with an Introductory Module and continues with five modules, each focusing on one recommendation (figure 1). The current module, Module 2, provides a deep dive into the **representations**. Recommendation.

Figure 1. Course Sequence



The course is specifically designed for **teachers of mathematics intervention in grades 3–6**. This includes teachers with different roles, such as interventionists, Title I teachers, math specialists, general educators, and special educators. Participants will be able to apply the strategies in a variety of intervention settings, including separate intervention classes, intervention/enrichment blocks, and designated times for intervention during core mathematics classes. Similarly, the course will support participants who use a variety of intervention programs/curricula or who do not have a program.

The full course is intended to provide **28 hours** of professional learning during one or two school years. It uses a **hybrid format** that combines online learning, Professional Learning Community (PLC) sessions, and opportunities for classroom implementation. The course focuses on key **Number and Operations** topics, such as **fractions**, that are a high priority for mathematics intervention.

[1] Fuchs, L. S., Newman-Gonchar, R., Schumacher, R., Dougherty, B., Bucka, N., Karp, K. S., Woodward, J., Clarke, B., Jordan, N. C., Gersten, R., Jayanthi, M., Keating, B., and Morgan, S. (2021). *Assisting students struggling with mathematics: intervention in the elementary grades* (WWC 2021006). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. Retrieved from http://whatworks.ed.gov/.

Module Overview

This module focuses on the recommendation for representations from the WWC Practice Guide *Supporting Students Struggling with Mathematics: Intervention in the Upper Elementary Grades*. You will explore evidence-based strategies for using representations effectively with students in mathematics intervention. Supporting students' use of representations (concrete, semi-concrete, and abstract) helps build their understanding of important mathematical concepts and processes.

Recommendation for Representations

Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.

Implementation Steps

1. Provide students with the concrete and semi-concrete representations that effectively represent the concept or procedure being covered.
2. When teaching concepts and procedures, connect concrete and semi-concrete representations to abstract representations.
3. Provide ample and meaningful opportunities for students to use representations to help solidify the use of representations as "thinking tools."
4. Revisit concrete and semi-concrete representations periodically to reinforce and deepen understanding of mathematical ideas.

WWC Guide, pp. 21-28

Professional Learning Goals

Participants will:

- Build knowledge of the recommendation for using representations to support students' learning of mathematics concepts and procedures.
- Build knowledge of evidence-based strategies and how to implement them effectively with students who struggle with mathematics.
- Strengthen your ability to plan for and implement the strategies with your students.

Key Questions

Participants will explore these questions:

1. **What** is the WWC Guide's recommendation for representations?
2. **Why** is understanding multiple representations important for student learning?
3. What are **strategies** for **how to** implement the recommendation?
4. What are **ways to apply** the recommended strategies with your students?
5. What are **potential challenges** and ways to address them?

Mathematics Content Focus

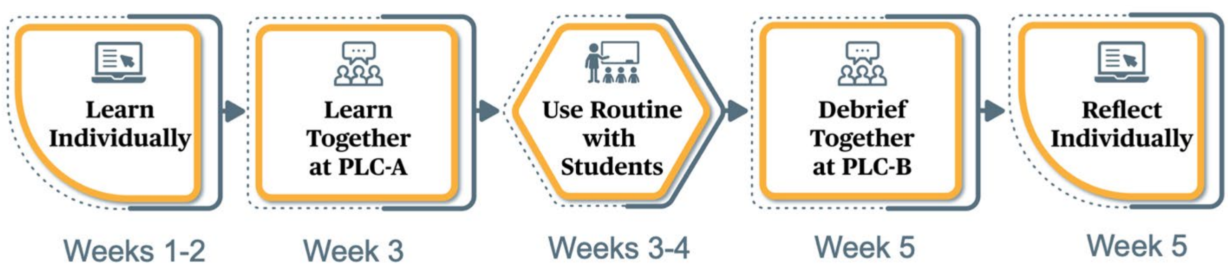
The module focuses on evidence-based strategies for representations that are applicable across mathematics content areas. It provides examples and activities for using representations to build students' understanding of high-priority fraction and decimal topics, including:

- Representing and comparing fractions.
- Solving fraction addition and subtraction problems.
- Representing and comparing decimals.
- Solving decimal addition and subtraction problems.

Module Sequence

This 5-week module provides 6 hours of professional learning, including an Online Session, two PLC sessions, and opportunities to apply strategies with students (figure 2).

Figure 2. Sequence of activities in the module.



The sequence begins with learning individually about the recommendation in the Online Session by doing self-paced, asynchronous activities, such as videos and readings. This session will prepare you for participating in the PLC sessions. At PLC Session-A, you and your colleagues will discuss the recommendation, try strategies, and prepare to use an instructional routine. After the session, you will use the routine with students one or more times. At PLC Session-B, you will share experiences with the routine and discuss ways to strengthen strategies. The module closes with the opportunity to reflect individually.

Resources

The Representations Module includes the following resources for participants:

- **Online Component**: Provides activities and resources (see next section).
- **Participant Workbook**: Provides all the handouts for the module. It also includes the Routine Teaching Guide, which has resources for using the routine. The Appendices have reproducible student handouts and other resources.
- **Classroom Video Example**: The video titled *Instructional Routine: Is it True?* shows an intervention teacher and students using the routine to represent and compare fractions.
- **Video Demonstrations of Strategies:** In the Online Component, these two videos show how to use and connect concrete and numeric representations to model decimal addition and subtraction:
    - Connecting Representations: Strategies for Decimal Addition
    - Connecting Representations: Strategies for Decimal Subtraction
- **Debriefing Slides Template**: Participants will use this file to create several slides about their experiences teaching the routine. The same template is used for Modules 1–4.
- **Optional: Slides for Teaching the Routine**: These slides are an optional resource that participants can choose to use when they teach the routine to students. Alternatives are to present the information by using chart paper, whiteboards, or other approaches.
- **Glossary**: Provides relevant vocabulary terms with definitions for the course.

The **Mathematics Intervention Toolkit,** which includes all the course resources, is available for free at https://ies.ed.gov/ncee/rel/math-support-grades-3-6.

Online Component

The module's Online Component provides resources and activities. It is organized by tabs (figure 3).

Figure 3: Tab Menu

Description of tabs

- **Tabs 1–3, Online Session**: Provide self-paced, asynchronous activities to do independently. The Intro tab provides an overview of the module. On the Explore-A and Explore-B tabs, participants learn about the recommendation through videos, readings, and activities.
- **Tab 4. PLC Session-A**: Provides resources to use during or after the session.
- **Tab 5. Try It! Routine**: Provides information and resources to help participants prepare for and use the routine with students.
- **Tab 6. PLC Session-B**: Provides resources to use during or after the session.
- **Tab 7. Wrap-Up**: Provides short reflection activities.
- **Resources:** Provides a hyperlinked list of module resources and additional resources.

Mathematics Intervention Course Checklist

Use this checklist to keep track of your progress in the course.

	Introductory Module	Dates
☐	Kick-Off Session	
☐	Wrap-Up (Complete tab 4 of Online Component)	
	Module 1. Mathematical Language	
☐	Online Session (Complete tabs 1–3 of Online Component)	
☐	PLC Session-A	
☐	Try It!: Use Routine with Students	
☐	PLC Session-B	
☐	Wrap-Up (Complete tab 7 of Online Component)	
	Module 2. Representations	
☐	Online Session (Complete tabs 1–3 of Online Component)	
☐	PLC Session-A	
☐	Try It!: Use Routine with Students	
☐	PLC Session-B	
☐	Wrap-Up (Complete tab 7 of Online Component)	
	Module 3. Number Lines	
☐	Online Session (Complete tabs 1–3 of Online Component)	
☐	PLC Session-A	
☐	Try It!: Use Routine with Students	
☐	PLC Session-B	
☐	Wrap-Up (Complete tab 7 of Online Component)	
	Module 4. Word Problems	
☐	Online Session (Complete tabs 1–3 of Online Component)	
☐	PLC Session-A	
☐	Try It!: Use Routine with Students	
☐	PLC Session-B	
☐	Wrap-Up (Complete tab 7 of Online Component)	
	Module 5. Systematic Instruction	
☐	PLC Session	
☐	Course Wrap-Up (Complete tab 3 of Online Component)	

Website URL: https://ies.ed.gov/ncee/rel/math-support-grades-3-6.

Module Checklist: Representations

Use this checklist to keep track of the module dates and your progress on the tasks.

Module Dates:____________ to ____________

- **Online Session**: Complete during this time span: ____________ to ____________
- **PLC Session-A**: Attend session on date: ____________ time: ____________
- **Try It! Routine**: Implement during this time span: ____________ to ____________
- **PLC Session-B**: Attend session on date: ____________ time: ____________
- **Wrap-Up**: Complete by date: ____________

Tasks

1-3. Online Session.

Complete the activities *before* PLC Session-A.

☐ Tab 1, Introduction. Read about the module's goals, key questions, and sequence.

☐ Tab 2, Explore-A Tab. Do online activities to learn about the recommendation.

☐ Tab 3, Explore-B Tab. Do more online activities to continue learning.

4. PLC Session-A.

☐ Participate in the PLC session: discuss the recommendation, try strategies, and prepare to use an instructional routine.

5. Try It! Routine.

Implement the routine *before* PLC Session-B.

☐ Use the routine one or more times with students.

☐ Prepare slides for sharing experiences. Use the Debriefing Slides Template.

6. PLC Session-B.

☐ Participate in the session: Share experiences using the routine by showing slides and focusing on the debriefing questions. Discuss common themes and plan next steps.

7. Wrap-Up.

Complete the reflection activities *after* PLC Session-B.

☐ Do the closing activities on Tab 7, Wrap-Up, to reflect on your learning in the full module.

Website URL: https://ies.ed.gov/ncee/rel/math-support-grades-3-6.

Handouts

Online Session Handouts

PLC Session-A Handouts

PLC Session-B Handouts

Wrap-Up Handouts

H1. Reference Sheet: Representations Recommendation

Directions: During the module, record notes below to create a helpful resource for future use.

1. What is the WWC Guide's recommendation?[2]

Recommendation for Representations

Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.

2. Why is understanding representations important for student learning?

Add your ideas to the list.

- Understanding and using concrete, semi-concrete, and abstract representations help students:
  - Build understanding of mathematical concepts and processes.
  - Make sense of and solve problem situations.
  - Show and explain their mathematical ideas and approaches.

3. How do you carry out the recommendation? What are the implementation steps?

The WWC Guide provides four main implementation steps (below) that involve the use of evidence-based strategies (see next page).

Main Implementation Steps

1. Provide students with the concrete and semi-concrete representations that effectively represent the concept or procedure being covered.
2. When teaching concepts and procedures, connect concrete and semi-concrete representations to abstract representations.
3. Provide ample and meaningful opportunities for students to use representations to help solidify the use of representations as "thinking tools."
4. Revisit concrete and semi-concrete representations periodically to reinforce and deepen understanding of mathematical ideas.

[2] The text for the representations recommendation and implementation steps are from the WWC Guide, pp. 21–28.

4. What are strategies for how to implement the representations recommendation with students struggling with mathematics?

As you go through the module, use this chart to list strategies you want to remember and apply.

Recommended Strategies
Carefully select representations. • When teaching mathematics concepts like fractions, use manipulatives that are proportional, so that the pieces accurately represent the relative sizes of different fractions.
Connect representations. • Ask guiding questions to help students see how concrete and semi-concrete representations connect to abstract representations, and how they represent a concept or process.
Use representations as thinking tools. • Model ways to use representations to think about ways to solve problems and have students share their thinking and approaches.
Revisit representations. • Have concrete representations available for students to use as needed; for example, have bins of manipulatives that students can choose to use.

H2. How to Carry Out the Recommendation

Recommendation for Representations

Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.

Read this excerpt from the WWC Practice Guide to learn about the recommendation and how to use concrete, semi-concrete, and abstract representations effectively (see definitions below). The guide describes four implementation steps; each involves the use of evidence-based strategies to support students struggling with mathematics.

What are concrete, semi-concrete, and abstract representations?

Representations illustrate the value of numbers and the relationship between quantities. Concrete and semi-concrete representations are powerful ways to make mathematics visible and more accessible for students. Creating visual models with concrete or semi-concrete representations may help students think through and solve problems more successfully because they help students understand the logic behind mathematical concepts and procedures.

- **Concrete representations** are three-dimensional (3D), physical materials or actions that students can organize, act upon, or manipulate to better understand mathematics content (e.g., regrouping with base 10 blocks, using fraction tiles to compare two fractions, role playing a problem situation). Concrete representations may help students better understand mathematical concepts when they physically model with the manipulatives or act out problem scenarios to make the underlying mathematical concepts visible and less challenging to interpret. For example, by counting out beans or connecting cubes, students learn one-to-one correspondence.
- **Semi-concrete representations** are two-dimensional (2D) visual depictions such as strip diagrams, simple drawings, tables, arrays, graphs, and number lines that may help students organize information. They can be used in conjunction with concrete representations to transition to more abstract representations. For example, students who struggle with addition, subtraction, multiplication, or division may find it useful to represent quantities with tick marks or by drawing dots. Because the number line is an essential semi-concrete representation in many interventions and standards, we devote a separate recommendation to using the number line effectively.
- **Abstract representations** are mathematical notations that can include numbers, equations, operations, relational symbols, and expressions (such as 4, 16, multiplication and equal signs, greater than or less than signs, as well as equations such as 4 x 4 = 16).

Source: Excerpt from *WWC Practice Guide*, pp. 29–39, with adaptations from the Toolkit authors.

Implementation Step 1. Carefully Select Representations

> Provide students with the concrete and semi-concrete representations that effectively represent the concept or procedure being covered.

Not all representations work for every mathematical concept. Choosing representations must be intentional and mathematically appropriate to be effective. It is critical to provide students with the representations that most accurately model the concept or procedure being addressed. Table 1 provides some examples of representations that work well for a sample of concepts and procedures.

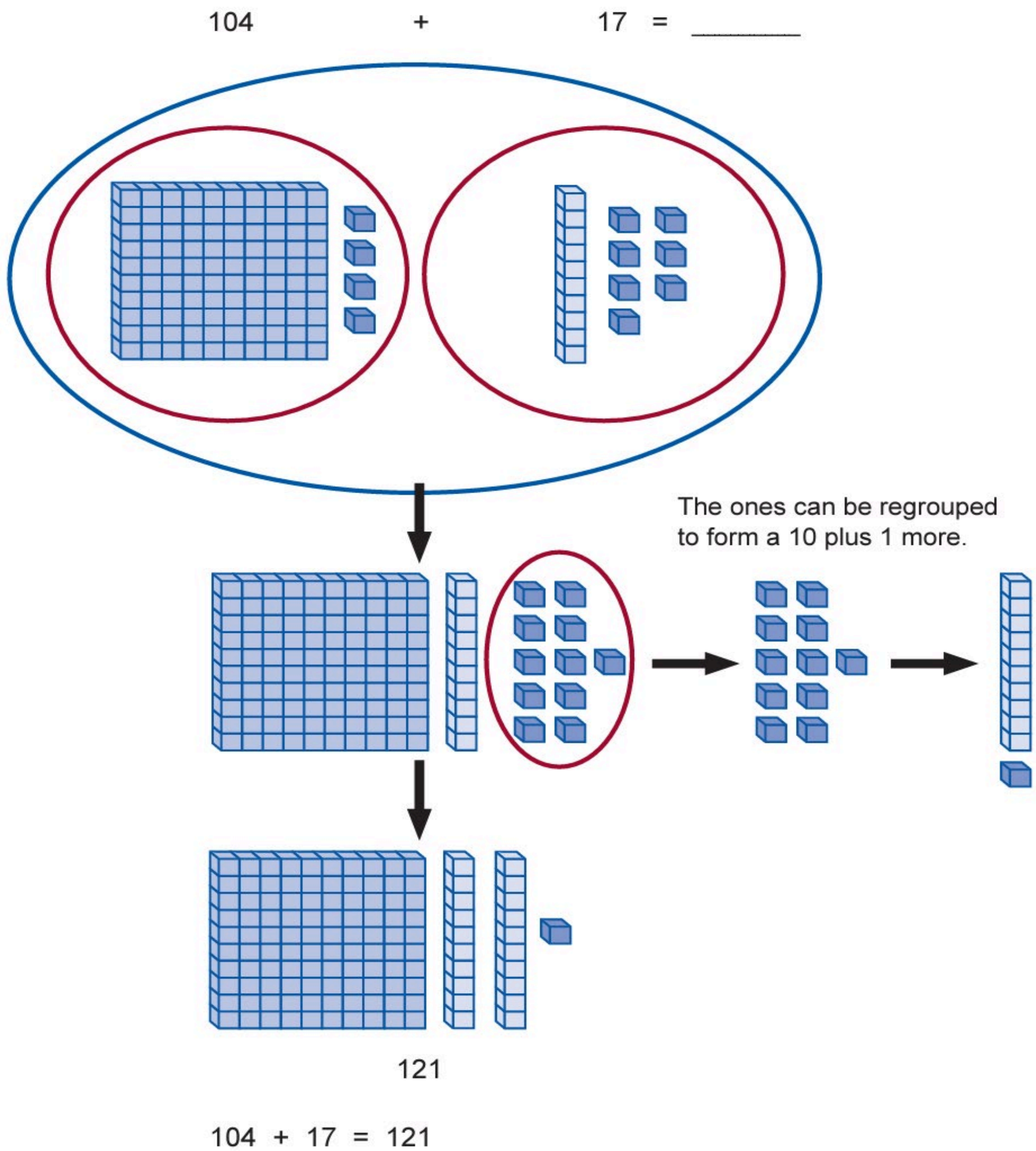
Table 1. Examples of common concrete and semi-concrete representations

Mathematics concepts and procedures	Concrete	Semi-concrete
Counting/skip counting Addition Subtraction Multiplication Division Equality	• base ten blocks • connecting cubes • Cuisenaire® rods • beads • two-colored counters • beans and cup • 1-inch tiles • Balances	• hundreds chart • five frames, ten frames, double ten frames • strip diagrams • arrays
Place value Decimals/operations with decimals	• base ten blocks • 1-inch tiles • connecting cubes • decimal squares	• place value chart • base ten pictures • hundreds chart • rational number wheels
Fractions/operations with fractions Data Ratios and proportions	• Cuisenaire® rods • connecting cubes • pattern blocks • fraction bars • fraction tiles • fraction circles • two-sided counters • 1-inch tiles	• table • number line • strip diagram • bar graph • line plot
Patterns Geometry Graphing Area/perimeter Volume/capacity Line symmetry Length measurement 2D shapes 3D shapes	• pattern blocks • 1-inch tiles • connecting cubes • rulers, tape measures • geometric solids • protractors, angle rulers • containers	• pictures of shapes • graph paper grids • number line • isometric paper

Note: This table is not a complete listing of all representations, nor does it list all matching mathematical concepts and procedures.

