IES Institute of Education Sciences

Regional Educational Laboratory Northeast & Islands

at Education Development Center

# Mathematics Intervention Toolkit: Number Lines Module

## Participant Workbook

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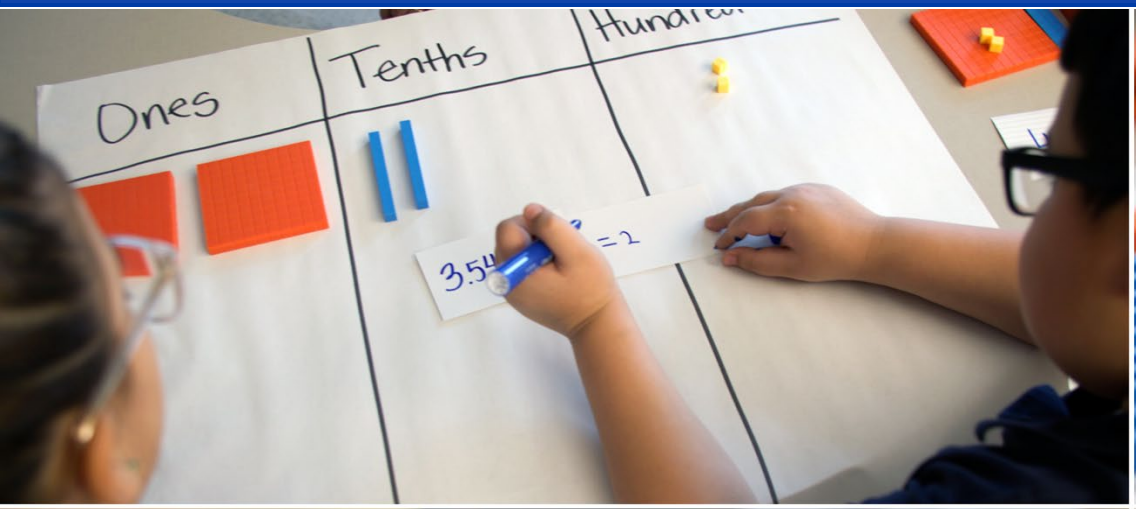
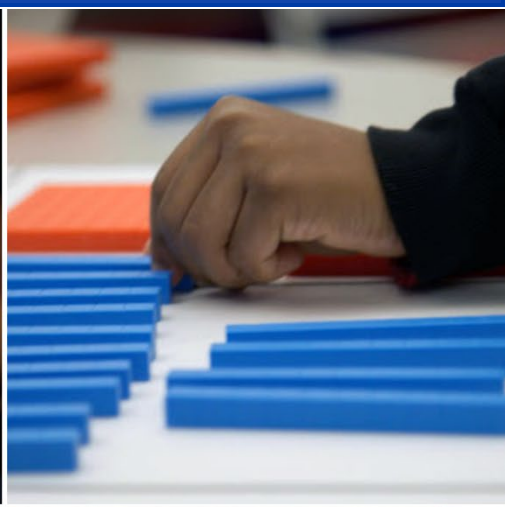
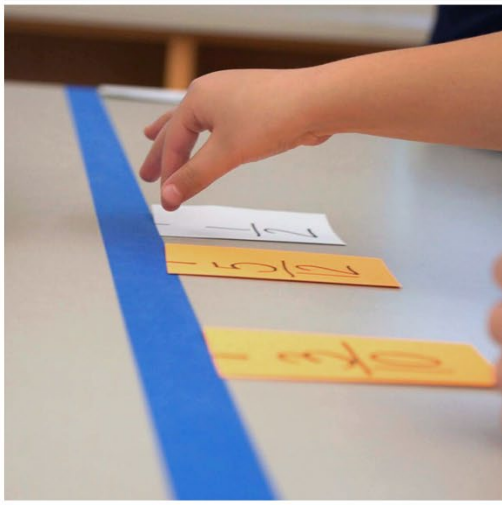




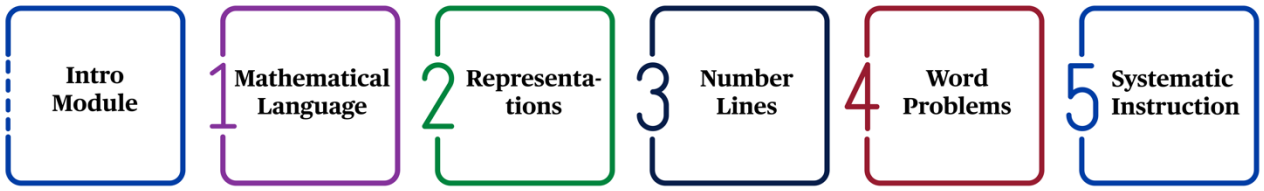
# Contents

# Introduction to the Course

**Welcome!** This professional development (PD) course is designed to build participants' knowledge and practices for supporting students struggling with mathematics. It focuses on the evidence-based recommendations of the What Works Clearinghouse Practice Guide *Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades* [1] (WWC Guide). These recommendations are based on a rigorous review and synthesis of research studies of effective intervention practices. The course is designed to connect this important research to participants' classroom practice.

The course has a series of **modules** to support in-depth professional learning. It starts with an Introductory Module and continues with five modules, each focusing on one recommendation (figure 1). The current module, Module 3, provides a deep dive into the recommendation for **number lines**.

**Figure 1. Course Sequence**




The course is specifically designed for **teachers of mathematics intervention in grades 3–6**. This includes teachers in different roles, such as interventionists, Title I teachers, math specialists, general educators, and special educators. Participants will be able to apply the strategies in a variety of intervention settings, including separate intervention classes, intervention/enrichment blocks, and designated times for intervention during core mathematics classes. Similarly, the course will support participants who use a variety of intervention programs/curricula or who do not have a program.

The full course is intended to provide about **28 hours** of professional learning during one or two school years. It uses a **hybrid format** that combines online learning, Professional Learning Community (PLC) sessions, and opportunities for classroom implementation. The course focuses on key **Number and Operations** topics, such as **fractions**, that are a high priority for mathematics intervention.


[1] Fuchs, L. S., Newman-Gonchar, R., Schumacher, R., Dougherty, B., Bucka, N., Karp, K. S., Woodward, J., Clarke, B., Jordan, N. C., Gersten, R., Jayanthi, M., Keating, B., and Morgan, S. (2021). *Assisting students struggling with mathematics: Intervention in the elementary grades* (WWC 2021006). U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. Retrieved from http://whatworks.ed.gov/.

# Module Overview

This module focuses on the WWC Guide's recommendation for number lines. Participants will explore evidence-based strategies for teaching number lines to students in mathematics intervention. Number lines are powerful tools for building students' understanding of the number system, including the magnitude of whole numbers, fractions, decimals, and other sets of numbers. Using number lines helps students to compare quantities, determine equivalencies, model operations, and represent word problems. This versatile representation supports students' learning of critical elementary standards, building an essential foundation for current and future study of mathematics.

**Recommendation for Number Lines**

Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics.

**Implementation Steps**

1. Represent whole numbers, fractions, and decimals on a number line to build students' understanding of numerical magnitude.
2. Compare numbers and determine their relative magnitude using a number line to help students understand quantity.
3. Use the number line to build students' understanding of the concepts underlying operations.

Source: WWC Guide, pp. 29–39

**Professional Learning Goals**

*Participants will:*

- Build knowledge of the WWC Guide's recommendation for using number lines to support students' learning of mathematics concepts and procedures.
- Build knowledge of evidence-based strategies for using number lines effectively with students struggling with mathematics.
- Strengthen their ability to plan for and implement strategies with students.

## Key Questions

*Participants will explore these questions:*

1. **What** is the WWC Guide's recommendation for number lines?
2. **Why** is understanding number lines important for student learning?
3. What are **strategies** for **how to** implement the recommendation?
4. What are **ways to apply** the recommended strategies with your students?
5. What are **potential challenges** and ways to address them?

## Mathematics Content Focus

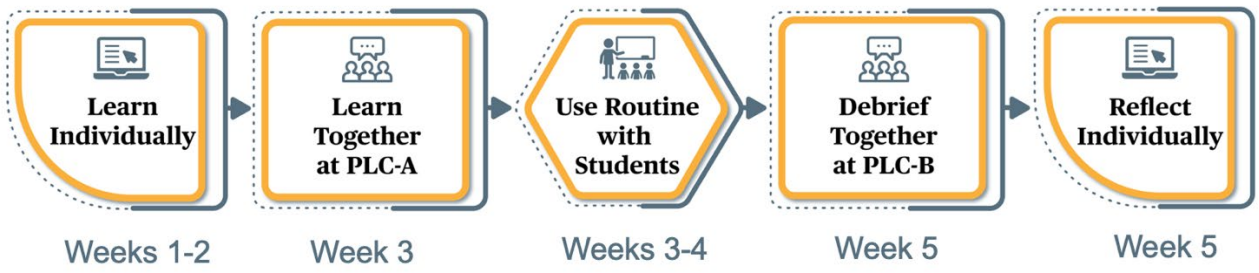
In this module, participants build knowledge and strategies for teaching **number lines** to students struggling with mathematics. The module's examples and activities focus on the following key concepts:

- Representing fractions on a number line.
- Using benchmark numbers to estimate the location of fractions on a number line.
- Using number lines to compare fractions and determine equivalencies.
- Using number lines to represent and solve fraction addition and subtraction problems.

# Module Sequence

This 5-week module provides 6 hours of professional learning, including an online session, two PLC sessions, and opportunities to apply strategies with students (figure 2).

**Figure 2. Sequence of Professional Development Activities in the Module**




The sequence begins with learning individually about the recommendation in the Online Session by doing self-paced, asynchronous activities, such as videos and readings. This session will prepare you for participating in the PLC sessions. At PLC Session-A, you and your colleagues will discuss the recommendation for number lines, try strategies, and prepare to use an instructional routine. After the session, you will use the routine with students one or more times. At PLC Session-B, you will share experiences with the routine and discuss ways to strengthen strategies. The module closes with the opportunity to reflect individually.

# Module Resources

The Number Lines Module includes the following resources for participants:

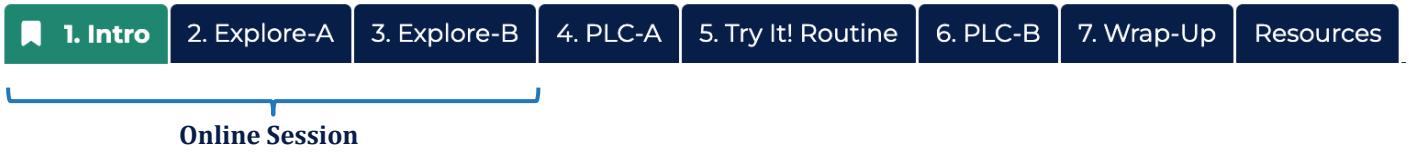
- **Online Component**: Provides activities and resources (see next section).
- **Participant Workbook** (current document): Provides all the handouts for the module. It also includes the Routine Teaching Guide, which has resources for using the routine. The appendices have reproducible student handouts and other resources.
- **Classroom Video Example**: Shows the routine for number lines in a grade 5 intervention class. There are two videos:
    - *Instructional Routine: Using a Benchmark Strategy to Place Fractions on a Number Line. Part 1*
    - *Instructional Routine: Using a Benchmark Strategy to Place Fractions on a Number Line, Part 2*
- **Video Demonstrations of Strategies**: In the Online Component, these two videos show how to use number lines to model fraction addition and subtraction:
    - *Using Number Lines to Model Operations: Fraction Addition*
    - *Using Number Lines to Model Operations: Fraction Subtraction*
- **Debriefing Slides Template**: Participants will use this file to create several slides about their experiences teaching the routine. The same template is used for Modules 1–4.
- **Optional: Slides for Teaching the Routine**: These slides are an optional resource that participants can choose to use when they teach the routine to students. Alternatives are to present the information by using chart paper, whiteboards, or other approaches.
- **Glossary**: Provides relevant mathematical terms with definitions for the course.