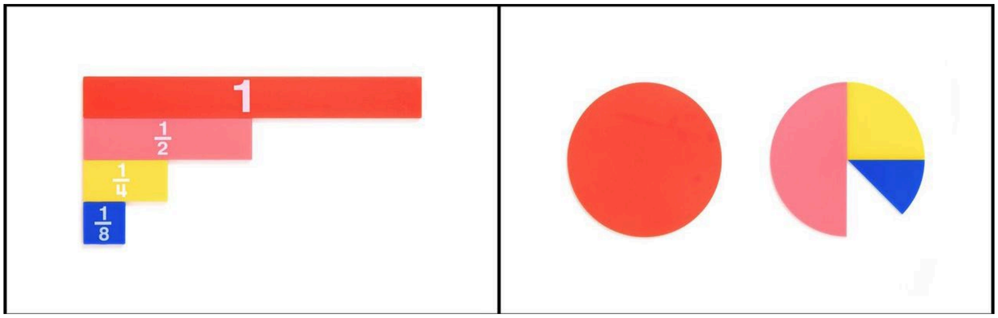
When appropriate, use representations that are **proportional**. For example, when teaching place value, the representation for ones should be one-tenth the size of the representation for tens, and the representation for tens should be one-tenth the size of the representation for hundreds. In example 1, the problem is depicted with base ten blocks, which represent the place value for ones, tens, and hundreds proportionally. Notice how the representations for place value depict regrouping accurately. The single-unit cubes, when grouped into a set of ten, match the size, shape, and length of the tens unit. Ten of the tens units match the size and shape of the hundreds unit. Representations that are proportional enhance students' understanding of the concept of place value.

Example 1. Teacher represents the addition problem with base ten blocks, which are proportional for showing place value and regrouping concepts.



Using proportional manipulatives is also important when representing fractions concepts and operations (example 2).

Example 2. Fraction bar and circle manipulatives have proportional pieces.



Source: Example from Toolkit authors.

Implementation Step 2. Connect Representations

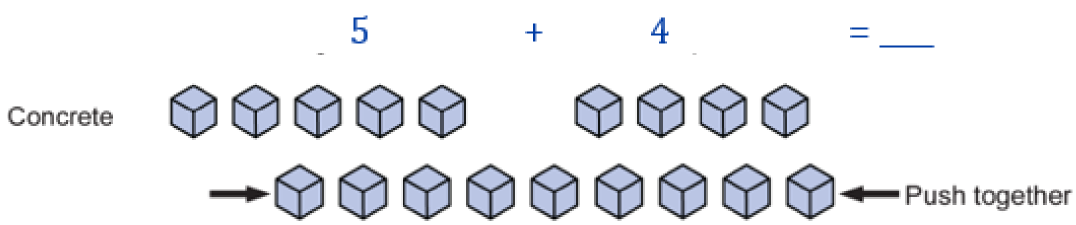
> When teaching concepts and procedures, connect concrete and semi-concrete representations to abstract representations.

Although in some circumstances choosing either a concrete or semi-concrete representation may be appropriate for teaching and representing a particular concept, most concepts and procedures can be effectively represented by connecting both a concrete and a semi-concrete representation to the abstract representation (e.g., mathematical notation). When demonstrating concepts and procedures with concrete and semi-concrete representations, present the mathematical notation simultaneously so that students can conceptualize the connection between the representations and the mathematics. It is also important to connect concrete and semi-concrete representations to each other when teaching the same concept or procedure. It is helpful to make these connections when introducing new material and when reviewing previously learned content. It is also important to make these connections when using familiar representations in a new way.

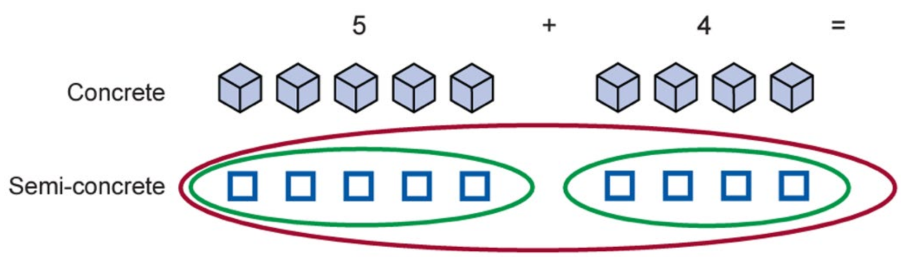
Example 3 demonstrates how the representations can be aligned vertically to demonstrate their connection. When teaching early addition concepts, a teacher may use counters as a concrete representation while also using pictures or sketches as a semi-concrete representation. Linking the counters to the pictorial representations and to the mathematical notation can help students solidify the concept of addition.

Example 3. Teacher shows how combining two groups relates to concrete and semi-concrete representations and to an equation

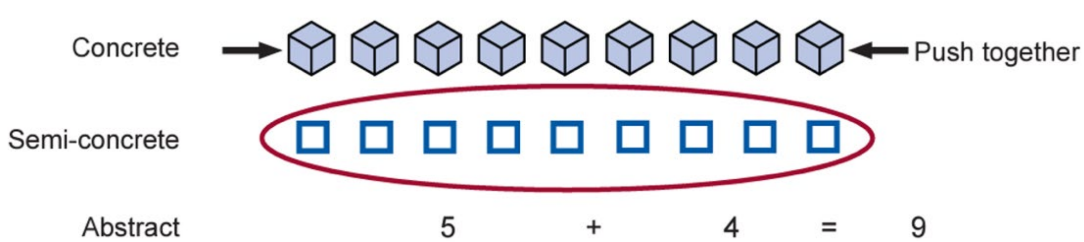
Teacher: When looking at this problem, we see that we need to add or combine the 4 and the 5. I can use counters to count out a group of 4 and another group of 5. Then I can combine them and count how many I have.



Teacher: I can also make a drawing to solve the problem. I will draw five squares to represent the 5 and four squares for the 4. I will draw two green loops around the amounts to show that I have a group of 5 squares and a group of 4 squares. Then, I will draw a large red loop to show that I'm combining all the squares. Then I can count how many squares I drew in all. I find that I drew 9 squares, so the answer to the problem 5 + 4 equals 9.



Here is the solution shown with the three representations stacked vertically. Connecting these representations can help build understanding of addition.

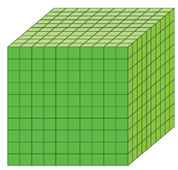
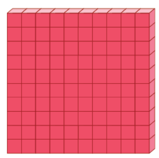
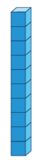


Source: Example adapted by Toolkit authors to provide more information on the visual representation.

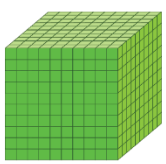
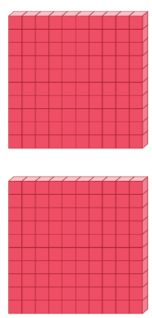
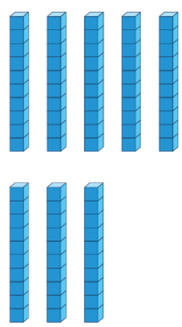
Example 4 depicts a teacher demonstrating how to use a familiar representation in a new way. In this example, upper-elementary students who are familiar with using base ten blocks to represent whole-number operations can use these materials to represent decimal place value. The teacher and students discuss how, by reassigning the value of the blocks, they can also use the blocks to represent decimals. The teacher is helping students conceptualize how this same tool can be used to represent decimal place value. This representation is still proportional when used for decimals. After introducing students to this way of using base ten blocks, the teacher might ask students to represent simple decimals, like 1.2 or 4.5.

Example 4. Teacher explains how to use base ten blocks to represent quantities with decimals

Teacher: When we are thinking about showing decimal amounts, the base ten blocks can be used again to represent different units. The hundreds piece will represent ones, the tens unit will represent tenths, and the unit cubes will represent hundredths. Then the cube that previously represented thousands will now represent tens.

Tens	Ones	Tenths	Hundredths
			
10	1	0.1	0.01

Teacher: Let’s use the base ten blocks to represent the number 12.89. To represent 1 ten, we would need 1 large cube. To represent 2 ones, we’d need 2 flat square pieces. To represent 0.8, we’d need 8 rod shape pieces. And to represent 0.09, we’d need 9 small unit cubes.

Tens	Ones	Tenths	Hundredths
			
10	2	0.8	0.09

12.89

Source: Example adapted from WWC Guide by Toolkit authors.

Implementation Step 3. Use Representations as “Thinking Tools”

> Provide ample and meaningful opportunities for students to use representations to help solidify the use of representations as “Thinking Tools.”

Students will need many opportunities to work with representations before they will successfully use them to model concepts and procedures and solve problems. Allowing students to use representations only a few times will not be enough. Over multiple uses, students will begin to more deeply understand mathematics concepts and will grasp how representations can be used as “thinking tools” in mathematics. The goal is for representations to help students better understand mathematics concepts and procedures and for students to grow comfortable using representations as tools to model problems and build their understanding.

Representations can be used when students explain their thinking. At first, students may need help articulating how they used the representations to depict the mathematical ideas. Pose prompting questions to help students explain how they represented the concepts or procedures. As students become more comfortable using representations, routinely ask them to use the representations to explain their solution approach. This helps reinforce the mathematics not only for the student explaining their thinking but also for the students who are listening to the explanation the student is giving.

Implementation Step 4. Revisit Representations

> Revisit concrete and semi-concrete representations periodically to reinforce and deepen understanding of mathematical ideas.

Students will use representations in high school (and beyond) such as algebra blocks, geometric models, and computer-based transformation tools for rigid motions.

Systematically revisit concrete and semi-concrete representations to reinforce and deepen students’ understanding of mathematical ideas. Also, if students are not able to correctly solve problems, or if they feel uncertain about how to approach a problem, encourage them to use a concrete or semi-concrete representation to represent or model the situation. Make concrete and semi-concrete representations available for students to use as necessary. Representations will help students become comfortable with the mathematics.

H3. Video Observations

Background Information: The video, *Instructional Routine: Is it True?*, shows an intervention teacher and grade 4 students using a routine that has recommended strategies for representations.

Directions: In the module, you will watch the video two times:

- In the Online Session, watch individually to take a close look at strategies in action. Write notes on this handout below.
- At PLC Session-A, you and your colleagues will rewatch the video so that it's fresh in your minds for the discussion and for preparing to use the routine.

Video Watching Norms

- Observe, without judging, the teacher and students.
- Look for ideas to apply in your practice.

Focus Question

As you watch, focus on this question and write notes:

1. What **instructional strategies** for representations do you notice in the video?

Ideas to Share at PLC Session-A

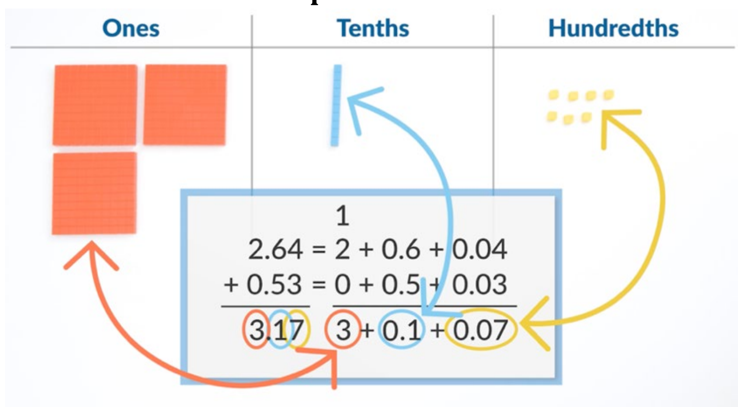
2. What are **one or two ideas** from the video that stood out for you?

H4. Decimal Addition with Base Ten Blocks

Overview: This PD activity for teachers builds on the video, *Connecting Representations: Strategies for Decimal Addition* (on the Explore-B tab).

Goal: Model decimal addition problems with base ten blocks, and use strategies to visually connect the concrete representation and the abstract numeric representation.

Example from Video



Decimal Addition Mat

Ones	Tenths	Hundredths

Materials

- Base ten blocks, markers, and colored pencils.
- Decimal addition mats: Draw two mats on large sheets of paper. Make the spaces large enough for base ten blocks.

Part 1. Try Approaches from the Video with New Problems

1. **Represent and solve: 1.37 + 1.62 =**

   a. **Model and solve the problem** by using base ten blocks and the addition mat.
      - Set up the problem by showing the two addends.
      - Combine the addends to determine the sum.
      - Did you need to use regrouping? ___ Yes ___ No

   b. **Label** your concrete representation with numbers by writing on the mat or on sticky notes. Write the problem and the solution in **expanded form** on the mat.

   c. **Show the connections between** the concrete and numeric representations. For example, draw arrows to connect numbers with base ten blocks.
      - List what you did to show the connections between the representations.

2. **Represent and solve: 1.38 + 0.87 =**

   a. **Model and solve the problem** by using base ten blocks and the addition mat.

      - Set up the problem by showing the two addends.
      - Combine the addends to determine the sum.
      - Did you need to use regrouping? ___ Yes ___ No

   b. **Label** your concrete representation with numbers by writing on the mat or on sticky notes. Write the problem and the solution in **expanded form** on the mat.

   c. **Show the connections between** the concrete and numeric representations. For example, draw arrows to connect numbers with base ten blocks.

      - List what you did to show the connections between the representations.

Part 2. Reflect on the Approaches

1. What are the **benefits** of using these approaches for modeling decimal addition with base ten blocks and visually connecting the representations? List ideas below.

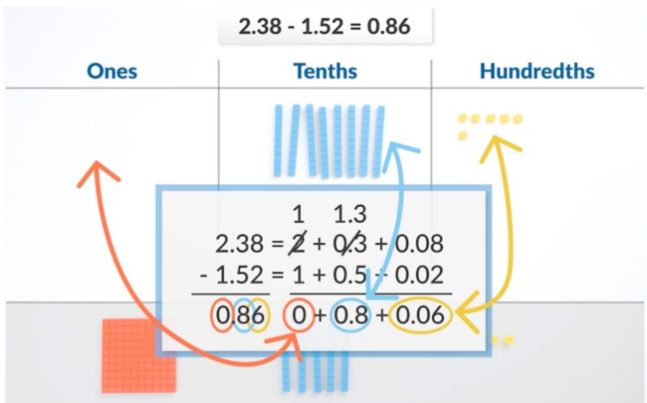
2. What are your **suggestions** for using these approaches with your students for decimal addition and for connecting representations?

H5. Decimal Subtraction with Base Ten Blocks

Overview: This PD activity for teachers builds on the video, *Connecting Representations: Strategies for Decimal Subtraction* (Explore-B tab).

Goal: Model decimal subtraction problems with base ten blocks and use strategies to visually connect the concrete representation and the abstract numeric representation.

Example from Video



Materials:

- Base ten blocks, markers and/or colored pencils.
- Decimal subtraction mats: Create two mats on large sheets of paper. Shade the bottom row.

Decimal Subtraction Mat

Ones	Tenths	Hundredths

Part 1. Try Approaches from the Video with New Problems

1. **Represent and solve: 3.74 - 1.31 =**
   a. **Model and solve the problem** by use base ten blocks and the subtraction mat.
      - Build the first quantity of **3.74** (minuend) in the top row of the mat.
      - Take away **1.31** (subtrahend) by moving the blocks to the shaded row. The amount remaining in the top row is the difference or answer.
      - Did you need to use regrouping? ___ Yes ___ No

   b. **Label** your concrete representation with numbers by writing on the mat or on sticky notes. Write the problem and the solution in **expanded form** on the mat.

   c. **Show the connections between** the concrete and numeric representations. For example, draw arrows to connect numbers with base ten blocks.
      - List what you did to show the connections between the representations.

2. **Represent and solve: 1.38 + 0.87 =**

   a. **Model and solve the problem** by using base ten blocks and the subtraction mat.

      - Use base ten blocks to build **1.38** (minuend) in the top row of the mat.
      - Take away **0.87** (subtrahend) by moving the blocks to the shaded row. The amount remaining in the top row is the difference, or answer.
      - Did you need to use regrouping? ___ Yes ___ No

   b. **Label** your concrete representation with numbers by writing on the mat or on sticky notes. Write the problem and the solution in **expanded form** on the mat.

   c. **Show the connections between** the concrete and numeric representations. For example, draw arrows to connect numbers with base ten blocks.

      - List what you did to show the connections between the representations.

Part 2. Reflect on the Approaches

1. What are the **benefits** of using these approaches for modeling decimal subtraction with base ten blocks and visually connecting the representations? List ideas below.

2. What are your **suggestions** for using these approaches with your students for decimal subtraction and for connecting representations?

H6. Challenges and Suggestions

Directions: Read about potential challenges and the expert panel's advice.[3] Then, choose **one challenge** (A, B, or C) to focus on, and add your responses to the questions.

Challenge A

"My students just play with concrete representations and can't concentrate on the mathematics."

Panel's Advice

Explain the expectations for appropriately using concrete representations as a learning tool. Students will need instruction in how to think about and use representations effectively. Showing the connection between representations and the mathematics may not be enough for students to be able to use them independently. Students may also need instruction in how representations can be used to think through and solve a problem. Without instruction, students may not know how to use the representation in a way that will aid their understanding. Instruction can be done by modelling and talking through the strategies for solving a problem, or it can be done by facilitating a discussion with students about how concepts and procedures can be represented. Students can provide their thoughts and model their use for the class.

Add Your Ideas

1. What are specific expectations for using the manipulatives that you have used or would like to use with students?

2. What are ways to help students use manipulatives as tools for representing and solving mathematics problems?

Sources: WWC Guide, pp. 27–28; questions from the Toolkit authors.

Challenge B

"I connected the abstract concepts and procedures to concrete and semi- concrete representations and then faded them, but I don't think my students fully understand the concepts."

Panel's Advice

Only fade out concrete and semi-concrete representations as students can fully explain the concept and are consistently accurate with doing the work abstractly. If students do not fully understand the concept, then fading out is not appropriate. Revisit concrete and semi-concrete representations periodically to clarify and reinforce the connection between abstract notations and concrete or semi-concrete representations. By periodically revisiting the connections, students will often experience deeper understanding and new insights about those connections. Even after the abstract representations are used more consistently, it may be helpful to revisit concrete and semi-concrete representations to build and clarify the underlying mathematical concepts.

Add Your Ideas

1. What are ways to provide ample and meaningful opportunities for students to use concrete and semi-concrete representations to build understanding of mathematics concepts?

2. What are ways to revisit concrete and semi-concrete representations to build and reinforce mathematics concepts?

Challenge C

"My students are confused because different representations are used in different classes."

Panel's Advice

Consistency in the types of representation shared in core classroom instruction and during intervention sessions, throughout the year and across grades, is critical. Consistent use of representations is particularly important for students who are struggling to grasp the concept or operation. Using a core set of representations across settings and grades helps reinforce instruction on the same concepts as they grow in sophistication. This level of consistency can be part of a whole-school agreement where the goal is to align mathematics instruction through use of cohesive representations, language, notation, rules, and generalizations across grade levels.

Add Your Ideas

1. How does this challenge relate to the use of representations at your school? What is the level of consistency within and across grades for teaching mathematics?

2. What are your suggestions for how to improve consistency in the use of representations within and across grades? Who needs to be involved in creating a shared list of representations?

H7. Reflection for Online Session

Directions: Reflect on your learning by writing responses to these questions. Bring your completed handout to PLC Session-A so you can refer to the ideas you recorded.

1. What are **two important ideas** that you learned about the recommended strategies for using representations?

2. What is **one strategy** (from this Online Session) that you are interested in trying with your students or that you currently use? How is this strategy helpful for students?

3. What is **one idea, strategy, or question** about using representations with students that you want to discuss with colleagues at PLC Session-A?

H8. Use Questions and Sentence Starters

This resource has examples of questions and **sentence starters** to support students with connecting representations and explaining their ideas. You can adapt the questions to specific representations and problems by replacing the [words in brackets].

Connect representations

- Where is the quantity ___ in your [model] and in the [equation]?
- How does each part of your [model] connect to the [equation]? Point to the parts.
- How does your [diagram] represent the [word problem]?

Use representations to explain ideas

- What does your model show about how the sizes of [the two fractions] compare?
- Why does your answer make sense? Point to your model to show as you explain.
- **You can see in my [model] that the fraction ___ is [greater than/less than] the fraction ___.**
- **My [model] shows that the solution is ___ because…**

Describe strategies for using representations

- How did you use the [base ten blocks] to find the [sum]?
- How did making a [drawing] help to represent the [fractions] and find the [difference]?
- How did you use a [number line] to solve the problem?
- **One thing I learned about using [specific manipulative] is…**
- **When you use [fraction tiles] to [add fractions], it's helpful to…**

H9. Video Discussion

Directions: We will rewatch the video, *Instructional Routine: Is it True?*, so it is fresh in our minds for the discussion and for preparing to use the routine ourselves. This handout has new questions.

Video Watching Norms

- Observe, without judging, the teacher and students.
- Look for ideas to apply in your practice.

Focus Questions

As you watch, focus on these **new** questions and write notes.

1. What do you notice about how the teacher supports students with explaining their ideas and using the representations?

2. What do you notice about how students:
   a. explain why the statement is false?
   b. change the statement to make it true?

Ideas to Apply

3. What are one or two ideas from the classroom video that you would like to try with your students? Why?

H10. Walk-Through of Routine: Example Problem

This handout has an example problem, sentence starters, and vocabulary chart to use with the walk-through script (**H11**). Note that this handout is **not** intended for use with students.

Step 1. Predict, Represent, and Compare

Is it true? 0.26 > 0.3

Choice of representations: You can choose to build quantities with base ten blocks, draw base ten blocks, shade grids (on next page), or another approach.

Step 2. Show and Explain

When you explain your reasons, use the sentence starter and words from the vocabulary chart.

Sentence starter: The statement is ___ because…

Vocabulary Chart

Less than <	Greater than >	Equals =
Place	Value	Digit
Ones	Tenths	Hundredths
Decimal	Compare	Difference

Step 3. Make It True!

Ways to make the false statement true:

a. Change one symbol. = < >

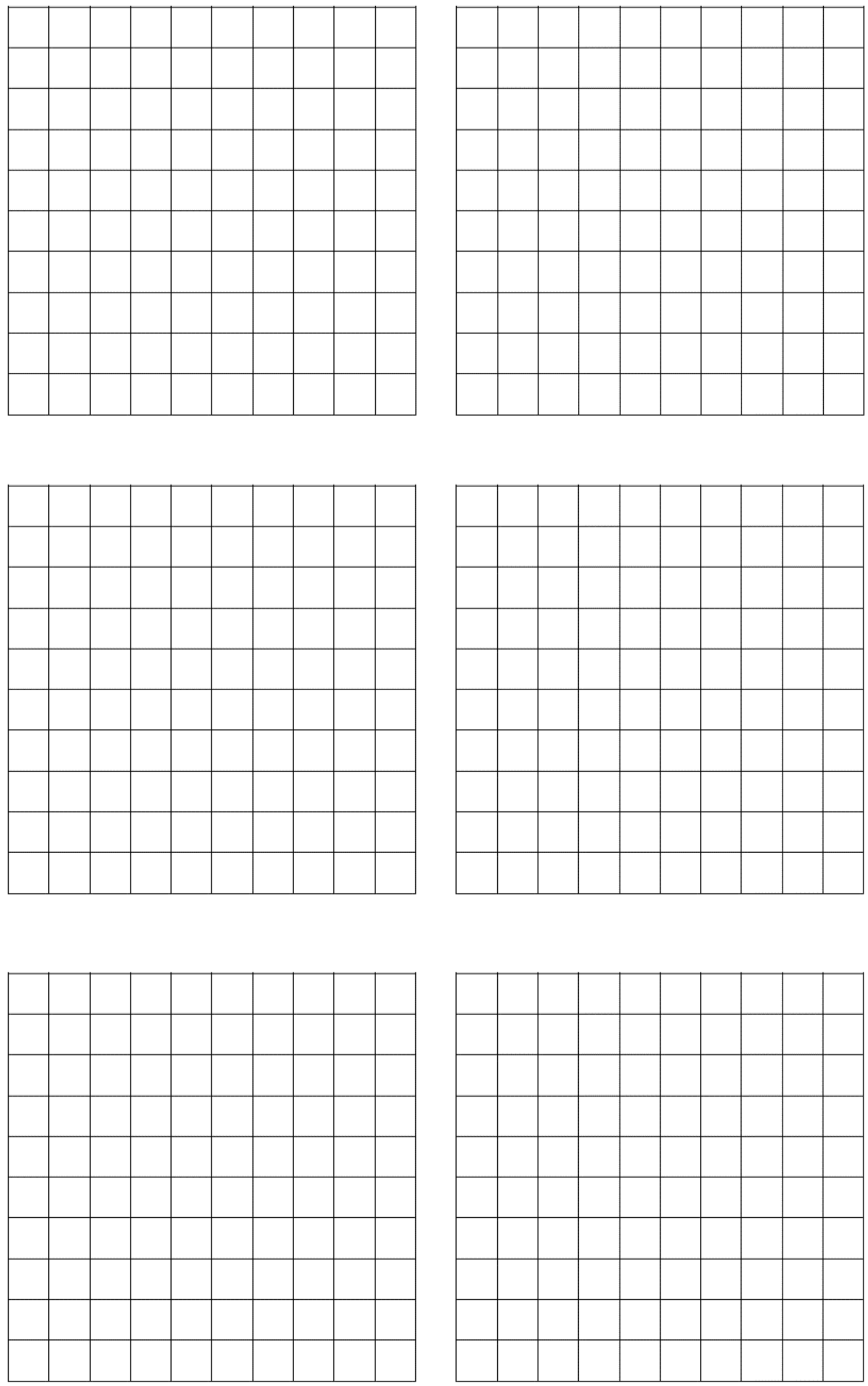
b. Change one of the numbers.

c. Change one symbol and one number.

- Use **one** of these ways to change the false statement to make it true.
- Write your **revised statement**: ____
- Change your **model** to match your revised statement.
- Use the **sentence starter** to explain your reasons:

We changed _______ to ______ because…

Grids for Representing Decimals



H11. Walk-Through of Routine: Script

Routine: Is It True?

Steps:

Predict, represent, and compare.

1. Show and explain.
2. Make it true!

Mathematics Focus: Comparing decimals.

Purpose: To get familiar with the routine by going through the steps in the roles of teachers and students. We will use a script to read aloud what a teacher and students might say and do. The script is designed for this PD activity and is **not** intended for teaching students. After we finish the walk-through, we will discuss ways to apply the routine with students.

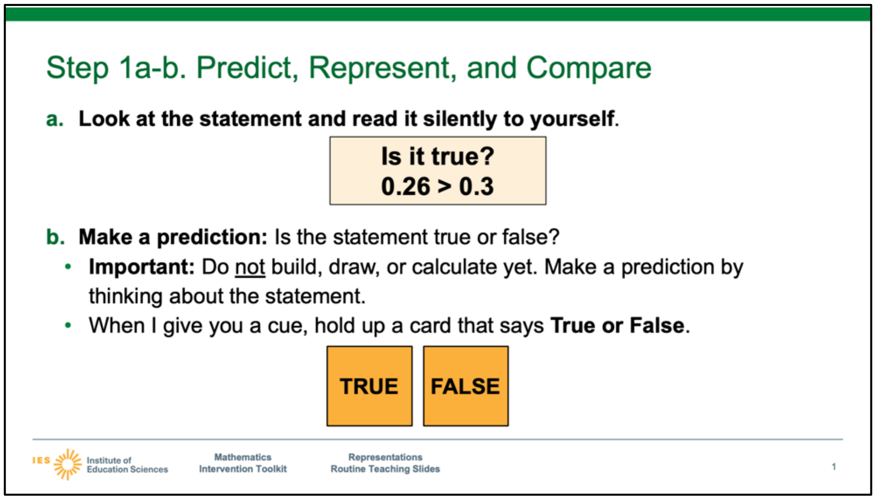
The script is set up as follows:

- The script is labeled with the roles of "Teacher" and "Student" to provide examples of what the teacher or students might **say**. You can read the text aloud or say it in your own words.
- *Text in italics* describes actions that the teacher and students will **do**.
- [Brackets] provide information for doing the walk-through.
- **Slide images** are used to orient you to what is happening in each step as you go through the script. These slides are **not** intended for use with students.

Roles:

- Teacher: We will take turns in the role of teacher. The script provides suggestions about when to switch the teacher role.
- Students: Everyone will take on the role of students when they are not the teacher.

Step 1. Predict, Represent, and Compare



Teacher: Today we are going to do an activity where you get to determine if a statement is true or false. Let's start. I'm going to show you a statement that compares two numbers.

Show the statement but do not read it aloud:

Is the statement true? 0.26 > 0.3

Teacher: Read the statement to yourself silently.

Pause briefly to allow students time to read the statement.

Teacher: Now, I want you to make a prediction: Is the statement **true** or **false**? Think about the statement. Do **not** build, write, draw, or calculate yet, and do not say your prediction aloud.

I've given you two cards. One card says *True*, and the other card says *False*. When I tell you to do so, you will hold up your prediction.

Give students a little time to think.

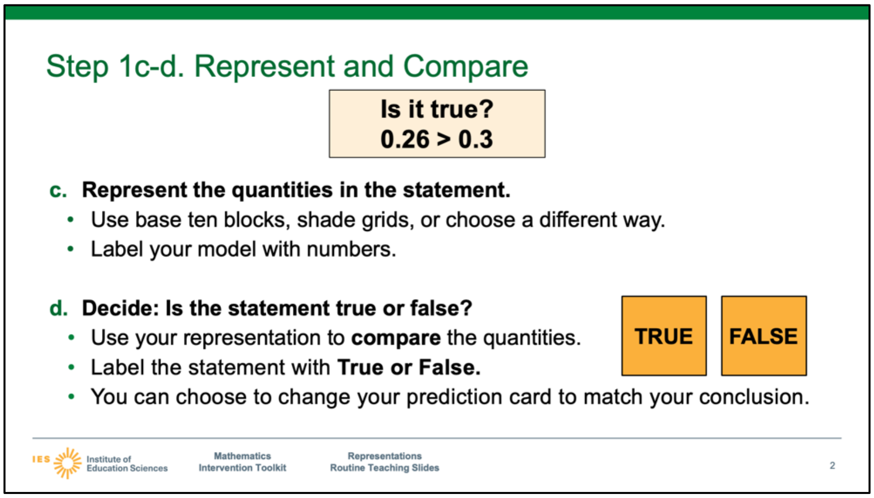
Teacher: Do you predict that the statement is true or false? Hold up a card.

Students: *Hold up their cards.*

Teacher looks at the cards to get a sense of students' initial predictions about the statement. Do not ask students to give reasons at this time.

Teacher: We are **not** going to talk about your reasons now, but we will discuss them in a little bit. First, we will build representations with base ten blocks to determine whether the statement is true or false.

Step 1. Predict, Represent, and Compare, cont.



Teacher: Now your task is to represent the quantities in the statement. Let's go over the directions. You have a choice of what kind of representation to use. You can build with base ten blocks or draw them on a mini-whiteboard and then label your model with numbers. Or you can shade grids on the handout **(H10)** and label them with numbers.

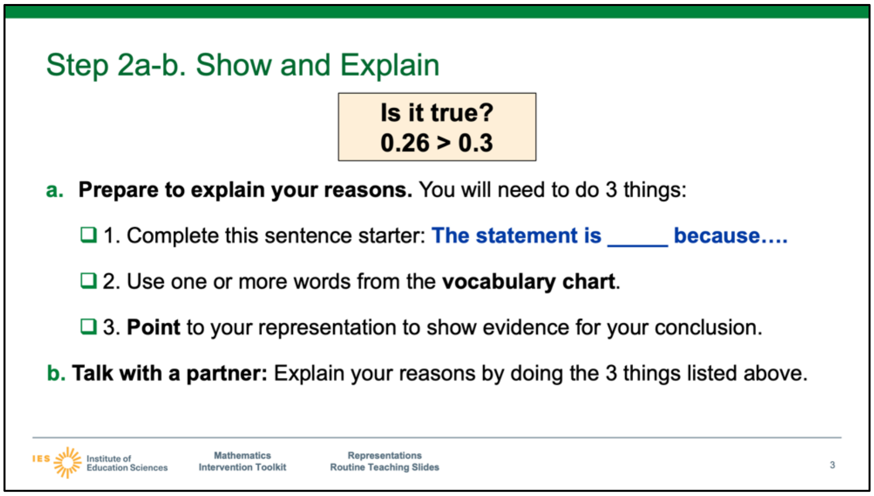
After you build your model, use the representation to decide whether the statement is true or false. Show your conclusion by placing the card that says *True* or the card that says *False* next to your model. You can change the prediction that you made if you want to.

Students choose a representation to model the quantities and label them with numbers.

[Note for walk-through: Give participants a little time to represent the statement.]

[Walk-through continues on the next page.]

Step 2. Show and Explain



[Switch to a new teacher]

Teacher: Next you will explain the reasons for your answers. Before we share with partners, let's go over the directions for communicating your explanations.

Use this sentence starter: **The statement 0.26 > 0.3 is __ because…**

I'd like you to use at least one word from this **vocabulary chart** when you explain your reasoning. Let's read the words together.

Students and Teacher: *Read the words on the vocabulary chart shown above.*

Teacher: It's also important to point to your model when you are talking to show evidence for your conclusions.

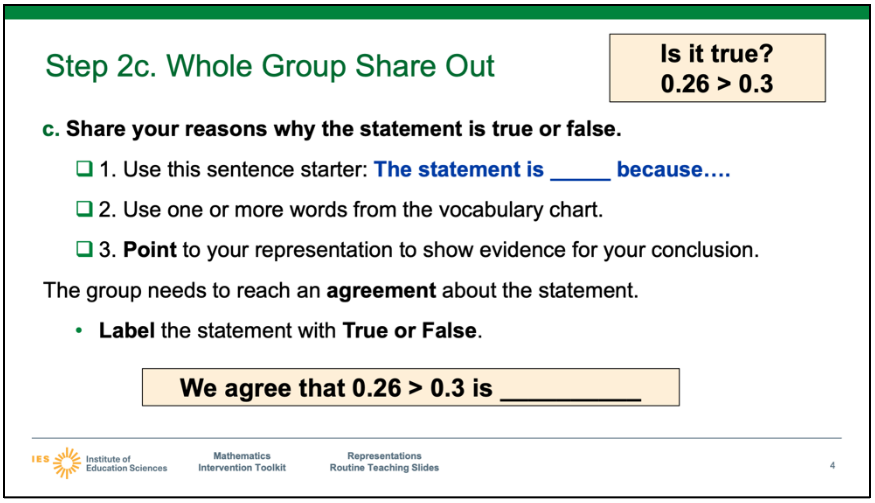
Teacher: Who will restate these directions for the class? What are the three things that you need to do when you explain your reasons?

Student 1: We need to try to point to our model and use mathematics words and the sentence starter when we are explaining.

Teacher: That's right. Okay, you can get started working with your partner now.

[Walk-through continues on the next page.]

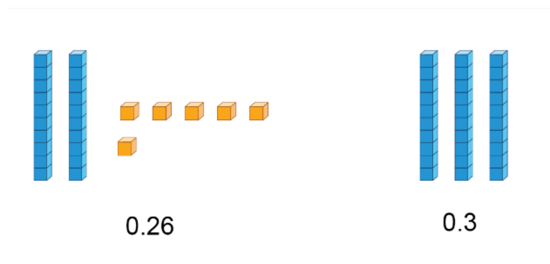
Step 2. Show and Explain, cont.



Note to Read Aloud:

We will not read a script for the partner discussions. Here's what happens in this step. Students work with a partner, and the teacher circulates to observe and ask questions. We will continue the walk-through with the whole-group discussion by reading an example teacher–student dialogue.

Teacher: Let's come back together to share with the whole group. Let's start with [Student 1]. Is the statement 0.26 > 0.3 true or false? Why?



Student 1: We think that the statement is false. Here is our model. That's what our model shows.

Teacher: Okay. I can see you built a model. Can you use your model to tell us some more about how you know the statement is false? And remember to use the sentence starter and some of the vocabulary in the chart.

Student: The statement is false because here is 26 (*point to representation of 0.26*) and here is 3 (*point to the representation of 0.3*).

Teacher: Is this 3? (*Point to the 3 rods*).

Student: Yes.

Teacher: Three would be 3 flats or 3 wholes. *(Show a flat).* You have this represented correctly, but what does one rod represent? The vocabulary chart might help you.

Student: Oh, right, this is *3 tenths*. A rod is *1 tenth*.

Teacher: That's right. Each rod represents 1 tenth because 10 of these rods make 1 whole. So, your model shows 3 tenths. Keep going to explain how you know that 3 tenths is greater than the other quantity you built.

Student: Okay, so we can see these 3 tenths are greater than the 2 tenths and the 6 hundredths. So, the statement is false.

Teacher: And how do you know this is greater *(gesture to the 3 rods)*?

Student: Because 26 hundredths has only 2 rods and 6 hundredths, and that's less than 3 rods. The third rod is equivalent to 10 tenths, and this is only 6 tenths (*pointing to the six small cubes*). Here's a way I can show you. (*Student pushes the 6 cubes together in vertical alignment*). These are shorter than the rod, so the number is less.

Teacher: Thank you. Okay, we heard [Student 1's name] explain that this statement is false. Do others agree or disagree? Show a thumbs up or thumbs down.