The **Mathematics Intervention Toolkit,** which includes all the course resources, is available for free at https://ies.ed.gov/ncee/rel/math-support-grades-3-6.

## Online Component

The module's Online Component provides resources and activities for the module. It is organized by tabs (figure 3), which are numbered to show the sequence.

**Figure 3: Tab Menu**




### Description of tabs:

- **Tabs 1–3, Online Session**: Provide self-paced, asynchronous activities for participants to do independently. The Intro tab provides an overview of the module's goals and key questions. On the Explore-A and Explore-B tabs, participants learn about the recommendation through videos, readings, and activities.
- **Tab 4. PLC Session-A**: Provides resources to use during and after the session.
- **Tab 5. Try It! Routine**: Provides information and resources to help participants prepare for and use the routine with students.
- **Tab 6. PLC Session-B**: Provides resources to use during and after the session.
- **Tab 7. Wrap-Up**: Provides short reflection activities.
- **Resources:** Provides a hyperlinked list of module resources and additional resources.

# Mathematics Intervention Course Checklist

Use this checklist to keep track of your progress in the course.

| | Introductory Module | Dates |
|---|---|---|
| ☐ | Kick-Off Session | |
| ☐ | Wrap-Up (Complete tab 4 of Online Component) | |
| | **Module 1. Mathematical Language** | |
| ☐ | Online Session (Complete tabs 1–3 of Online Component) | |
| ☐ | PLC Session-A | |
| ☐ | Try It!: Use Routine with Students | |
| ☐ | PLC Session-B | |
| ☐ | Wrap-Up (Complete tab 7 of Online Component) | |
| | **Module 2. Representations** | |
| ☐ | Online Session (Complete tabs 1–3 of Online Component) | |
| ☐ | PLC Session-A | |
| ☐ | Try It!: Use Routine with Students | |
| ☐ | PLC Session-B | |
| ☐ | Wrap-Up (Complete tab 7 of Online Component) | |
| | **Module 3. Number Lines** | |
| ☐ | Online Session (Complete tabs 1–3 of Online Component) | |
| ☐ | PLC Session-A | |
| ☐ | Try It!: Use Routine with Students | |
| ☐ | PLC Session-B | |
| ☐ | Wrap-Up (Complete tab 7 of Online Component) | |
| | **Module 4. Word Problems** | |
| ☐ | Online Session (Complete tabs 1–3 of Online Component) | |
| ☐ | PLC Session-A | |
| ☐ | Try It!: Implement Routine with Students | |
| ☐ | PLC Session-B | |
| ☐ | Wrap-Up (Complete tab 7 of Online Component) | |
| | **Module 5. Systematic Instruction** | |
| ☐ | PLC Session | |
| ☐ | Course Wrap-Up (Complete tab 3 of Online Component) | |

**Website URL**: https://ies.ed.gov/ncee/rel/math-support-grades-3-6.

# Module Checklist: Number Lines

Use this checklist to keep track of the module dates and your progress on the tasks.

**Module Dates:** _____ – _____

- **Online Session:** Complete during this time span: _____ to _____
- **PLC Session-A:** Attend session on date: _____ time: _____
- **Try It! Routine:** Implement during this time span: _____ to _____
- **PLC Session-B:** Attend session on date: _____ time: _____
- **Wrap-Up:** Complete by date: _____

## Tasks

### 1–3. Online Session.

**Complete the activities *before* PLC Session-A.**

- ☐ Tab 1, Introduction. Read about the module's goals, key questions, and sequence.
- ☐ Tab 2, Explore-A tab. Do online activities to learn about the recommendation.
- ☐ Tab 3, Explore-B tab. Do more online activities to continue learning.

### 4. PLC Session-A.

- ☐ Participate in the PLC session: Discuss the recommendation, try strategies, and prepare to use an instructional routine.

### 5. Try It! Routine.

**Implement the routine *before* PLC Session-B.**

- ☐ Use the routine one or more times with students.
- ☐ Prepare slides for sharing experiences. Use the Debriefing Slides Template.

### 6. PLC Session-B.

- ☐ Participate in the session: Share experiences using the routine by showing slides and focusing on the debriefing questions. Discuss common themes and plan next steps.

### 7. Wrap-Up.

**Complete the reflection activities *after* PLC Session-B.**

- ☐ Do the closing activities on tab 7, Wrap-Up, to reflect on your learning in the full module.

**Website URL**: https://ies.ed.gov/ncee/rel/math-support-grades-3-6.

# Handouts

**Online Session Handouts**



**PLC Session-A**



**PLC Session-B**



**Wrap-Up**

# H1. Reference Sheet: Number Lines Recommendation

## Directions

During the module, record notes below to create a helpful resource for future use.

### 1. What is the WWC Guide's recommendation?[2]

**Recommendation for Number Lines**

Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics.