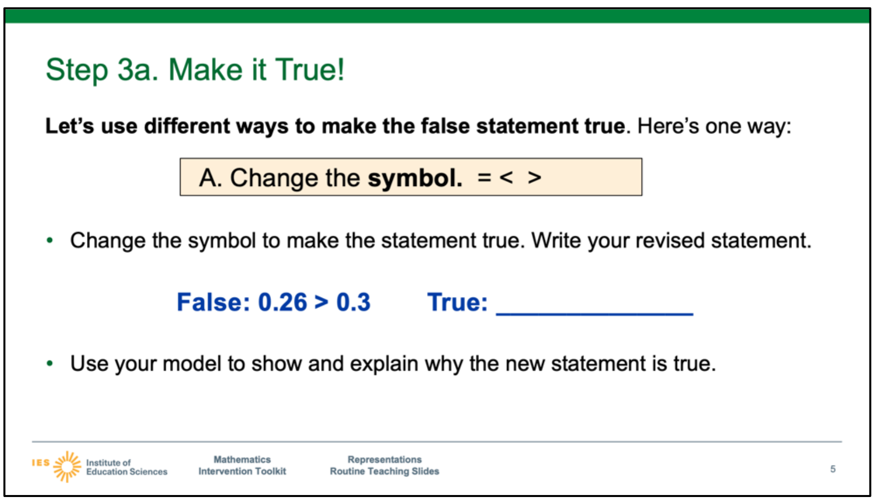
Students hold up their thumbs.

Teacher: How were your reasons similar or different from [Student 1]?

Student 2: We represented ours the same way, and we know that both representations have 2 rods, and so we just looked at the other part and could see that this one has 1 more tenth and 26 hundredths only has 6 hundredths more. I know that 6 hundredths is less than 1 tenth which is 10 hundredths, so it's not greater than it.

Teacher: We all agree that the statement is false. Let's all label the statement with the word **false**.

Step 3a. Make it True by Changing the Symbol



[Switch to a new teacher.]

Teacher: We determined that the statement **0.26 > 0.3** is false. Now we will change the statement to make it true.

I'd like you to **change one symbol.** You can use these symbols: An equals sign, a greater than sign, or a less than sign. What symbol could you use to make the statement true? Write the statement with the new symbol on your whiteboard.

Give students time to write.

Teacher: Hold up your whiteboards. Let's see what symbols you used to make the statement true.

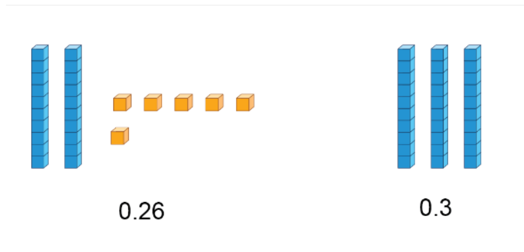
Students hold up whiteboards.

Teacher: [Student 2], please explain how you changed the statement to make it true. Please use the sentence starter: **I changed __ to __ because…**

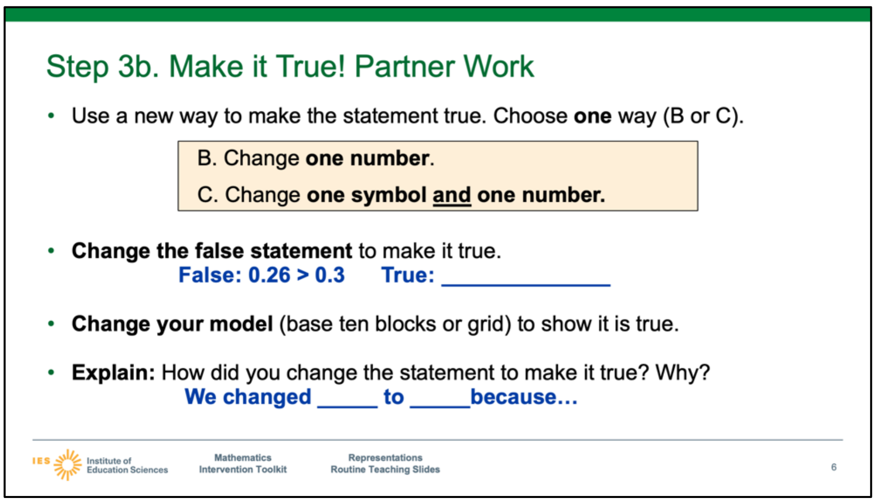
Student 2: I changed the greater than sign to a less than sign because 26 hundredths is less than 3 tenths. You can see in the model that 26 hundredths has 2 tenths and 6 hundredths which, in total, is less than 3 tenths.”

0.26 > 0.3 False

0.26 **<** 0.3 True



Step 3b. Partner Work



Teacher: Let’s try a different way to change the statement **0.26 > 0.3** to make it true. You can choose to change one of the numbers or to change one symbol and one number.

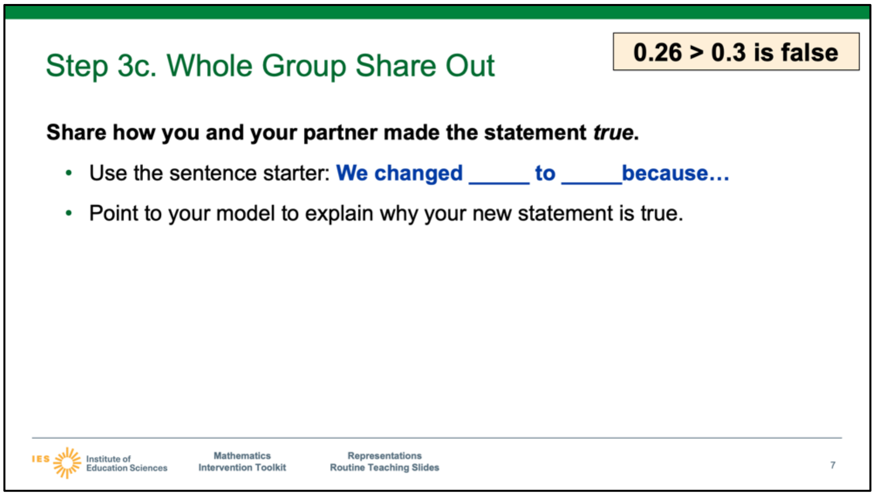
I’d like you to work in pairs to change the statement to make it true. Change your model to match your new statement. Then write your revised statement on one of your whiteboards.

Give students time to work with partners and to write their revised statement on their whiteboards.

Note for walk-through:

If time allows, have participants work in pairs to make the statement true. If time is short, skip to the whole group share out.

Step 3c. Whole Group Share Out

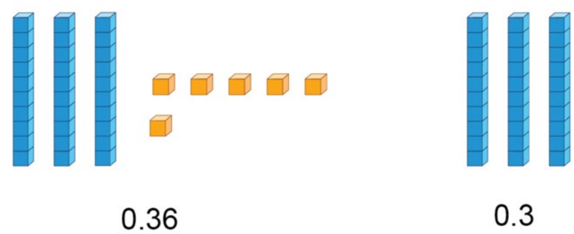


Teacher: Please hold up your whiteboards.

Students hold up their whiteboards.

Teacher: I see that you used different numbers to make the statement true. Who would like to explain what you did and why? Remember to use the vocabulary and sentence starter: **We changed __ to __ because…** and point to your model as you are explaining.

0.36 > 0.3 True



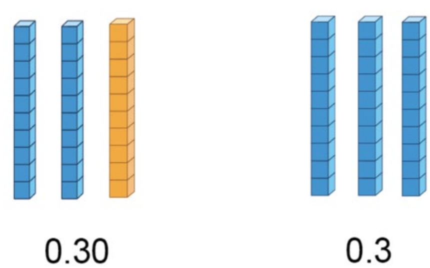
Student 2: We changed 26 hundredths to 36 hundredths (*point to model*) to make the statement true because we know that 36 hundredths has 1 more tenth than 26 hundredths. You can see in our model that we added one more rod (*point to blue rod that was added*). 36 hundredths (*point to model*) is greater than 3 tenths (*point to model*).

Teacher: Thank you for sharing what you did. Who would like to share a different way?

Student 2: We changed a number and a symbol. We added 4 cubes and changed the symbol from the greater than to the equals sign.

Teacher: Okay. Can you point to your model to show us what you did?

0.30 = 0.3 True



Student: We changed 26 hundredths to 30 hundredths (*point to model*), and we changed the symbol to an equals sign. You can see here that there are 10 hundredths (*point to model*) and that is equivalent to 1 tenth. So, 2 tenths and 10 hundredths (*point to model*) is the same as 3 tenths (*point to model*).

The end of the walk-through!

H12. Debriefing Questions and Protocol

Debriefing Questions

1. Description of how you implemented the routine. Who? When? What?
2. Show or describe one or two student work examples. What did you notice in students' work about their mathematics understandings and difficulties?
3. What worked well to support students' mathematics learning and ability to communicate their ideas?
4. What was challenging for students?
5. What would you do again or do differently in the future? Why?

Debriefing Protocol

Part 1: Sharing Experiences

1a. Each person takes a turn presenting their experiences using the routine.

- *Presenter* describes experiences and answers the debriefing questions (~6 min.).
- *Group members* are active listeners during the presentation (avoid interruptions).
- *Timekeeper* gives a 1-minute warning and says when time is up.

1b. After each presenter finishes, group members can:

- Ask clarifying questions and note ideas they would like to discuss in Part 2. (~1–2 min.)

Time*:* About 7–8 minutes per presenter; total time varies by number of presenters.

Part 2: Group Discussion of Common Themes and Suggestions

The group discusses the following questions.

1. Think about the ideas shared by your colleagues. What's one idea that you want to use with this routine or apply in other mathematics activities?
2. Based on what you learned about students' understanding and challenges with using mathematical language, what next steps will you take?
3. What other strategies, challenges, or questions related to the routine would you like to discuss further?

Time*:* About 6–10 minutes for group discussion.

Source: Toolkit authors

H13. Recap Strategies for Representations

Directions: Review this list of strategies and add more ideas. Star * strategies that you want to try or to strengthen in your practice.

1. **What are recommended strategies for representations?[4]**

- **a.** Use concrete representations, such as fraction tiles or base ten blocks, to build understanding of mathematical ideas.
- **b.** Use concrete materials that are proportional for place value concepts, fractions, and decimals.
- **c.** Use semi-concrete representations, such as drawings or diagrams, to build understanding of mathematical ideas.
- **d.** Explicitly connect concrete, semi-concrete, and abstract representations to build understanding of mathematical ideas.
- **e.** Use questioning to help students understand how a model represents a mathematical concept and to support students in creating and explaining representations.
- **f.** Provide ample and meaningful opportunities for students to work with concrete and semi-concrete representations to model concepts and procedures.
- **g.** Support students in using representations as thinking tools to solve problems.
- **h.** Have students use their representations to explain their solution or approach, such as by pointing to parts of their model.

2. **What things should you avoid? What should you do instead?**

- Using representations in a rigid sequence of concrete, then semi-concrete, then abstract.
  - Instead, flexibly use two or more representations in ways that support the learning goals. You do not always need to start with concrete or connect concrete to semi-concrete as an interim step before using abstract representations.
- Putting concrete materials away too quickly or not using them at all in higher grades.
  - Instead, keep concrete materials available and emphasize connections between the representations. The use of concrete materials is appropriate at all grade levels.

[4] This list of strategies is adapted from the WWC Guide by the Toolkit authors. The list is not exhaustive.

H14. Strengthen Strategies

Directions: Use the prompts to reflect on current practices, brainstorm ideas, and plan actions.

1. Brainstorm Together

How will you strengthen your strategies for teaching representations? Choose prompts and list ideas.

a. Try the strategy of . . .

b. Increase . . .

c. Decrease . . .

d. Make sure to . . .

e. Find out about students' understanding of ___ by . . .

f. Build on students' strengths with ___ by . . .

g. Address students' challenges with ___ by . . .

2. Set an Implementation Intention

Look over your ideas above and choose one or two to focus on. Work individually to write an implementation intention and plan specific actions to take.

Implementation Intention
Set a **goal** to strengthen your instructional strategies for supporting students with understanding and using representations. What will you focus on?
Plan **two actions** to take. What will you do? When?

H15. Self-Reflection Form: Representations

Introduction

This form provides an opportunity to reflect on your learning and current understanding of the recommendation for representations. This self-reflection can help you celebrate progress and guide you in identifying areas for continued growth in your professional learning. It is not intended to be evaluative and will not be submitted or shared with others unless you choose to do so.

The form has two parts. In **Part 1**, you are asked to self-assess your understanding of specific strategies from the module on a scale of 1–3 (1: A little understanding; 2: Some understanding; and 3: A strong understanding). Participants are not expected to have a "strong understanding" of every strategy at the end of a module. Please be assured that it's fine to select "some" or "a little" understanding. Your learning is evolving, and you will have opportunities to strengthen your understanding of the strategies in the subsequent modules and in your classroom. Continuing to use strategies with your students is a critical step in deepening your understanding and skills.

In **Part 2**, you are asked to reflect on the strategies and select one that you would like to improve. Focusing on one strategy is helpful for planning concrete next steps that are manageable to carry out. If you would like support, reach out to your facilitator and colleagues in the course. You may also want to revisit resources on specific strategies in the Participant Workbook and the online component.

This self-reflection form captures your thinking at one point in time. It's helpful to revisit the form later in the course to consider how your understanding has changed and to plan ways to continue your professional learning of the recommended strategies.

Part 1. Reflect on your understanding of strategies.

Directions: The table has a list of recommended strategies for teaching representations. Read each strategy and self-assess your level of understanding by using this **rating scale**:

1. **A Little Understanding:** Have a vague sense of the strategy.
2. **Some Understanding:** Able to explain the strategy in general terms.
3. **A Strong Understanding:** Able to explain the strategy in detail and give examples.

NA. Have not learned about the strategy yet.

Strategies What is your current level of understanding for each strategy?	**Select Your Rating**
a. How to use concrete representations, such as fraction tiles or base ten blocks, to build understanding of mathematical ideas.	○ ○ ○ ○ 1 2 3 NA
b. How to use semi-concrete representations, such as drawings or diagrams, to build understanding of mathematical ideas.	○ ○ ○ ○ 1 2 3 NA
c. How to connect concrete, semi-concrete, and abstract representations, such as equations, to build understanding of mathematical ideas.	○ ○ ○ ○ 1 2 3 NA
d. How to visually show the connections between two representations (such as drawing arrows to connect the same quantities in each representation).	○ ○ ○ ○ 1 2 3 NA
e. How to use questioning strategies to support students in connecting representations.	○ ○ ○ ○ 1 2 3 NA
f. Concrete representations: How to provide meaningful opportunities for students to work with manipulatives to model concepts and procedures.	○ ○ ○ ○ 1 2 3 NA
g. Semi-concrete representations: How to provide meaningful opportunities for students to make drawings or other visuals to model concepts and procedures.	○ ○ ○ ○ 1 2 3 NA
h. How to support students in using representations as thinking tools to solve problems.	○ ○ ○ ○ 1 2 3 NA
i. How to support students in using representations to explain their solution or approach.	○ ○ ○ ○ 1 2 3 NA

Part 2. Reflect on your professional learning and use of strategies.

Directions: Look over the strategies on the prior page and then answer these questions.

1. What is one strategy that you find particularly helpful for building students' understanding and use of representations? Why?

2. What is one strategy that you would like to improve in your practice? What are two actions you will take to strengthen your use of this strategy?

3. Reflect on your learning in this module on the recommendation for representations. What are one or two ways that your knowledge and practices have changed?

Source: Questions from Toolkit authors

Routine Teaching Guide

This section has information and resources for teaching the routine *Is It True?* on using representations to compare and explain. Routine handouts are labeled with the letter R and a number.

Contents

Appendix A has additional resources for the routine including response cards and an extensive answer key with pictures of example representations.

Slide Decks for Routine

- Debriefing Slides Template for Sharing Experiences: This is the same file as the prior module.
- Optional Slides for Teaching the Routine

Introduction to the Routine

This Routine Teaching Guide provides information, suggestions, and resources to support teachers in planning and teaching the routine with students and then sharing their experiences. The routine uses multiple strategies from the WWC Guide's recommendations for representations. It has a consistent sequence of steps to allow for ease of implementation by teachers and students. It is applicable to many mathematics topics, giving teachers flexibility for integrating it into their intervention classes.

In this routine, the teacher works with students to build their abilities to use concrete and semi-concrete representations to determine if a statement is true or false and, in the case of false statements, build or draw a representation of a true statement. The routine engages and supports students in building, drawing, writing, and connecting representations.

At PLC Session-A, you and your colleagues will learn how to use the instructional routine by watching and discussing a classroom video example, doing a walk-through, and discussing ways to use the routine with students. Everyone is expected to use the routine with students one or more times and then to share experiences at PLC Session-B.

Frequently Asked Questions (FAQs) about the Routine

1. What recommended strategies does the instructional routine incorporate?

The routine focuses on the WWC Guide recommendation for using representations implementation Step 2: "When teaching concepts and procedures, connect concrete and semi-concrete representations to abstract representations" and implementation Step 3: "Provide ample opportunities for students to use the representations to help solidify the use of representations as 'thinking tools'." The routine also uses strategies to support student communication from the recommendation for mathematical language. Here is a list of recommended strategies in the routine:

- **Modeling quantities** with concrete, semi-concrete, and abstract representations.
- **Connecting** concrete and semi-concrete representations to abstract representations of mathematics concepts and processes.
- Using representations as **thinking tools** to make sense of, solve, and revise problems.
- **Explaining ideas** by using visual representations (also helpful for listeners).
- Asking **questions** to support creating and connecting representations.
- Using **mini whiteboards** for students to create, show, and explain their representations.
- Using **response cards, sentence starters, and vocabulary charts** to support student communication.
- Using **strategically designed problems** to focus on and address misconceptions and difficulties with fraction and decimal topics.

2. What mathematics content does the routine focus on?

The routine focuses on building and connecting different representations of numbers. The routine provides flexibility for using the same format with a variety of mathematics topics, including equivalence, comparison, addition, and subtraction for fractions and decimals.

3. How do you integrate the routine with your intervention program?

The routine is designed to reinforce and deepen understanding of representing and comparing quantities. It also provides helpful formative information about students' understandings and difficulties with these concepts. To plan when to use the routine in your sequence of instruction, consider these prerequisites. The statements include equalities and inequalities, so it is important that students have had prior experience with using inequality symbols (<, >) to compare quantities.

You have a choice of problem sets for fractions and decimals to use with this routine, and as you gain familiarity, you can create additional statements to suit your identified learning goals**.** Each problem set includes one true statement and two false statements. Select a problem set that is a good fit for your students and the instructional sequence in your program. If you are unsure, select two related problem sets so you can adjust the level of challenge as needed.

The instructional routine will take about 30–40 minutes during one intervention class session. You can adjust the routine to fit your class time.

Using the Routine with Students

Expectations

- Use the routine with students **at least one time** *before* PLC Session-B.
- It's okay to **adjust the routine** to fit your teaching situation. The five-step routine takes about 30–40 minutes. If you have shorter intervention classes, you can shorten the time for Step 3.
- After you use the routine, prepare to share experiences at PLC Session-B by using the **Debriefing Slides Template** to create about five slides.

Suggestions

- The routine is likely to take more time the first time you use it. That's okay!
- Use the Two-Page Overview **(R1)** when you teach. It outlines each step. Feel free to put the routine into your own words.
- It's helpful to use a routine more than one time. Some teachers said they felt unsure the first time and that they felt more comfortable when they used the routine again. Consider using the routine more than once, such as with different intervention sections.
- Keep in mind that the routine is also new for your students. Encourage and support their participation.

R1. Two-Page Overview of Routine

Routine: Is it True? Total time: 30–40 min.

Step 1. Predict, represent, and compare. (Whole Group and Individual) ~10 min.

a. **Show the statement** and ask students to read it to themselves silently.

- Do **not** read the numbers out loud because that might provide clues to students about the values of the quantities.
- If the statement has an inequality symbol (<, >), check to make sure students know how to interpret it.

b. Ask students to make a **prediction** about whether the statement is true or false. Give directions that they should **not** build, draw, or calculate at this time.

- Give a little time for students to think individually. Then ask students to hold up cards with *True* or *False* to show their prediction.
- Do **not** ask students to explain their reasons at this time because it might give too much away to other students.

c. Ask students to work individually to **represent the quantities** in the statement by using a concrete or semi-concrete representation. (Each problem set has suggested representations, such as base ten blocks, fraction tiles, drawings, and hundreds grids.)

d. Then, students use their representations to **compare the quantities** and **decide** if the statement is true or false.

- Ask students to write their conclusion (*True* or *False*) next to the statement.
- Give students an opportunity to change their prediction card to match their conclusion.

Step 2. Show and explain. (Pairs and Whole Group) ~10 min.

a. Give students directions for explaining their reasons to a partner. They need to:

- Use the sentence starter: **The statement is ___ because…**
- Use one or two words from the **vocabulary chart**.
- Point to their representation to show why the statement is true or false.

b. Have students work with a partner to take turns explaining their reasons.

c. With the whole group, have students show and explain their representations. Facilitate a discussion to reach consensus on whether the statement is true or false.

Note: If the statement is **false**, continue to Step 3, which focuses on making it true. If the statement is **true**, you can end here or use a new problem (false) to do Steps 1–3.

Step 3. Make it True!	(Whole Group, Individual, and Pairs) 10–15 min.

Note: Step 3 only applies to false statements.

a. Introduce the task of changing the false statement to make it true. Ask students: How could you make the statement true by changing the **symbol**?

- Give students individual think time to change the statement and write it down.
- Then, have students share their revised statements with the whole group.

b. Next, ask students to use a **new way** to change the false statement into a true statement.

- Explain the two ways: Change one of the numbers or change the symbol **and** a number.
- Have students work with a partner to change the statement and to change their representation (manipulatives, drawing, etc.) to fit the new statement.

c. Bring the whole group together, and have pairs share their approaches and reasons.

- Ask students to explain how they made the statement true by pointing to their model and using this sentence starter:

  We made the statement true by changing ___ to ___ because…

Suggestions

Use mini whiteboards for students to write the original statement, build or draw a model, label it with numbers, record their conclusion (True or False), and write their revised statement.

Different Options for Step 3, Make It True!:

- Use the sequence in the overview above. First, have students change the symbol. Then, have students change the statement a different way to make it true.
- Present all three ways at one time: Change the symbol, change a number, and change a number **and** a symbol. Ask students to each choose one way to make the statement true.
- Instead of presenting the three ways, ask students to come up with their own ideas for how to make the statement true.

If your intervention time is less than 30 minutes, consider these options:

- Shorten Step 3 to have students make the statement true in just one way instead of two ways.
- Use two class sessions. At the first session, focus on Steps 1 and 2 only. At the second session, use a new problem, spending less time on Steps 1 and 2 and more time on Step 3.

Problem Sets (R5) provide a variety of problems for fraction and decimal topics.

Answer Key provides examples of responses, visual models, and ways to make statements true.

R2. Planner for Routine

1. List Ideas to Apply for Teaching the Routine

a. What ideas from today's session do you want to apply with your students? What do you want to make sure to do when you use this routine?

2. Make Specific Plans

Suggested Prerequisites for the Routine

Students should have prior knowledge and experiences with 1) the fraction or decimal topic of the chosen problems, 2) using inequality symbols (<, >), and 3) the specific concrete or semi-concrete representations. For example, if you want students to use base ten blocks to represent decimals in the routine, it's important they have prior experiences with this model.

a. **When** will you use the routine with your students before PLC Session-B?

b. **Which problem(s) do you want to use with students?** The routine is designed to reinforce and deepen understanding of mathematical concepts and processes that have already been introduced to students. The problems are grouped by the mathematics topics below.

- Which topic(s) would be a good fit to your students' needs and instructional sequence?

  Fractions: ___ Comparison and Equivalence ___ Addition ___ Subtraction

  Decimals: ___ Place Value ___ Comparison ___ Addition ___ Subtraction

- Look at the problems for your chosen topic (**R5**) and select one or two problems to use. Start with statements that are *false* so that students will be able to make them true.

c. **What concrete or semi-concrete representations** do you want students to use in the routine?

See the suggested representations for your chosen problem set (**R5**)**.** Select representations that are already familiar to your students. You can select one representation or give students a choice.

3. Consider Potential Challenges and Ways to Address Them

a. How will you provide additional support if students have difficulty representing the quantities or operations, or doing other parts of the routine? (See handout **R3** for suggestions.)

R3. Suggestions for Addressing Potential Challenges

Directions: Look over the suggestions. Star * ideas you want to try and add ideas.

1. **If students have difficulty representing the quantities or operations**, here are suggestions for providing additional support and example questions/prompts:

    a. Ask questions to help students get started:
    - Have students restate the directions in their own words: “What is the task asking you to do? Tell me in your own words.”
    - Prompt students to choose one quantity to start with. Ask: “Which quantity would you like to build first with the manipulatives?”
    -

    b. If students have made an error in representing a quantity, consider these suggestions:
    - Ask students to read the statement aloud. If they say the quantities or symbols incorrectly, restate them using correct terms. Hearing the quantities may help students picture them. Hearing the symbols read correctly may support students in making sense of the statement.
    - Ask students to talk about their representation, such as “Tell me about how you used the base ten blocks to represent the quantities. Point to the blocks to show me what you did.” Prompts such as these may help them notice an error and self-correct.
    -

2. **If students have difficulty explaining their reasons,** here are suggestions for providing additional support and example questions/prompts:

- Encourage students to explain what they built and how they used it to choose an answer.
- Suggest vocabulary words in the chart that might be helpful to use in their explanation.
- Using the student’s representation, model for the students how to complete the sentence starter, and use mathematical vocabulary.
-

R4. Preparation and Materials Checklist

Use this list to prepare for using the routine with students before PLC Session-B.

☐ **1. Select one or two problems** that you will use from the handout **R5, Is it True? Problem Sets**.

- Start with a false statement so you can complete all three steps of the routine.

☐ **2. Choose which manipulative or semi-concrete representation** students will use, or offer them a choice of representations. See handout **R5, Is it True? Problem Sets** for suggested representations to use.

- Select representations that are already familiar to students.
- Asking students to use the same representation has the benefit of building and reinforcing understanding of that representation.
- Giving students a choice of representations has the benefit of students showing different models during step 2, which can make the discussion more interesting.

☐ **3. Prepare how you will display the statement** to students, such as writing it on chart paper or a whiteboard or printing copies to place on the table.

☐ **4. Prepare one pair of response cards per student** that say *True* and *False*. In Step 1, students will be asked to hold up a card with their prediction about whether the statement is true or false. Print and cut the *True* and *False* cards (**Appendix A**) or write the words *True* or *False* on index cards or paper plates.

☐ **5. Prepare sentence starters.** Print and cut the sentence starters (**Appendix A**) or write them on chart paper or a whiteboard for students to see.

☐ **6. Prepare vocabulary charts** for students to use when they explain their reasons. There are sample vocabulary charts for the different mathematics topics and problems in handout **R5**. You can choose to change, add, or cut terms from the vocabulary chart to customize the chart for your students. Print and cut the vocabulary charts to give to students or post them on chart paper for students to reference.

R5. Is it True? Problem Sets

Choices of Mathematics Topics

Select an option below to go to the page with problems for that specific topic.

1. Fraction Comparison and Equivalence
2. Fraction Addition
3. Fraction Subtraction
4. Decimal Place Value
5. Decimal Comparison
6. Decimal Addition
7. Decimal Subtraction

Tips for Selecting Problems

- Students should have prior knowledge and experiences with the following: 1) the fraction or decimal topic, 2) using inequality symbols (<, >), and 3) the specific concrete or semi-concrete representations that they will use in the routine. For example, if you want students to use base ten blocks to represent decimals in the routine, it's important they have prior experiences with this model.
- Start with a false statement so you can complete all three steps of the routine.

Answer Keys are in **Appendix A**. These answer keys provide example representations, explanations, and ways to make false statements true.

1. Fraction Comparison and Equivalence Problems

	Is it true?	Answer
1A.	$\frac{3}{4} < \frac{5}{8}$	False
1B.	$\frac{4}{5} < \frac{7}{10}$	False
1C.	$\frac{5}{6} = \frac{7}{8}$	False
1D.	$\frac{4}{3} < \frac{5}{6}$	False
1E.	$\frac{3}{8} > \frac{3}{5}$	False
1F.	$\frac{4}{5} > \frac{4}{12}$	True
1G.	$\frac{2}{3} = \frac{8}{12}$	True

Suggested Representations:

- Fraction tiles, fraction circles, drawings, and number lines.

  Tip: You may want to provide number line templates that are already partitioned but unlabeled for students to use.

Goals: Here are example goals that you can use as written or adapt.

Mathematics Learning Goals:

- Represent fractions accurately by using fraction tiles, fraction circles, or number lines.
- Compare fractions to determine their relationship.

Language Goal:

- Show and explain whether a fraction expression is true or false by using manipulatives, visual representations, words, numbers, and symbols.

Vocabulary Chart

less than <	greater than >	equals =
equivalent	numerator	denominator
fraction	compare	closer to

2. Fraction Addition Problems

	Is it true?	Answer
2A.	$\frac{2}{3}+\frac{1}{6}>1$	False
2B.	$\frac{5}{4}+\frac{5}{6}=\frac{10}{10}$	False
2C.	$\frac{4}{6}=\frac{3}{4}+\frac{1}{2}$	False
2D.	$1<\frac{3}{4}+\frac{3}{8}$	True

Suggested Representations:

- Fraction tiles, fraction circles, drawings, and number lines.

Goals: Here are example goals that you can use as written or adapt.

Mathematical Learning Goals:

- Represent adding fractions accurately by using fraction tiles.
- Compare fractions to determine their relationship.
- Compare fractions to benchmark numbers.

Language Goal:

- Show and explain reasoning about whether the fraction expression is true by using manipulatives, visual representations, words, numbers, and symbols.

Vocabulary Chart

less than <	greater than >	equals =
equivalent	numerator	denominator
fraction	compare	closer to
benchmark number	sum	add

3. Fraction Subtraction Problems

	Is it true?	Answer
3A.	$\frac{11}{12} - \frac{1}{6} < \frac{1}{2}$	False
3B.	$\frac{4}{6} = \frac{5}{8} - \frac{1}{2}$	False
3C.	$\frac{5}{6} - \frac{1}{12} < \frac{1}{2}$	False
3D.	$\frac{3}{4} - \frac{3}{8} < \frac{1}{2}$	True

Suggested Representations:

- Fraction tiles, fraction circles, drawings, and number lines.

Goals: Here are example goals that you can use as written or adapt.

Mathematical Learning Goals:

- Represent subtracting fractions accurately by using fraction tiles.
- Compare fractions to determine their relationship.
- Compare differences to benchmark numbers.

Language Goal:

- Show and explain reasoning about whether the fraction expression is true by using manipulatives, visual representations, words, numbers, and symbols.

Vocabulary Chart

less than <	greater than >	equals =
equivalent	numerator	denominator
fraction	compare	closer to
benchmark number	difference	subtract

4. Decimal Place Value

	Is it true?	Answer
4A.	2.05 = 2 ones and 5 tenths	False
4B.	0.47 = 4 ones and 7 tenths	False
4C.	1.08 = 10 tenths and 8 hundredths	True

Suggested Representations:

- Base ten blocks or grids for representing decimals. Place value mats.

Goals: Here are example goals that you can use as written or adapt.

Mathematical Learning Goals:

- Represent decimals using base ten blocks or shading decimals grids.
- Compare decimals by interpreting the value of the digits of each place, including zeros.

Language Goal:

- Show and explain reasoning by using manipulatives, visual representations, words, numbers, and symbols.

Vocabulary Chart

compare	digit	equals =
ones	tenths	hundredths
equivalent	greater than	less than

5. Decimal Comparison Problems

	Is it true?	Answer
5A.	$0.4 < 0.38$	False
5B.	$0.26 > 0.3$	False
5C.	$0.06 = 0.60$	False
5D.	$0.50 = 0.5$	True

Suggested Representations:

- Base ten blocks or grids for representing decimals. Place value mats.

Goals: Here are example goals that you can use as written or adapt.

Mathematical Learning Goals:

- Represent decimals using base ten blocks or shading decimals grids.
- Compare decimals by interpreting the value of the digits of each place, including zeros.

Language Goal:

- Show and explain reasoning by using manipulatives, visual representations, words, numbers, and symbols.

Vocabulary Chart

less than <	greater than >	equals =
equivalent	ones	tenths
hundredths	compare	digit

6. Decimal Addition Problems

	Is it true?	Answer
6A.	$1.65 + 1.7 < 3$	False
6B.	$1.25 + 0.9 < 2$	False
6C.	$\frac{4}{10} + \frac{3}{100} = 0.07$	False
6D.	$\frac{32}{100} + \frac{4}{10} = 0.36$	False

Suggested Representations:

- Base ten blocks or grids for representing decimals. Place value mats.

Goals: Here are example goals that you can use as written or adapt.

Mathematical Learning Goals:

- Represent and add decimals by using base ten blocks or shading grids.
- Represent fractions with denominators of tenths and hundredths using base ten blocks.

Language Goal:

- Show and explain reasoning by using manipulatives, visual representations, words, numbers, and symbols.

Vocabulary Chart

greater than >	less than <	equals =
equivalent	ones	tenths
hundredths	sum	digit

7. Decimal Subtraction Problems

	Is it true?	Answer
7A.	$3.25 - 1.40 > 2$	False
7B.	$3.7 - 0.22 < 3$	False
7C.	$0.79 - 0.6 > 0.5$	False

Suggested Representations:

- Base ten blocks or grids for representing decimals. Place value mats.

Goals: Here are example goals that you can use as written or adapt.

Mathematical Learning Goals:

- Represent decimals using base ten blocks or shading decimals grids.
- Use base ten blocks to model and perform subtraction.

Language Goal:

- Show and explain reasoning by using manipulatives, visual representations, words, numbers, and symbols.

Vocabulary Chart

less than $<$	greater than $>$	equals =
equivalent	ones	tenths
hundredths	difference	digit

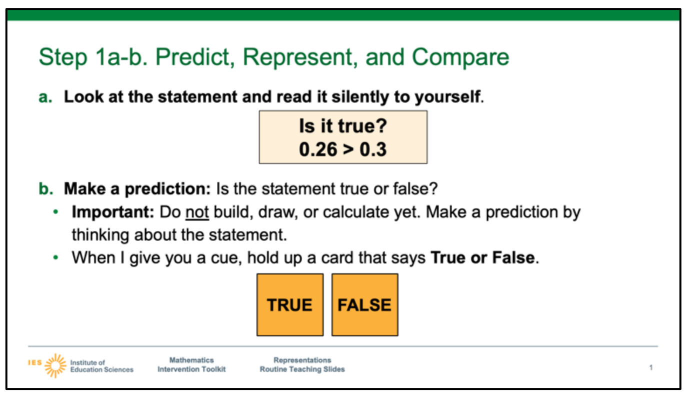
R6. Detailed Teaching Notes for the Routine

This section provides notes for teaching the routine with the decimal comparison problem from the walk-through. It includes pictures of slides to provide key information for the steps as you read through the teaching notes. These slides are not needed for teaching the routine. It works well to teach the routine by writing the statement on chart paper or a whiteboard and give students copies of the sentence starters and vocabulary chart. If you want to use slides, you can adapt the slide deck, Optional Slides for Teaching the Routine, for your chosen problems. The notes are organized as follows:

- **SAY:** Provides information about what to say and do. It is *not* meant to be a script. Use these examples to communicate the ideas in your own way.
    - Regular text indicates things to **say**.
    - *Italicized text* indicates things for the teacher or students to **do.**
- **NOTES:** Provides directions and information for teachers. Some slides do not have any notes.
- **TIPS:** Provides suggestions for implementing specific steps. Some slides do not have any tips.

The examples below have pictures of the slides for problem **5B** in the decimal comparison problem set. You can change the text to fit the problem you have selected.

Step 1a–b (Slide 1)



NOTES: You will display the statement on a whiteboard, chart paper, or sheet of paper on the table. Do not read the numbers aloud, as you might give clues about the quantities' values to students.

SAY: Today we are going to work on an activity in which you get to decide whether a statement is true or false. Let's start with the statement.

Reveal statement.

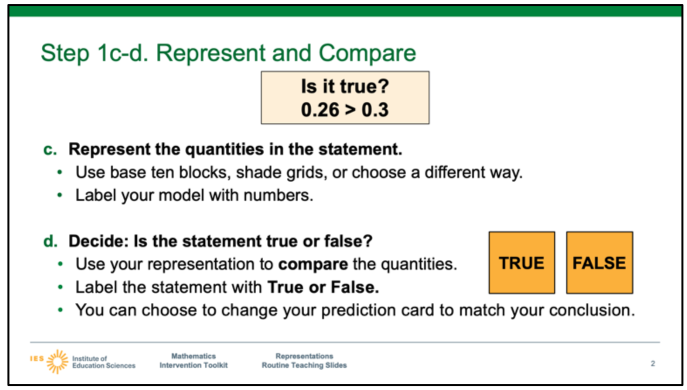
Read the statement to yourself silently.

Pause briefly to give students time to read the statement.