### 2. Why is understanding number lines important for student learning?

*Add your ideas to the list.*

Understanding and using number lines helps students to:

- Represent and understand the magnitudes of whole numbers, fractions, and other numbers.
- Compare quantities and determine equivalencies.
- Build a strong foundation for future mathematics topics, such as positive and negative numbers and coordinate grids.

### 3. How do you carry out the recommendation? What are the implementation steps?

The WWC Guide provides three main implementation steps (below) that involve the use of evidence-based strategies (see next page).

**Implementation Steps**

1. Represent whole numbers, fractions, and decimals on a number line to build students' understanding of numerical magnitude.
2. Compare numbers and determine their relative magnitude using a number line to help students understand quantity.
3. Use the number line to build students' understanding of the concepts underlying operations.

[2] The text for the number lines recommendation and implementation steps are from the WWC Guide, pages 29–39.

## 4. What are strategies for how to implement the recommendation with students struggling with mathematics?

As you go through the module, use this chart to list strategies you want to remember and apply.

| Recommended Strategies |
| --- |
| **Represent numbers on the number line.**<br>• Place fraction tiles on a number line to determine where to locate fractions. Use a number line that has the same length from 0 to 1 as the fraction tile for 1 whole.<br>• |
| **Compare numbers on the number line.**<br>• Compare fractions with benchmark numbers, such as 0, 1/2, and 1, to estimate where to place fractions on a number line.<br>• |
| **Model operations on a number line.**<br>• Place fraction tiles on a number line to build the first addend and then add on the second addend. The combined length shows the sum.<br>• |

# H2. How to Carry Out the Recommendation

**Recommendation for Number Lines**

Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics.

Read this excerpt from the WWC Guide to learn about implementing the recommendation for number lines. The WWC Guide describes three main implementation steps; each involves the use of evidence-based strategies to support students struggling with mathematics. It includes sections on strategies for the-early elementary grades (K-2) and for upper elementary (3-6) to provide perspective on how number line ideas progress through the grades. Focusing on the progression across the grades builds understanding of number lines' breadth and flexibility for representing and performing operations with whole numbers, fractions, and decimals. The information on early elementary concepts and strategies can help upper elementary teachers provide support to students struggling with foundational number line ideas. They can also make connections to students' prior experiences with number lines to build their understanding of grade-level content. In addition, teachers can use this information to create review activities.

## Implementation Step 1. Represent Numbers

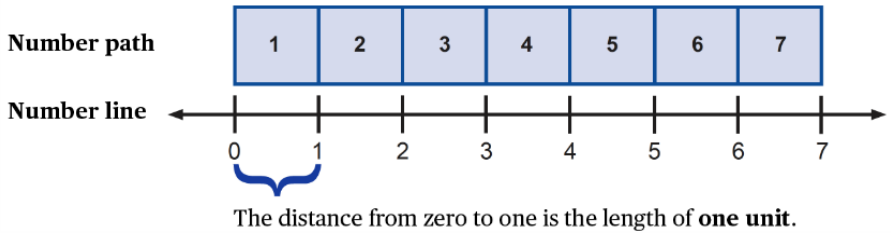
Represent whole numbers, fractions, and decimals on a number line to build students' understanding of numerical magnitude.

### Early elementary (grades K–2)

Before using a number line, introduce students to concrete versions of a number line. Introduce students to number lines by first using a number path that students can walk on or by playing board games; these representations may help students begin to form a visual image of a number line. When using these number lines, focus on the length of the units and the equivalency of the length unit. The distance between the positions of zero and one establishes the length of the unit and is the same distance between all consecutive whole numbers.

After exposing students to the concrete number line with a series of individual units lined up on a path, connect that idea to a number line on paper or projected on a screen. **Example 1** depicts how a concrete number path with a series of individual units corresponds to the tick marks on a number line. Ask students to identify similarities and differences between the two representations. Draw their attention to the distance from zero to one and how that distance is the same length as one unit. This connection will help students understand that the 1 on a number line is not merely a tick mark but also represents the full one-unit distance from zero. Discuss with students how the concrete units represent the same numbers as a number line does.

**Example 1. Connect concrete units to a number line to represent positive whole numbers.**




Explain the basic characteristics of number lines, such as those listed in the box below. Show students how each tick mark is equidistant from the previous tick mark and represents a unit of the length of 1. Notice how the whole numbers appear in the same predictable counting pattern. That is, when moving to the right on a horizontal number line, the magnitude of the number increases by one unit, just like counting by ones. Explain how numbers decrease by one unit as they move to the left and show students how zero is to the left of 1 on a horizontal number line. Introduce them to other counting patterns on the number line—such as skip-counting by twos, fives, or tens—by showing that the length of a new unit composed of 2, 5, or 10 units can be repeated across the distance of the number line.

**Characteristics of Number Lines Using Whole Numbers**

**Characteristics:**

- Each whole number on a number line is equidistant from the next whole number.
- Whole numbers are represented in a predictable sequence.
- Number lines increase or decrease to infinity; you can always add or subtract one more unit.
- Display number lines with arrows on both ends to show that the units go infinitely in both directions.
- On a horizontal number line, the numbers increase in value as you move to the right, and numbers decrease in value as you move to the left.
- On a vertical number line, the numbers increase in value as you move up, and numbers decrease in value as you move down.
- Number lines can be presented with a sequence of numbers that represent numbers in a predictable way (e.g., 0, 10, 20, 30 or 0, 2, 4, 6, 8, 10).