Is this true? Is this quantity (*point to 0.26*) greater than this quantity (*point to 0.3*)? I want you to make a prediction. Think about the statement. Do **not** build, write, draw, or calculate yet, and do not say your prediction aloud. I've given you two cards. One card says *True*, and the other card says *False*. When I tell you to do so, you will hold up your prediction. *Give students a little time to think.*

Do you predict that the statement is true or false? Hold up a card. *Students hold up their cards but do not explain. (Hold off on having students explain until later in the routine after making representations.)*

Step 1c–d (Slide 2)

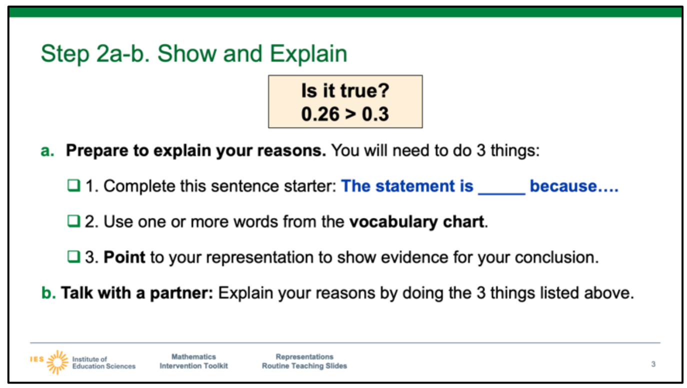


SAY: Use the base ten blocks or shade grids to model the quantities. Label your model with numbers. Then, use your model to decide whether the statement is true or false. Show your conclusion by placing either the True card or the False card next to your model.

Tips for Step 1:

When you first ask students for a prediction, do not linger. The purpose is to get their general sense of the quantities before they have a chance to represent the statement. Let them know they will have a chance to revisit their prediction and revise if they want.

Step 2a-b (Slide 3)



NOTES: Prepare students to explain their reasoning to a partner and then move into partners.

SAY: Next, you will explain the reasons for your answers. Before we share with partners, let's go over the directions for communicating your explanations. Use this sentence starter:

The statement is ____ because...

I'm also going to give you this **vocabulary chart** so that you can reference these words when you are working on your explanation. Let's read the words together.

Students read the words on the chart. You may also want to show optional slide 9 which has the vocabulary chart.

It's also important to point to your model when you are talking to show evidence for your conclusions. Who will restate these directions for the class? What are the three things that you need to do when you explain your reasons?

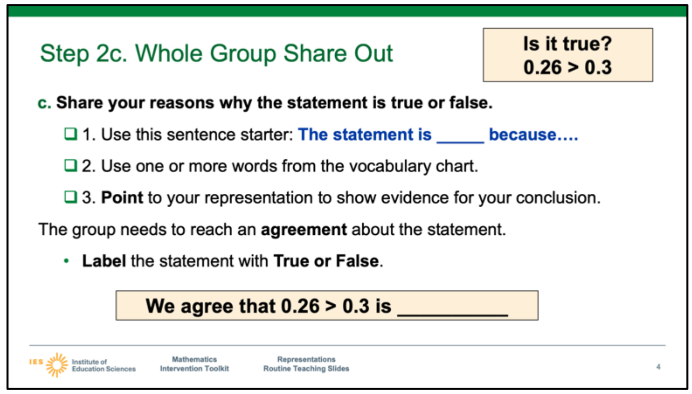
Student restates the directions. Give students time to prepare what they will say. Then, move students into pairs.

Now, you will share with a partner. When you show your model and explain your reasons, remember there are three things you need to do. Use this sentence starter to share your conclusion about whether the statement is true or false. When you explain, include one or more mathematical terms from the vocabulary chart and point to your representation to show evidence for your conclusion.

As students work in pairs, check in to listen and provide support as needed. When they are ready, bring the pairs back for a whole-group share out. One of the goals of the share out is to reach consensus about whether the statement is true or false. Be sure that students label their statement in preparation for the next step.

Make sure that you have the original statement written down on your paper or whiteboard and label it *True* or *False*.

Step 2c (Slide 4)



NOTES: Students will share their representations and reasons with the whole group by using the same process as they did with their partners.

SAY: Let's come together to share with the whole group. Remember to use the sentence starter to share your conclusion about whether the statement is true or false. When you explain, use one or more mathematical terms from the vocabulary chart and point to your representation to show evidence for your conclusion.

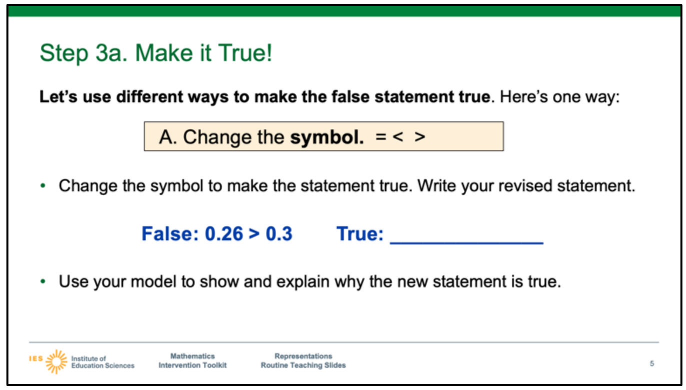
One of the goals of the whole-group share out is to reach consensus on whether the statement is true or false. Label the statement with True *or* False *on chart paper or a whiteboard. Be sure that students also label their statement with* True *or* False *on their mini-whiteboard or paper in preparation for the next step.*

Make sure that you have the original statement written down on your paper or whiteboard then label it *True* or *False*.

Tips for Step 2:

- Remind students to use mathematical vocabulary and to point to their representation as they are explaining. This strategy can help students with their explanations and can support other students in following the explanations of their classmates.
- Remind students that they can change their True/False card by their statement if they wish.

Step 3a (Slide 5)



SAY: We determined that the statement (*read statement*) is false. Let's make sure you have the statement labeled as false. Now we will make the statement true by changing the symbol. Think about what symbol you could use to make the statement true. Write the statement with the new symbol on your whiteboard. Write the word *True* next to your revised statement.

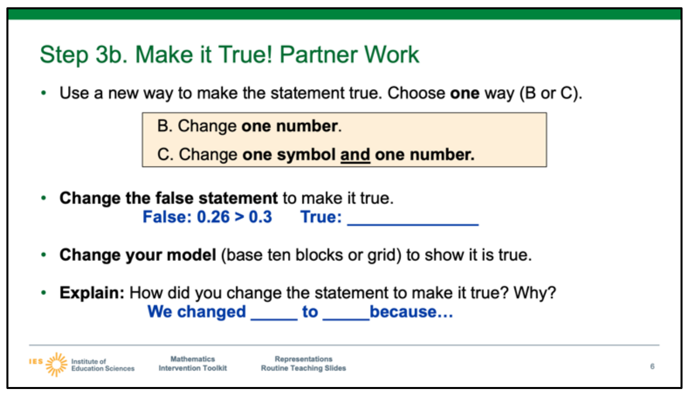
Give students time to write.

Hold up your whiteboards. Let's see what symbols you used to make the statement true.

Check to see if students have used the less than symbol.

Who would like to explain what symbol they used and why? Use your model to explain why the new statement is true. *Student explains.*

Step 3b (Slide 6)



NOTES: Next, have students work with a partner to make the original false statement true by changing a number or by changing *both* a number and a symbol. Students will change their model to represent their new statement.

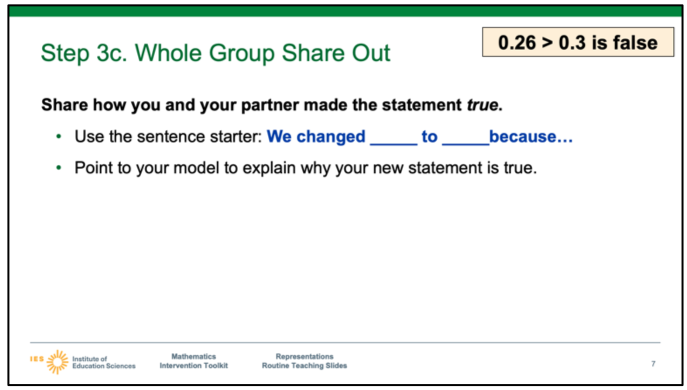
SAY: Let's try a different way to change the statement (*read statement*) to make it true. This time, you can change one number to make the statement true, or you can change one number and one symbol.

Work with your partner to come up with a new true statement using those rules. Write your revised statement on your whiteboard. Also be sure to build a model to represent the statement. I'd like you to use this sentence starter when you explain how you made the statement true.

We changed __ to __ because…

Check in with pairs as they work.

Step 3c (Slide 7)



NOTES: Have the pairs share out to the whole group. Remind students to use the sentence starter and their model when they explain.

SAY: Let's hear how you and your partner made the statement true.

Each pair shares. Use follow-up prompts and questions as needed to support students.

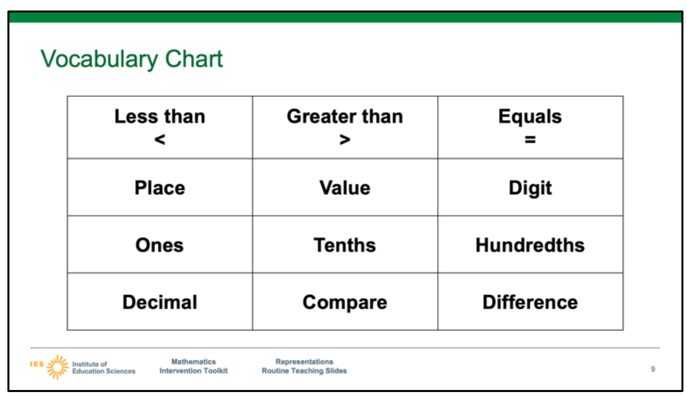
Tips for Step 3:

- In Step 3b, make sure that students are working from the original false statement. Have them erase the true statement they made, leaving the original statement with its label *False*. When students make their new statement, remind them to label it *True*.
- Ask follow-up questions to elicit more details about their thinking, and check for understanding, such as:
  - How did you use the [base ten blocks or shade grids] to model your new true statement?
  - How does your model show that this statement is true?
  - What strategy did you use to come up with a new true statement?

Optional Slide

Slide 9

This optional slide has a larger version of the vocabulary chart for the example problem. You may want to show it during Step 2.



Appendix A: Routine Resources and Answer Key

This appendix has additional resources for using the routine with students. It also has an answer key with pictures of example representations for the problems.

Response Cards: True and False

Print and cut these cards to give one set to each student. Or write the words on index cards.

True	False
True	False
True	False
True	False

Source: Toolkit authors

Sentence Starters

Provide a copy of the sentence starters to each pair of students or individual student.

For Step 1.

The statement is ____________ because . . .

The statement is ____________ because . . .

For Step 3.

We changed __________ to __________because . . .

We changed __________to __________ because . . .

Answer Key for Is it True? Problems

This key provides examples of concrete or semi-concrete representations, explanations, and ways to change the statements to make them true. Note that there are other possible representations and approaches for each problem. Brackets and **dark orange font** are used to indicate text for answers and examples.

The answers are organized by problem type:

1. Fraction Comparison and Equivalence Problems

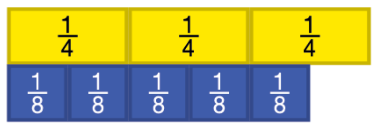
1A. Is the statement true? $\frac{3}{4} < \frac{5}{8}$ [FALSE]

Predict. *When students are asked to predict, why might they think the false statement is true?*

Students might incorrectly predict that this false statement is true because they are comparing the numerators and denominators as separate whole numbers. Since 3 is less than 5 and 4 is less than 8, they conclude, in error, that $\frac{3}{4}$ is less than $\frac{5}{8}$.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because when I built each fraction with fraction tiles, I could see that $\frac{3}{4}$ is longer than $\frac{5}{8}$. This means that $\frac{3}{4}$ is greater than $\frac{5}{8}$.]



Make it true! *Examples:*

[Change the symbol: $\frac{3}{4} > \frac{5}{8}$ Change a number: $\frac{3}{4} < \frac{7}{8}$]

1B. Is the statement true? $\frac{4}{5} < \frac{7}{10}$ [FALSE]

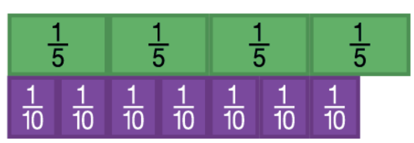
Predict. *When students are asked to predict, why might they think the false statement is true?*

Students might incorrectly think that $\frac{7}{10}$ is greater than $\frac{4}{5}$ because they are overgeneralizing from whole numbers that 7 and 10 are greater numbers than 4 and 5.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because when I used fraction tiles to build each fraction, I saw that $\frac{7}{10}$ is shorter. I would need one more $\frac{1}{10}$ piece to make them equal. This means that $\frac{7}{10}$ is less than $\frac{4}{5}$.]

Make it true! *Examples:*



[Change the symbol: $\frac{4}{5} > \frac{7}{10}$ Change a number: $\frac{4}{5} < \frac{9}{10}$]

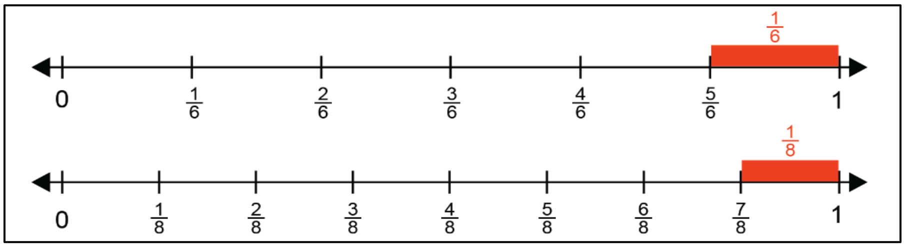
1C. Is the statement true? $\frac{5}{6} = \frac{7}{8}$ [FALSE]

Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might incorrectly think that $\frac{5}{6}$ is equal to $\frac{7}{8}$ because each fraction is missing one part to make a whole. Students may not realize that the size of the missing parts is different.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because on a number line, $\frac{5}{6}$ is $\frac{1}{6}$ away from 1, and $\frac{7}{8}$ is $\frac{1}{8}$ away from 1. Since $\frac{7}{8}$ is closer to 1, it is greater than $\frac{5}{6}$.]



Make it true! *Examples:*

[Change the symbol: $\frac{5}{6} < \frac{7}{8}$ Change a number: $\frac{5}{6} > \frac{7}{\mathbf{10}}$]

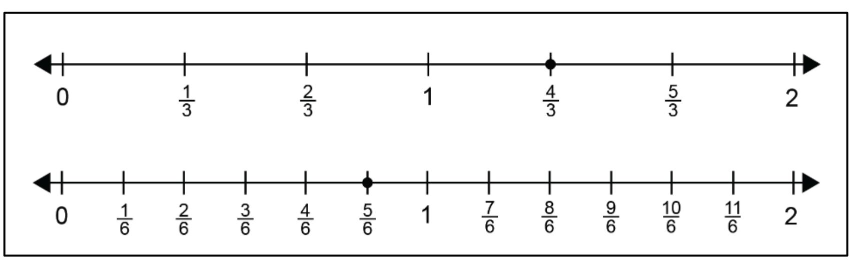
1D. Is the statement true? $\frac{4}{3} < \frac{5}{6}$ [FALSE]

Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might incorrectly think that $\frac{4}{3}$ is less than $\frac{5}{6}$ because they are using whole-number thinking to compare the numerators and denominators separately—4 is less than 5 and 3 is less than 6.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because when I placed the fractions on number lines, it shows that $\frac{4}{3}$ is greater than 1 and $\frac{5}{6}$ is less than 1.]



Make it true! *Examples:*

[Change the symbol: $\frac{4}{3} > \frac{5}{6}$ Change a number: $\frac{4}{3} < \frac{4}{\mathbf{2}}$]

1E. Is the statement true? $\frac{3}{8} > \frac{3}{5}$ [FALSE]

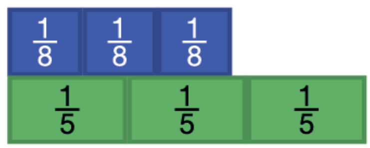
Note: This problem is in the classroom video, *Instructional Routine: Is it True?*.

Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might incorrectly think that $\frac{3}{8}$ is greater because they are comparing denominators as though they are whole numbers, and 8 is greater than 5.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because when I used fraction tiles to build each fraction and lined them up to compare them, I noticed that $\frac{3}{5}$ is longer than $\frac{3}{8}$. I also noticed that there are three pieces in each fraction but the $\frac{1}{5}$ pieces are bigger than the $\frac{1}{8}$ pieces so that's why three of the $\frac{1}{5}$ pieces are greater.]



Make it true! *Examples:*

[Change the symbol: $\frac{3}{8} < \frac{3}{5}$ Change a number: $\frac{3}{8} > \frac{3}{\mathbf{10}}$]

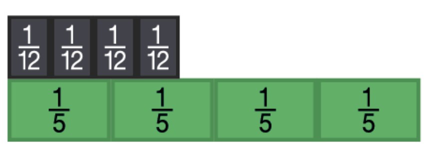
1F. Is the statement true? $\frac{4}{5} > \frac{4}{12}$ [TRUE]

Predict. *When students are asked to predict, why might they think this true statement is false?*

Students might incorrectly predict that $\frac{4}{5}$ is less than $\frac{4}{12}$ because they are comparing the denominators as though they are whole numbers rather than fractions, and 5 is less than 12.

Represent and explain. *Example response and model:*

[The statement is <u>true</u> because when I used fraction tiles to build each fraction and lined them up to compare them, I noticed that $\frac{4}{5}$ is longer than $\frac{4}{12}$. I also noticed that there are four pieces in each fraction but the $\frac{1}{5}$ pieces are bigger than the $\frac{1}{12}$ pieces so that's why four of the $\frac{1}{5}$ pieces are greater.]



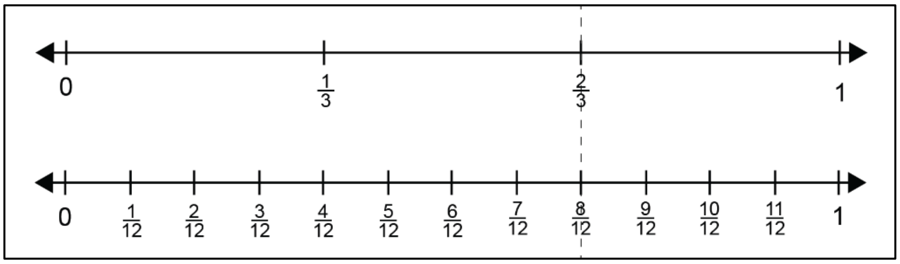
1G. Is the statement true? $\frac{2}{3} = \frac{8}{12}$ [TRUE]

Predict. *When students are asked to predict, why might they think this true statement is false?*

Students might think that fractions cannot be equivalent if they have different numbers.

Represent and explain. *Example response and model:*

[The statement is true because when I located the fractions on a double number line, I saw that the two fractions are at the same location. This means they are equivalent fractions.]



2. Fraction Addition Problems

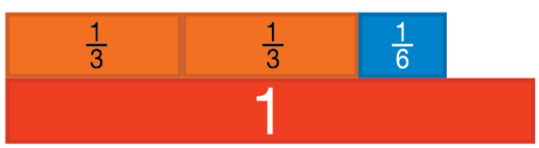
2A. Is the statement true? $\frac{2}{3} + \frac{1}{6} > 1$ [FALSE]

Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might incorrectly predict that $\frac{2}{3} + \frac{1}{6}$ is greater than 1 if they have an inaccurate sense of the size of these fractions, such as estimating that $\frac{2}{3}$ is very close to 1.