**Number lines can be:**

- Three-dimensional/concrete representations (e.g., Cuisenaire Rods®, a thermometer, or a ruler).
- Two-dimensional/semi-concrete representations (e.g., a drawing of a number line, a picture of a ruler, a scale on a thermometer, or a coordinate grid).
- Virtual number lines on a screen or as a mental image in a student's head.

## Upper elementary (grades 3–6)

Once students understand the concept of a fraction with concrete representations, show students how to represent fractions on a number line by connecting linear concrete representations to the semi-concrete number line. Demonstrate the location of fractions on the number line, starting with familiar fractions that are less than one. Ask students to fold a strip of paper that is the same length as a number line from 0 to 1 in half to see two equal parts. Discuss with students how this folded strip represents partitioning a 0–1 segment of the number line into two equal parts. Ask students to mark the location of one-half on the paper strip and then, use it to mark one-half on a number line. Then ask them to partition a strip of paper into four equal parts to represent the locations of one-fourth, two-fourths, and three-fourths on a 0–1 line segment. Ask students to demonstrate how the number line can be partitioned into additional parts, by showing a larger denominator, like eighths.

Reinforce the idea that the denominator represents the number of partitions in one whole. Include partitioning the number line with odd denominators which may be more difficult for students to partition equally. Number lines can be used to demonstrate the pattern of unit fractions and their corresponding magnitude.

To ensure that students do not assume all fractions are less than one, expand the 0–1 segment to 0–2 to depict fractions equal to and greater than one. Show students that whole numbers can be represented as fractions and that similar fractions are located between other whole numbers. **Example 2** shows two number lines. The first includes fractions equal to and greater than one so that a student can see the pattern of counting by fourths as each number increases by one-fourth. The second number line shows fraction equivalencies to the first number line with familiar fractions written between two whole numbers, which is how rulers are designed to measure length. Discuss how fractions greater than one can be expressed in two ways: With a numerator that is larger than the denominator as in the first number line and as a way to measure length, as in the second number line, which includes a whole number and a fraction less than one. This comparison of numbers expands students' ideas of fractions and measurement.

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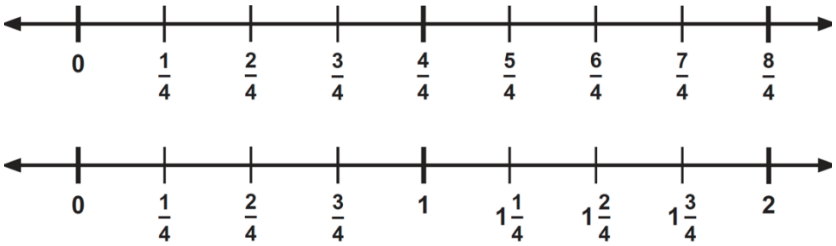
**Example 2. Fractions equal to, greater than, and less than 1.**




Once students conceptually understand and can articulate that the point where a fraction is located on the number line represents the length of units from zero to that position, then the same teaching steps can be used to concretely introduce the concept of equivalent fractions. Show students how equivalent fractions are positioned at the same point on the number line by sequentially partitioning a number line into different sized units. Introduce new denominators over several lessons.

In **Example 3,** the number line is first partitioned in halves, then into fourths, and then into eighths. Explain that some fractions are positioned at the same location on the number line and are therefore equivalent (for example, $\frac{1}{4}$ and $\frac{2}{8}$ are equivalent, $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ are equivalent, and $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent).

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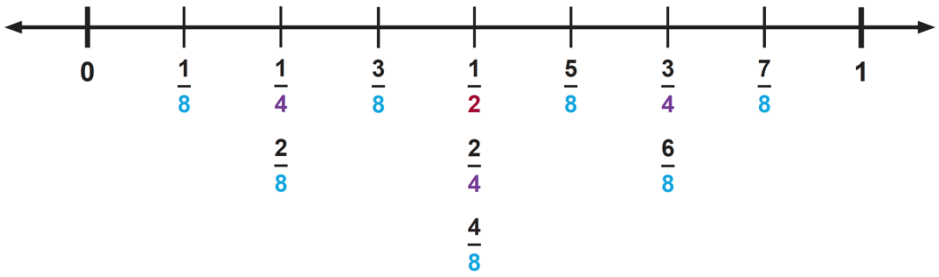
**Example 3. Equivalent fractions are positioned at the same point on the number line.**



**Incorporate other linear representations** to show how two fractions with different denominators can be equivalent and occupy the same distance on the number line. **Example 4** shows how Cuisenaire® Rods can be aligned with a number line to reinforce the equivalencies on a number line.

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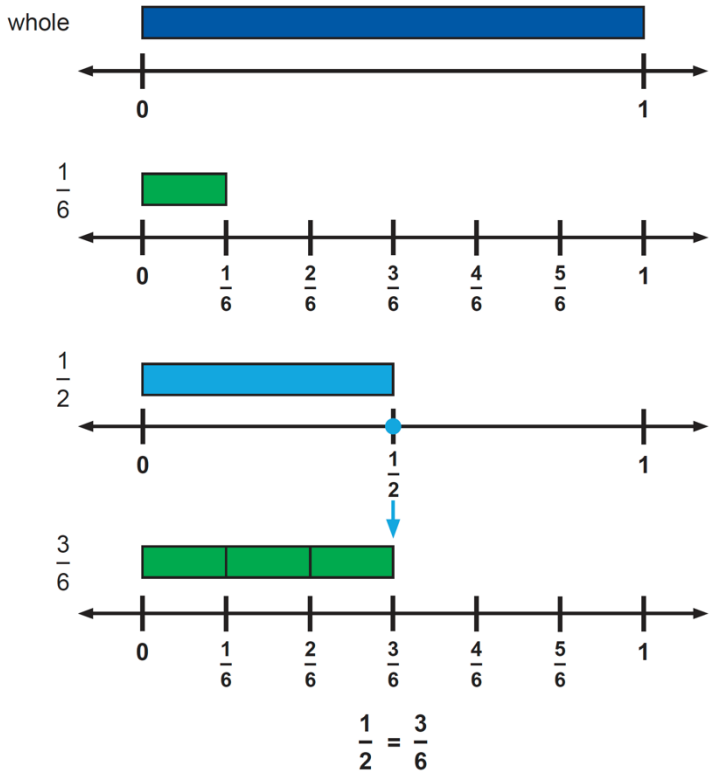
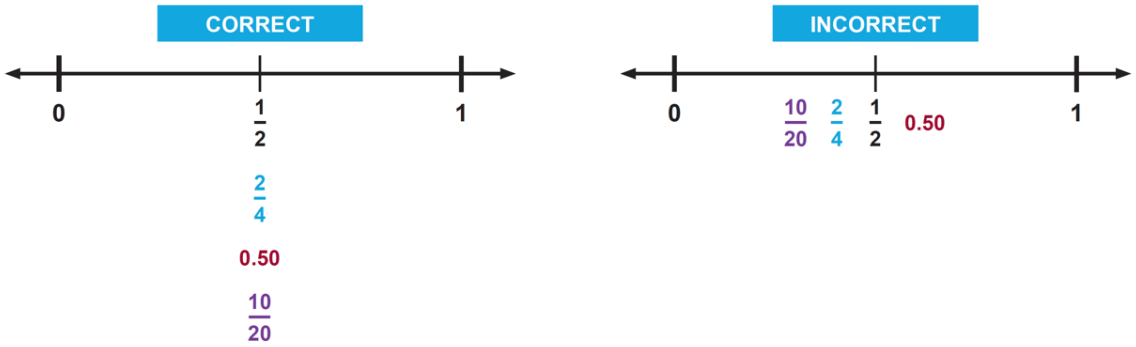
**Example 4. Connecting a concrete representation of a length to a number line.**



**Expand the idea of equivalencies** to include decimals and percentages so that students understand that these rational numbers are also equivalencies and there is an infinite number of equivalencies at any point on the number line. Reinforce this idea by writing the equivalent fractions underneath each other to show the precise position, rather than side by side when on the same number line. **Example 5** displays equivalent numbers underneath each other to ensure that the label aligns to the same tick mark.

**Example 5. Label tick marks that represent the same equivalencies vertically at the same position on the number line, rather than side by side.**




## Implementation Step 2. Compare Numbers

Compare numbers and determine their relative magnitude using a number line to help students understand quantity.

### Early elementary (grades K–2)

Use number lines to teach the relative magnitude of whole numbers. Start by putting two numbers on a number line using equal units. Explain that each number's distance from zero represents the number's magnitude. Whole numbers with greater magnitude are further to the right. Whole numbers with lesser magnitude are further to the left and therefore closer to 0. Explain how to compare numbers and determine which is greater based on which is more equal units away from zero (farther to the right when working with positive whole numbers).

### Upper elementary (grades 3–6)

Use number lines to compare the magnitude of fractions and decimals. Reinforce for students that fraction and decimal magnitude, like whole-number magnitude, is represented by how far to the right or left of zero a number is positioned. Before comparing fractions, students must understand that fractions have an infinite number of equivalencies, as shown in **Example 3**.

Comparing the relative magnitude of fractions with a number line supports students' understanding of fraction values and how they relate to one another. Help students compare fraction magnitude by thinking about "benchmark numbers," starting with 0, $\frac{1}{2}$, and 1 when thinking of fractions between 0 and 1. Benchmark numbers can be located on a number line, and then other fractions can be compared with the benchmark numbers to describe which fractions are greater than or less than other fractions. This knowledge can then be used as a strategy for students when they evaluate fraction magnitude during other activities, such as when comparing two fractions and showing the relationship using a greater-than or less-

than sign or ordering a set of fractions from least to greatest or greatest to least. Evaluating fraction magnitude can also be useful when predicting the approximate magnitude of a computational solution.

### Early and upper elementary (grades K–6)

Provide students with practice determining the magnitude of whole numbers, fractions, and decimals using a number line. This type of activity will build familiarity with number lines. Students receiving intervention will need ample practice using number lines to become more proficient in estimating magnitude. Present students with a number line with two points marked near the end (for example, 0–1, 0–2, 0–100, or 0–1,000) and ask them to estimate magnitude for whole numbers and fractions.

**In Example 6**, students work as a group to estimate fraction magnitude using benchmark numbers. Four fractions are given on index cards for students to place on the 0–1 number line. Here's a description of how students might use benchmark numbers, reasoning, and estimation to place the fractions:

- First, the students place the benchmark number one-half on the number line before estimating the magnitude of each fraction.
- For $\frac{7}{12}$, a student reasons that the fraction is a little to the right of $\frac{1}{2}$ because $\frac{7}{12}$ is just $\frac{1}{12}$ greater than $\frac{6}{12}$, which is equivalent to $\frac{1}{2}$.
- The next student also uses one-half as a benchmark for $\frac{3}{8}$, reasoning that the fraction is just $\frac{1}{8}$ less than $\frac{4}{8}$, which is also equivalent to $\frac{1}{2}$.
- For $\frac{1}{5}$, a student places it closer to 0 because it is a unit-fraction and less than $\frac{1}{2}$.
- For $\frac{9}{10}$, a student places it close to 1 because it is just $\frac{1}{10}$ away from $\frac{10}{10}$ or the whole.
- Ask students to discuss their reasoning for making the placement of each fraction with the group and pose additional questions to explore the depth of the conceptual understanding as necessary.