Represent and explain. *Example response and model:*

[The statement is false because when I added $\frac{2}{3} + \frac{1}{6}$ with fraction tiles, I saw that the total length is less than 1.]



Make it true! *Examples:*

[Change the symbol: $\frac{2}{3} + \frac{1}{6} < 1$ Change a number: $\frac{2}{3} + \frac{\mathbf{3}}{\mathbf{6}} > 1$]

2B. Is the statement true? $\frac{5}{4} + \frac{5}{6} = \frac{10}{10}$ [FALSE]

Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might incorrectly predict that $\frac{5}{4} + \frac{5}{6}$ is equal to $\frac{10}{10}$ by adding the numerators (5 + 5) and denominators (4 + 6) separately. They may not realize that they need to use common denominators.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because when I used fraction tiles to build the sum and compared it to $\frac{10}{10}$, I noticed that $\frac{5}{4} + \frac{5}{6}$ is longer than $\frac{10}{10}$, so that means it's greater than $\frac{10}{10}$.]

Make it true! *Examples:*

[Change the symbol: $\frac{5}{4} + \frac{5}{6} > \frac{10}{10}$ Change a number: $\frac{5}{4} + \frac{5}{6} = \frac{25}{12}$]

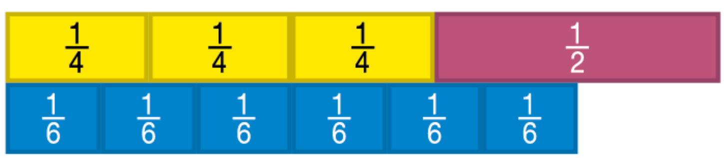
2C. Is the statement true? $\frac{4}{6} = \frac{3}{4} + \frac{1}{2}$ [FALSE]

Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might incorrectly think that $\frac{3}{4} + \frac{1}{2}$ is equal to $\frac{4}{6}$ by adding the numerators (3 + 1) and the denominators (4 + 2).

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because when I used fraction tiles to model $\frac{3}{4} + \frac{1}{2}$, I noticed that the sum is longer than $\frac{4}{6}$. This means that $\frac{3}{4} + \frac{1}{2}$ is greater than $\frac{4}{6}$.]



Make it true! *Examples:*

[Change the symbol: $\frac{3}{4} + \frac{1}{2} > \frac{4}{6}$ Change a number: $\frac{3}{4} + \frac{1}{4} = \frac{4}{6}$]

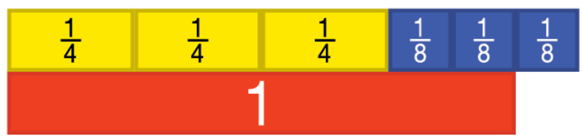
2D. Is the statement true? $1 < \frac{3}{4} + \frac{3}{8}$ [TRUE]

Predict. *When students are asked to predict, why might they think this true statement is false?*

Students sometimes think of all fractions as small, so the sum of two fractions is small and less than 1. Also, if students have the misconception of adding the numerators and the denominators, they will predict that the sum is $\frac{6}{12}$, which is less than 1.

Represent and explain. *Example response and model:*

[The statement is <u>true</u> because when I used fraction tiles to model $\frac{3}{4} + \frac{3}{8}$, I noticed that the total length of the tiles is less than 1. This means $\frac{3}{4} + \frac{3}{8}$ is less than 1.]



3. Fraction Subtraction Problems

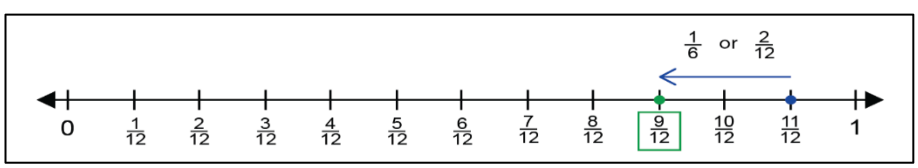
3A. Is the statement true? $\frac{11}{12} - \frac{1}{6} < \frac{1}{2}$ [FALSE]

Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might think that because they are subtracting, the answer will be very small. They may incorrectly assume that it will be less than $\frac{1}{2}$.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because $\frac{11}{12}$ is almost one whole, and I'm only subtracting $\frac{1}{6}$, which is a small amount, so I know the difference will still be greater than $\frac{1}{2}$. I showed this on a number line by starting at $\frac{11}{12}$ and moving $\frac{1}{6}$ or $\frac{2}{12}$ to the left on the number line. I ended up at $\frac{9}{12}$, which is greater than $\frac{1}{2}$.]



Make it true! *Examples:*

[Change the symbol: $\frac{11}{12} - \frac{1}{6} > \frac{1}{2}$ Change a number: $\frac{11}{12} - \frac{4}{6} < \frac{1}{2}$]

3B. Is the statement true? $\frac{4}{6} = \frac{5}{8} - \frac{1}{2}$ [FALSE]

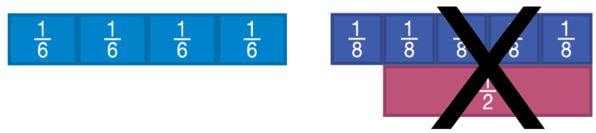
Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might subtract the numerators and subtract the denominators and get the answer $\frac{4}{6}$.

Represent and explain. *Example response and model:*

[The statement is false because when I built $\frac{5}{8}$ with fraction tiles and subtracted $\frac{1}{2}$, the amount left was $\frac{1}{8}$, and I know that $\frac{1}{8}$ is not equal to $\frac{4}{6}$.]

$\frac{4}{6}$



$\frac{5}{8} - \frac{1}{2} = \frac{1}{8}$

Make it true! *Examples:*

[Change the symbol: $\frac{4}{6} < \frac{5}{8} - \frac{1}{2}$ Change a number: $\frac{4}{6} = \frac{7}{6} - \frac{1}{2}$]

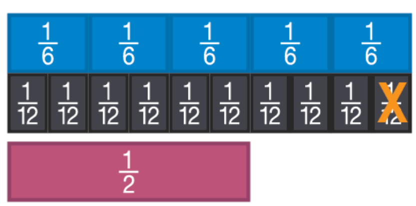
3C. Is the statement true? $\frac{5}{6} - \frac{1}{12} < \frac{1}{2}$ [FALSE]

Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might think that subtraction makes the answer smaller, so the difference will be less than $\frac{1}{2}$.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because when I built $\frac{5}{6}$ with fraction tiles and subtracted $\frac{1}{12}$, the amount left was $\frac{9}{12}$ and that's greater than $\frac{1}{2}$. To subtract $\frac{1}{12}$, I needed to change the sixths into twelfths: $\frac{5}{6}$ is equivalent to $\frac{10}{12}$.]



Make it true! *Examples:*

[Change the symbol: $\frac{5}{6} - \frac{1}{12} > \frac{1}{2}$ Change a number: $\frac{5}{6} - \frac{6}{12} > \frac{1}{2}$]

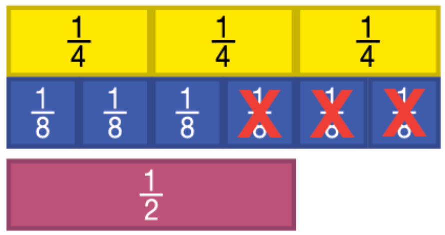
3D. Is the statement true? $\frac{3}{4} - \frac{3}{8} < \frac{1}{2}$ [TRUE]

Predict. *When students are asked to predict, why might they think this true statement is false?*

Students might subtract the numerators and then subtract the denominators and get an answer of $\frac{1}{4}$, which is less than $\frac{1}{2}$.

Represent and explain. *Example response and model:*

[The statement is true because when I used fraction tiles to subtract $\frac{3}{8}$, the amount left was less than $\frac{1}{2}$. To do the subtraction, I represented $\frac{3}{4}$ as $\frac{6}{8}$ so that I could subtract eighths.]



4. Decimal Place Value

4A. Is the statement true? 2.05 = 2 ones and 5 tenths [FALSE]

Predict. *When students are asked to predict, why might they think this false statement is true?*

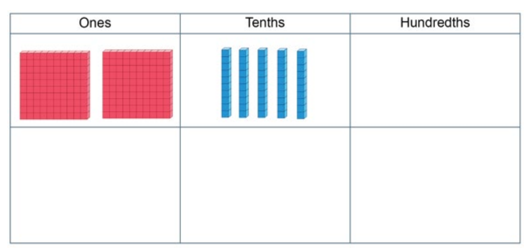
Students might incorrectly think that 2.05 is equal to 2 ones and 5 tenths by confusing the place value of tenths and hundredths.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because when I used base ten blocks to represent both quantities, I saw they are not equivalent. I modeled 2.05 using 2 ones and 5 hundredths. Then I put out 2 ones and 5 tenths, and I can see that this is not equal to 2.05.]

2.05

2 ones and 5 tenths



Make it true! *Examples:*

[Change the symbol: 2.05 **<** 2 ones and 5 tenths

Change a number (digit or word): 2.05 = 2 ones and **5 hundredths]**

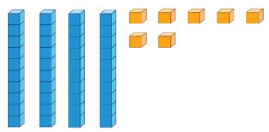
4B. Is the statement true? 0.47 = 4 ones and 7 tenths [FALSE]

Predict. *When students are asked to predict, why might they think this false statement is true?*

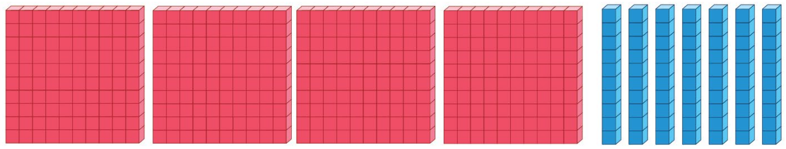
Students might incorrectly think that 0.47 is equal to 4 ones and 7 tenths by confusing the place value of digits in the tenths and ones places.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because when I used base ten blocks to represent both quantities, I saw they are not equivalent. I used 4 tenths and 7 hundredths to represent 0.47. Then, I built 4 ones and 7 tenths and saw that it was much greater.]



0.47



4 ones and 7 tenths

Make it true! *Examples:*

[Change symbol: 0.47 **<** 4 ones and 7 tenths Change a number: **4.70** = 4 ones and 7 tenths]

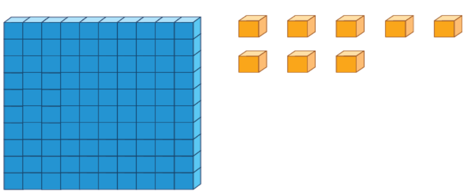
4C. Is the statement true? 1.08 = 10 tenths and 8 hundredths [TRUE]

Predict. *When students are asked to predict, why might they think this true statement is false?*

Students might incorrectly think that 1.08 is not equal to 10 tenths and 8 hundredths if they are confused about the value of digits in the tenths and hundredth places. For example, they might think it is 8 tenths instead of 8 hundredths.

Represent and explain. *Example response and model:*

[The statement is <u>true</u> because 10 tenths is equal to one whole and 8 hundredths is correctly written in the hundredths place. When I made a model with base ten blocks, I took 10 tenths, or rods, and put them next to each other to show that they make one whole. I also used 8 hundredths, or cubes, so that the model matches the words and the numbers in the statement.]



1.08 = 10 tenths + 8 hundredths

5. Decimal Comparison Problems

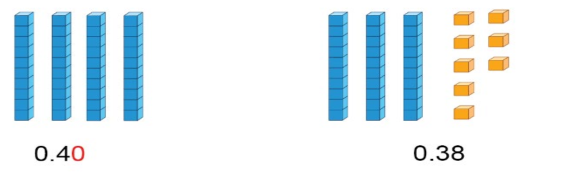
5A. Is the statement true? $0.4 < 0.38$ [FALSE]

Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might overgeneralize from whole numbers and think that 0.4 is less because it has fewer digits than 0.38.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because 4 tenths is equivalent to 40 hundredths, and this is greater than 38 hundredths. I used base ten blocks to represent tenths and hundredths and compared the quantities.]



Make it true! *Examples:*

[Change the symbol: 0.4 > 0.38 Change a number: 0.3 < 0.38]

5B. Is the statement true? $0.26 > 0.3$ [FALSE]

Note: This problem is in the walk-through.

Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might overgeneralize from whole numbers and think that 0.26 is greater than 0.3 because 0.26 has more digits than 0.3.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because when I built 26 hundredths and 3 tenths, I can see that 3 tenths is greater. It is 4 hundredths greater than 26 hundredths.]

Make it true! *Examples:*

[Change the symbol: 0.26 **<** 0.3 Change a number: **0.36** > 0.3]

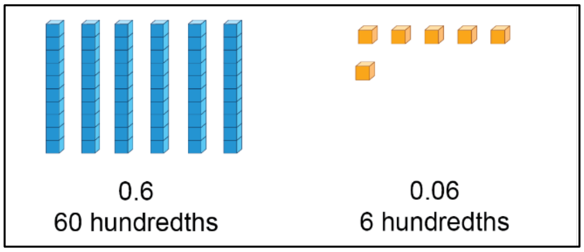
5C. Is the statement true? $0.60 = 0.06$ [FALSE]

Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might incorrectly think that the quantities are equal, overgeneralizing that because both have the digit "6" in them, they must be the same amount.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because 0.60 is 60 hundredths and 0.06 is 6 hundredths, so these two quantities are not equivalent. I used base ten blocks to represent and compare the quantities.]



Make it true! *Examples:*

[Change the symbol: 0.60 > 0.06   Change a number and a symbol: 0.60 < **0.76]**

Note: Changing **only** a number is not an interesting option for this problem (0.60 = 0.60). Instead, ask students to change a number **and** a symbol.

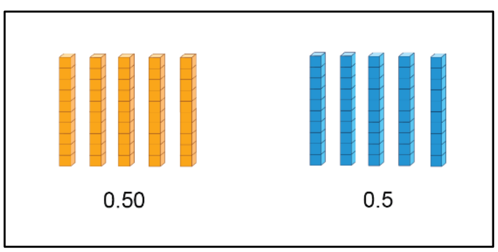
5D. Is the statement true? $0.50 = 0.5$ [TRUE]

Predict. *When students are asked to predict, why might they think this true statement is false?*

Students might incorrectly think that 50 hundredths is greater than 5 tenths, because 50 is greater than 5.

Represent and explain. *Example response and model:*

[The statement is <u>true</u> because 50 hundredths and 5 tenths are equivalent. I used base ten blocks to represent and compare the quantities, and I can see that each tenth is the same as 10 of the hundredth cubes.]



6. Decimal Addition Problems

6A. Is the statement true? $1.65 + 1.7 < 3$ [FALSE]

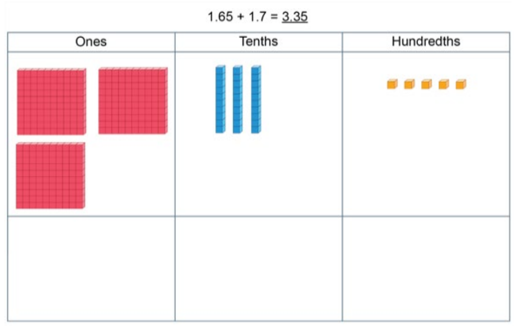
Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might incorrectly predict the sum is less than 3 if they only combine the digits in the ones place and don't attend to the digits in the tenths place.

Represent and explain. *Example response and model:*

[The statement is false because when I built the two addends with base ten blocks and combined them, I was able to regroup 10 tenths for a 1, so I had 3 in the ones place. I still had tenths and hundredths, so I knew the sum was greater than 3.

This model shows the sum after combining the two addends in the top row and regrouping 10 tenths for a 1.]



Make it true! *Examples:*

[Change the symbol: $1.65 + 1.7 > 3$ Change a number: $\mathbf{1.25} + 1.7 < 3$]

6B. Is the statement true? $1.25 + 0.9 < 2$ [FALSE]

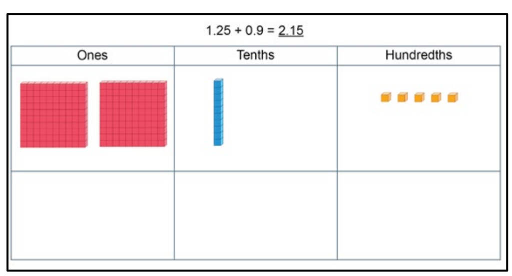
Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might incorrectly predict that the sum is less than 2 if they focus on only the digits in the ones place (1 and 0) and disregard the digits in the tenths and hundredths places.

Represent and explain. *Example response and model:*

The statement is <u>false</u> because when I built the two addends with base ten blocks and combined them, I was able to regroup 10 tenths for a 1 and had 2 ones in the ones place. I also had 1 tenth and 5 hundredths. This means the sum is greater than 2.

This model shows the sum after combining the two addends in the top row and regrouping 10 tenths for a 1.



Make it true! Examples:

[Change the symbol: $1.25 + 0.9 > 2$ Change a number: $\mathbf{1.05} + 0.9 < 2$]

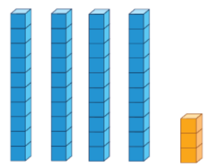
6C. Is the statement true? $\frac{4}{10} + \frac{3}{100} = 0.07$ [FALSE]

Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might incorrectly think the sum is equal to 7 hundredths if they just add the numerators and use hundredths as the denominator of the sum. They may not realize that 4 tenths is equivalent to 40 hundredths.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because when I built the two addends with base ten blocks, I noticed the collection of blocks was greater than 7 hundredths. The sum is 43 hundredths or 0.43.]



$$\frac{40}{100} + \frac{3}{100} = \frac{43}{100} \text{ or } 0.43$$

Make it true! *Examples:*

[Change the symbol: $\frac{4}{10} + \frac{3}{100} > 0.07$ Change a number: $\frac{\mathbf{4}}{\mathbf{100}} + \frac{3}{100} = 0.07$]

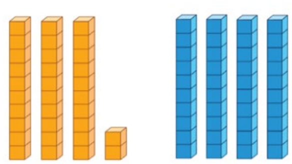
6D. Is the statement true? $\frac{32}{100} + \frac{4}{10} = 0.36$ [FALSE]

Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might incorrectly think the sum is equal to 36 hundredths if they add the numerators and do not attend to the different denominators.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because when I built the two addends with base ten blocks, I could see that the total was greater than 36 hundredths. The sum is 72 hundredths.]



$$\frac{32}{100} + \frac{40}{100} = \frac{72}{100} \text{ or } 0.72$$

Make it true! *Examples:*

[Change the symbol: $\frac{32}{100} + \frac{4}{10} > 0.36$ Change a number: $\frac{32}{100} + \frac{4}{10} < \mathbf{1}$**]**

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7. Decimal Subtraction Problems

7A. Is the statement true? $3.25 - 1.40 > 2$ [FALSE]

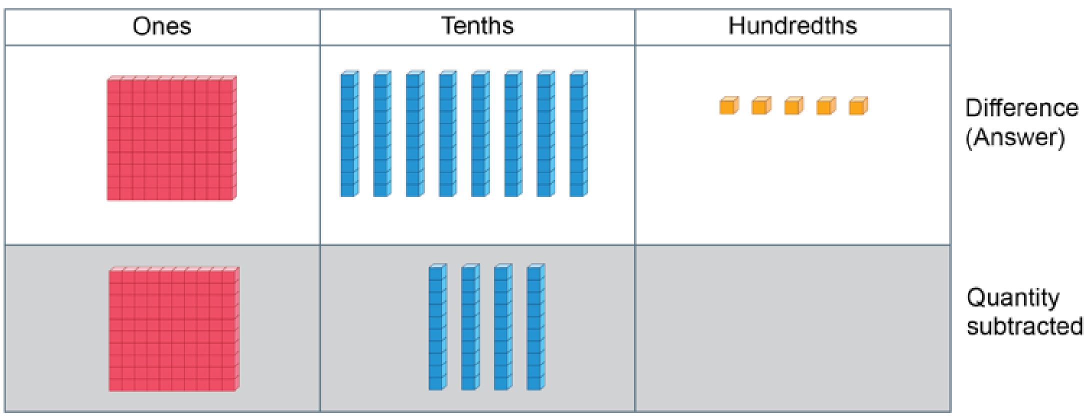
Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might incorrectly predict that the difference is greater than 2 if they estimate by focusing on the ones place and don't attend to the digits in the tenths and hundredths places.