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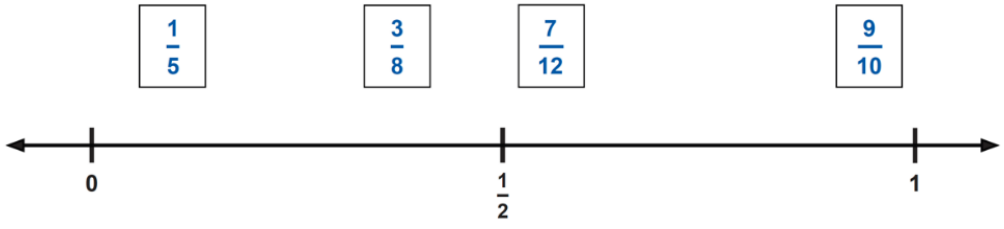
**Example 6. Students estimate the location of four fractions using benchmark numbers and place the fraction cards on the 0–1 number line.**



## Implementation Step 3. Model Operations

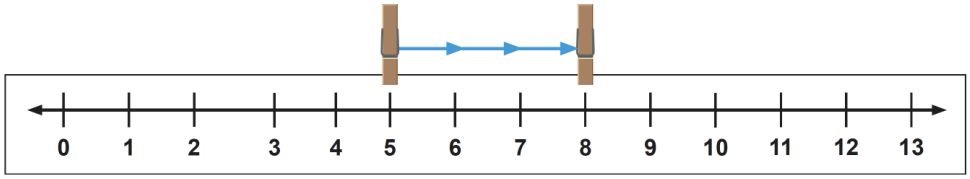
Use the number line to build students' understanding of the concepts underlying operations.

### Early elementary (grades K–2)

Show students how to use number lines for addition and subtraction of whole numbers. After comparing whole numbers, students begin to learn addition and subtraction by looking at the distance between whole numbers. In **Example 7**, students first determine that 8 is greater than 5. Then, students see how much greater 8 is than 5 by counting up. They can visibly see that 8 is 3 units to the right of 5. The focus on the unit length, or distance, is key rather than counting the tick marks.

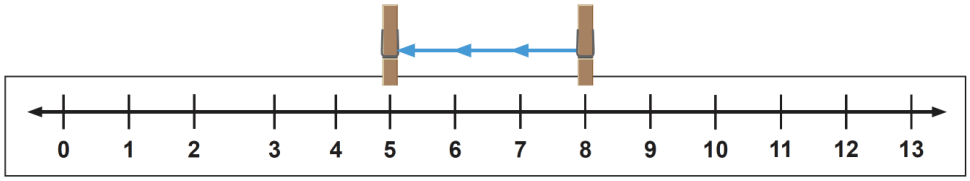
**Example 7. Show students how to use number lines to add and subtract whole numbers.**

When moving toward the **right**, students see that 5 units plus 3 more units is equal to 8 units.




5 + 3 = 8

When moving to the **left** students see that starting with 8 and moving 3 units to the left is equal to 5, showing the subtraction equation 8 - 3 = 5.




8 – 3 = 5

Always connect the equation to the number line when solving calculation problems with students. First, model a problem with the number line and have students write the corresponding equation. Then present an equation for students to model on the number line.

### Upper elementary (grades 3–6)

A number line is also a powerful visual for demonstrating addition and subtraction of fractions. Start by adding fractions with the same denominator. This can be done using one number line. When introducing addition and subtraction with unlike denominators, explain that number lines are particularly helpful. Number lines help make visible the concepts underlying addition and subtraction of fractions when the two addends have different denominators. Using double number lines can make the equivalencies more visible for students so that they can understand why finding an equivalent fraction is the necessary and correct approach for solving these types of problems.

**Example 8** depicts an addition problem where double number lines are used to show students that $\frac{1}{2}$ and $\frac{6}{12}$ are the same distance from 0, and therefore, the same magnitude. This illustration helps students see why they can use equivalent fractions when adding and subtracting fractions with unlike denominators.

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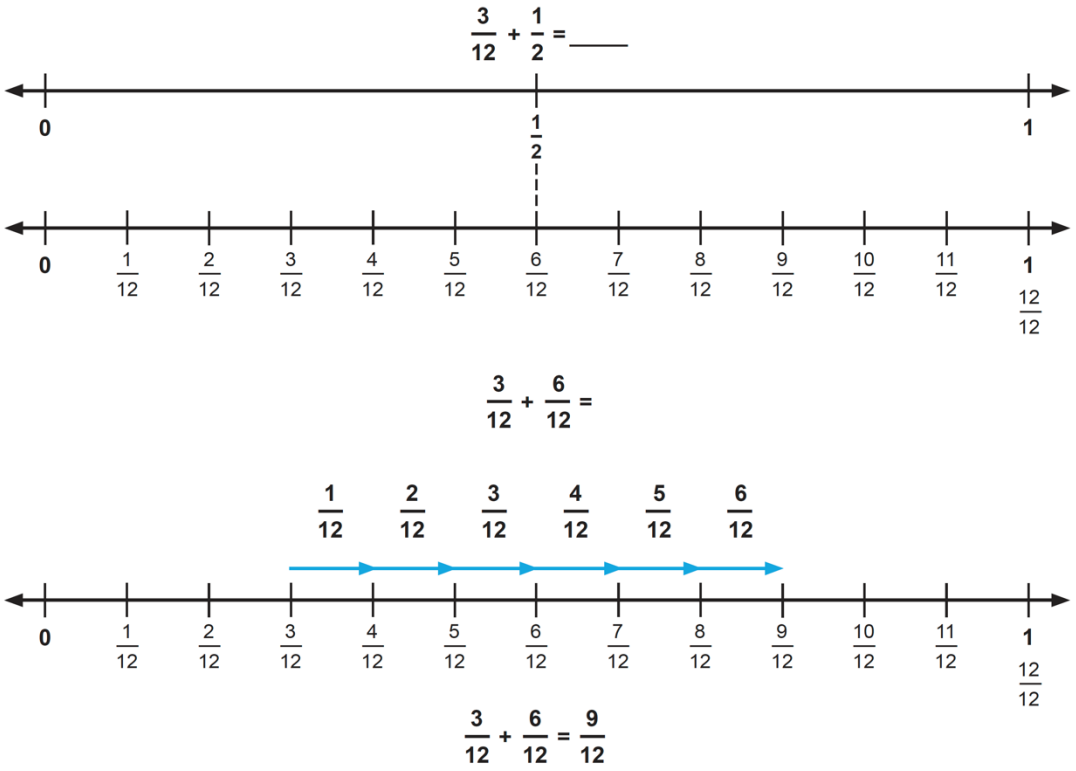
**Example 8. Use the number line to show students fraction addition.**