Represent and explain. *Example response and model:*

[The statement is false because when I built 3.25 with base ten blocks and subtracted 1.40, I was left with 1.85. This is less than 2.]

3.25 - 1.40 = 1.85



Make it true! *Examples:*

[Change the symbol: $3.25 - 1.40 < 2$

Change a number: $3.25 - \mathbf{0.40} > 2$]

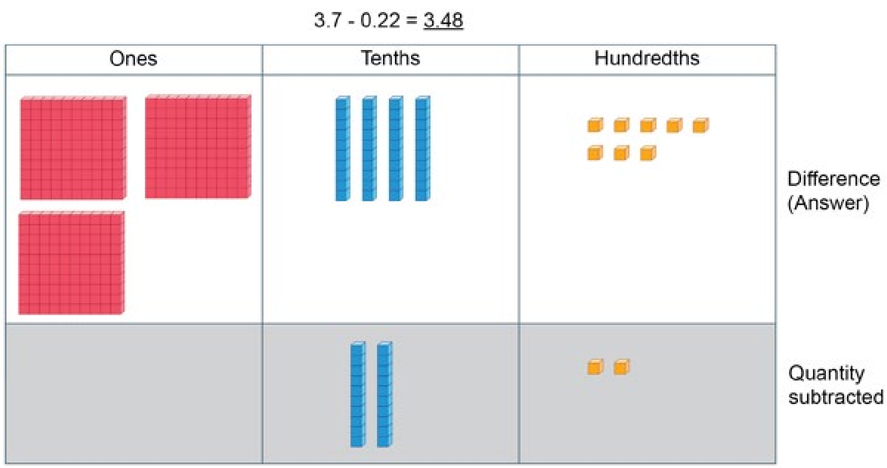
7B. Is the statement true? 3.7 — 0.22 < 3 [FALSE]

Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might incorrectly predict that the sum is less than 3 if they misalign the place values in their minds and treat 2 tenths as 2 ones, leading to an incorrect difference of about 1.

Represent and explain. *Example response and model:*

[The statement is <u>false</u> because when I represented 3.7 and took away 2 tenths and 2 hundredths, I had 3 ones, 4 tenths, and 8 hundredths left. That is greater than 3.]



Make it true! *Examples:*

[Change the symbol: 3.7 – 0.22 **>** 3 Change a number: 3.7 – **1.22** < 3]

7C. Is the statement true? 0.79 — 0.6 > 0.5 [FALSE]

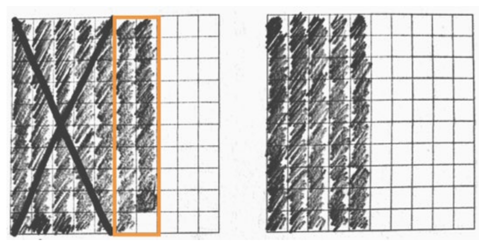
Predict. *When students are asked to predict, why might they think this false statement is true?*

Students might treat subtraction of decimals as though they are whole numbers and align the digits at the right. Then, they might get an incorrect answer of 0.73, which is greater than 0.5.

Represent and explain. *Example response and model:*

[The statement is false because when I shaded the grid to represent 79 hundredths and then took away 6 tenths, there were 19 hundredths left. This is less than 5 tenths.]

0.7 – 0.6 = 0.19 **0.5**



Make it true! *Examples:*

[Change the symbol: 0.79 -0.6 < 0.5 Change a number: 0.79 -**0.19** > 0.5]

Appendix B: Answer Keys for Handouts

This appendix has answer keys for two handouts. Example solutions are shown in brackets and dark orange font.

H4. ANSWER KEY: Decimal Addition with Base Ten Blocks

Overview: This professional development activity for teachers builds on the video, *Connecting Representations: Strategies for Decimal Addition* (Explore-B tab).

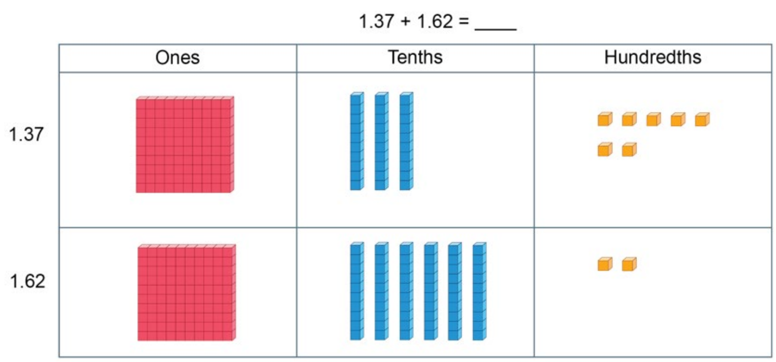
Example answers are shown in brackets and dark orange text. The pictures show example representations.

Part 1. Try Approaches from the Video with New Problems

1. **Represent and solve: 1.37 + 1.62 = [2.99]**

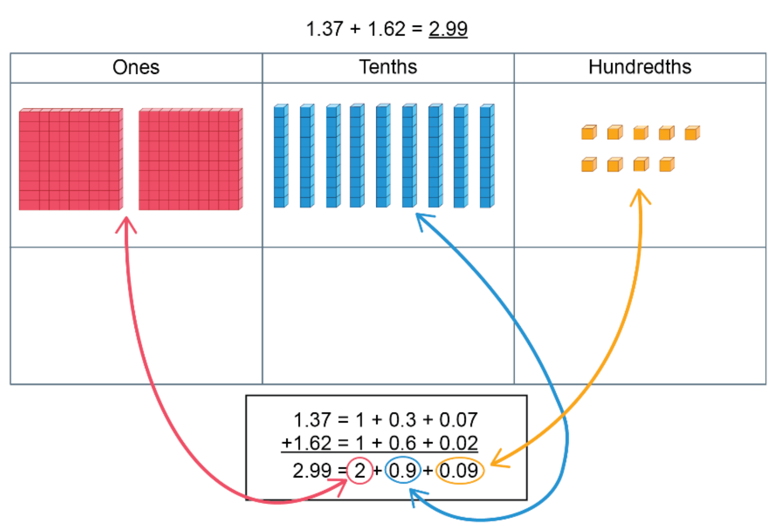
   a. **Model and solve the problem** by using base ten blocks and the addition mat.

   - Set up the problem by showing the two addends.



   - Combine the addends to determine the sum.
   - Did you need to use regrouping? __Yes _[X]_No

   b. **Label** your concrete representation with numbers by writing on the mat or on sticky notes. Write the problem and the solution in **expanded form** on the mat.



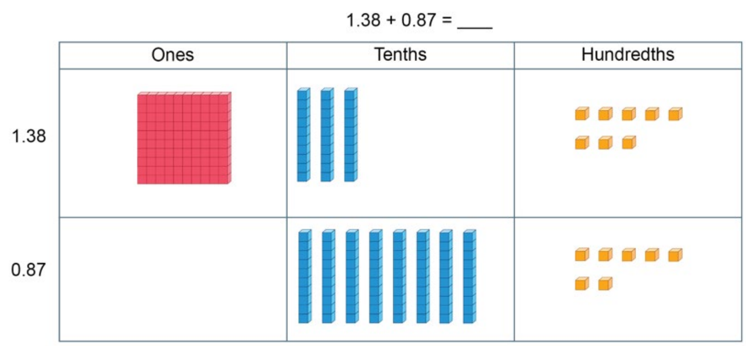
c. **Show the connections between** the concrete and numeric representations. List what you did to show the connections between the representations.

[To show the connections between the representations, I used colored arrows that match the specific color of each base ten block—ones with an orange arrow, tenths with a blue arrow, and hundredths with a yellow arrow. I circled the partial sums using the colors to connect to each quantity—ones, tenths, and hundredths. The sum is 2.99, which consists of 2 ones, 9 tenths, and 9 hundredths.]

2. **Represent and solve: 1.38 + 0.87 = [2.25]**

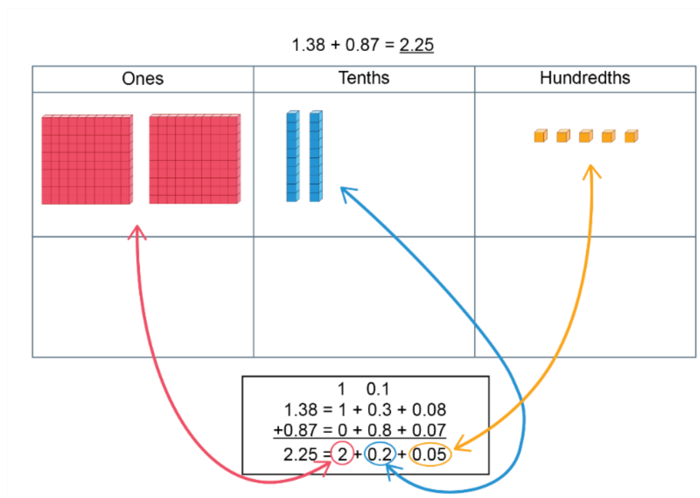
a. **Model and solve the problem** by using base ten blocks and the addition mat.

- Set up the problem by showing the two addends.



- Combine the addends to determine the sum.
- Did you need to use regrouping? _[X]_ Yes __No

b. **Label** your concrete representation with numbers by writing on the mat or on sticky notes. Write the problem and the solution in **expanded form** on the mat.



c. **Show the connections between** the concrete and numeric representations. For example, draw arrows to connect numbers with base ten blocks. List what you did to show the connections between the representations.

[I used colored arrows and circles to show the connections between the representations and the total quantity for each partial sum: 2 ones, 2 tenths, and 5 hundredths. This helps to illustrate that the sum is 2.25.]

Part 2. Reflect on the Approaches

1. What are the **benefits** of using these approaches for modeling decimal addition with base ten blocks and visually connecting the representations? List ideas below.

   [Answer will vary. Here are some examples:

   - Using base ten blocks allows students to construct each addend with concrete models.
   - Using base ten blocks helps students visualize the decomposition of each addend into ones, tenths, and hundredths.
   - Using base ten blocks and the place value mat helps students build understanding of place value concepts and when regrouping is appropriate.
   - Using the place value mat helps students use place value to organize and perform the operations with manipulatives.
   - Using expanded notation reinforces place value concepts.
   - Showing the connection between the representations by using colored circles and arrows helps students understand the quantities for the addends and sum.]

2. What are your **suggestions** for using these approaches with your students for decimal addition and for connecting representations?

   [Answer will vary. Here are some examples:

   - Be sure to provide many opportunities for students to represent decimals in different ways with the base ten blocks before moving to use the blocks for addition.
   - Having students draw what they did at each stage of the problem solving can bog them down or interrupt their thinking. Watch and listen to students while they build, and focus on having students connect their equation to the model at the end.
   - I think it would be helpful to have colored pencils or markers that match the colors of the base ten blocks available for students to use.
   - Having the base ten blocks separated into baskets or bins by type—ones, tenths, hundredths—will help students stay organized and easily find the quantities needed to represent each problem.]

H5. ANSWER KEY: Decimal Subtraction with Base Ten Blocks

Overview: This PD activity for teachers builds on the video, *Connecting Representations: Strategies for Decimal Subtraction* (Explore-B tab).

Part 1. Try Approaches from the Video with New Problems

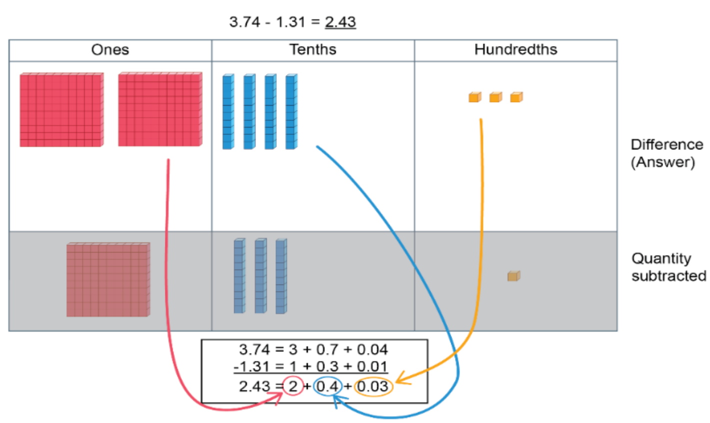
1. Represent and solve: 3.74 – 1.31 = [2.43]

a. **Model and solve the problem** by using base ten blocks and the subtraction mat.

- Build the first quantity of **3.74** (minuend) in the top row of the mat.
- Take away **1.31** (subtrahend) by moving the blocks to the shaded row. The amount remaining in the top row is the difference, or answer.
- Did you need to use regrouping? ___ Yes _[X]_ No

b. **Label** your concrete representation with numbers by writing on the mat or on sticky notes. Write the problem and the solution in **expanded form** on the mat.

[Below is the image of the mat at the completion of modeling this problem. The gray area shows the quantity that was subtracted. You can use this to check that the correct quantity was subtracted.]



c. **Show the connections between** the concrete and numeric representations. List what you did to show the connections between the representations.

[I used colored arrows and circles to show the connections between the representations and the difference. For example, I drew a yellow arrow to show that there are only 3 hundredths remaining. I drew a blue arrow to show that there are only 4 tenths remaining. I drew an orange arrow to show that there are only 2 ones remaining. The difference is 2.43.]

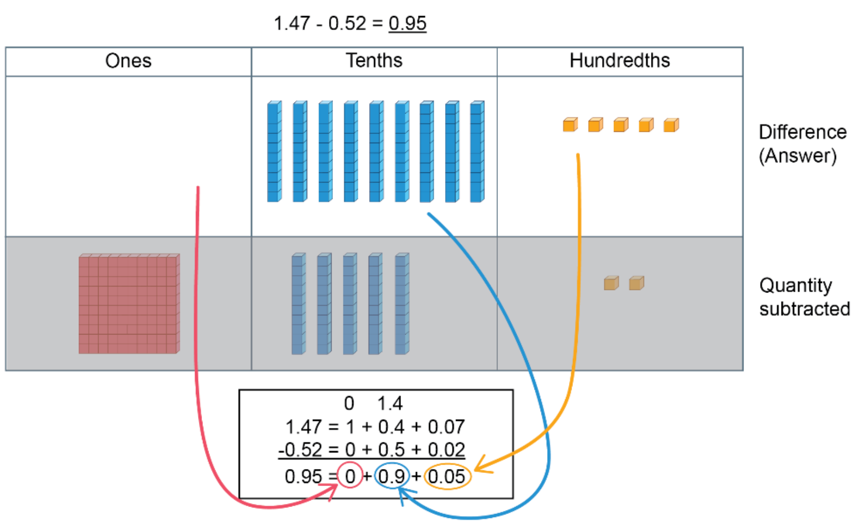
2. **Represent and solve: 1.47 – 0.52 = [0.95]**

   a. **Model and solve the problem** by using base ten blocks and the subtraction mat.

   - Use base ten blocks to build **1.47** (minuend) in the top row of the mat.
   - Take away **0.52** (subtrahend) by moving the blocks to the shaded row. The amount remaining in the top row is the difference, or answer.
   - Did you need to use regrouping? _[X]_Yes __No

   b. **Label** your concrete representation with numbers by writing on the mat or on sticky notes. Write the problem and the solution in **expanded form** on the mat.

   [Below is the image of the mat at the completion of modeling this problem. The gray area shows the quantity that was subtracted.]



   c. **Show the connections between** the concrete and numeric representations. List what you did to show the connections between the representations.

   [I used colored arrows and circles to show the connections between the representations and the difference. First, I drew a yellow arrow to show that once I subtracted 2 hundredths from 7 hundredths, there were 5 hundredths remaining. Then, I realized that I only had 4 tenths and needed to subtract 5 tenths, so I needed to regroup. I regrouped 1 one for 10 tenths. Once I did that, I had 14 tenths in all, and then I was able to subtract 5 tenths. I ended up with 9 tenths and used a blue arrow to connect this to the numeric representation 0.9. Then, since I had already regrouped, I had 0 ones remaining. The difference is 0.95.]

Part 2. Reflect on the Approaches

1. What are the **benefits** of using these approaches for modeling decimal subtraction with base ten blocks and visually connecting the representations? List ideas below.

   [Answer will vary. Here are a few examples:

   - Using base ten blocks allows students to construct the first quantity (minuend) with concrete models and then actively model taking away a second quantity (subtrahend), using trading as needed to build understanding of place value and regrouping.
   - Using base ten blocks helps students visualize the decomposition of the two quantities (minuend and subtrahend) into ones, tenths, and hundredths to perform subtraction.
   - Showing the connections between the representations helps students understand the quantities and difference.]

2. What are your **suggestions** for using these approaches with your students for decimal subtraction and for connecting representations?

   [Answer will vary. Here are some examples:

   - Spend time helping students understand how to use the subtraction mat and how and why the process for representing subtraction (separating or taking away) is different from the process for addition (combining). It will be important to reinforce that the bottom gray row is where to place the quantity being taken away.
   - Have colored pencils or markers that match the colors of the base ten blocks available for students to use.
   - Watch and listen to students as they are modeling the problem. The quantity in the gray area should match what they were to take away. Talk to students about this way of double checking that they subtracted the correct quantity.
   - Having the base ten blocks separated into baskets or bins by type—ones, tenths, hundredths—will help students stay organized and easily find the quantities needed to represent each problem.
   - Do this on chart paper to have room to draw the arrows. You can write the expanded notation at the top so it's closer to the answer.]