## Multiplication and Division with a Whole Number and a Fraction

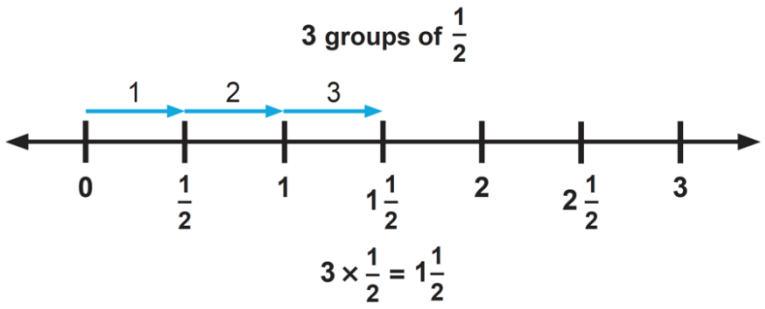
Concepts of multiplication and division can also be shown on a number line. When first introducing fraction multiplication and division, include whole numbers as one of the factors, divisors, or dividends. Start with a word problem to set the stage for understanding the concept underlying the operation. In these instances, a number line is useful for showing the problem because the number line effectively represents whole numbers and fractions. **Examples 9** and **10** show how number lines are used to depict multiplication and division with whole numbers as one of the operands.

---

**Example 9. Multiplication with a fraction and a whole number.**

Use a word problem to provide context for the operation:

> Arya runs a $\frac{1}{2}$-mile loop in her neighborhood. She does this loop 3 times each morning for exercise. How many miles does Arya run for exercise each morning?

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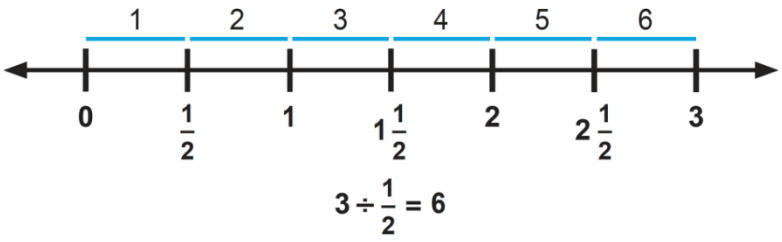
**Example 10. Division with a fraction and a whole number.**

Use a word problem to provide context for the operation:

> Twyla baked 3 large brownies. She cuts each brownie in half to share with friends. How many pieces of brownie does Twyla have to share?




In the above examples, the number line is a helpful representation for modeling multiplication and division of a whole number and a fraction. However, for multiplication and division with **two fractions less than 1** the number line is **not** the best representation, especially when fractions have large denominators.[3] Instead, use an area model for multiplication when the fractions are both less than one. (An example of an area model is provided in the handout, H7. Challenges and Suggestions.)

[3] This last paragraph is based on the WWC Guide, page 39, and was added to this handout by the Toolkit authors.

# H3. Video Observations

## Background Information

The two videos (Part 1 and 2) show an intervention teacher and grade 5 students using an instructional routine for number lines.

## Directions

- In the **Online Session**, watch the video, Instructional Routine: Using a Benchmark Strategy to Place Fractions on a Number Line (Part 1) individually to take a close look at strategies. Write notes on this handout below.
- At **PLC Session-A**, you and your colleagues will rewatch an excerpt of Part 1, and then watch Part 2 so that it's fresh in your minds for the discussion and for preparing to use the routine.

**Video Watching Norms**

- Observe, without judging, the teacher and students.
- Look for ideas to apply in your practice.

## Focus Question

As you watch, focus on this question and write notes:

1. What **instructional strategies for number lines** do you notice in the video?

## Ideas to Share at PLC Session-A

2. What are **one or two ideas** from the video that stood out for you?

# H4. Explore Example Activities

## Goal

Explore example activities for building students' understanding of the number line representation, including locating fractions, finding equivalent fractions, and partitioning. The activities incorporate strategies from the WWC Guide for connecting concrete linear representations (fraction tiles and paper strips) with number line diagrams.

## Directions

1. Look over three example activities. Then, choose **one** activity to focus on:

   ____**A.** Use fraction tiles to locate fractions on number lines.

   ____**B.** Use multiple number lines to find equivalencies.

   ____**C.** Fold paper strips to partition number lines.

2. Do the activity yourself to get firsthand experience with the tasks for students.
3. Consider ways to use the activity with students by answering the reflection questions below.

## Reflection Questions

Write responses to these questions *after* you try the activity.

1. How would this activity be helpful for building students' understanding of the number line representation for fractions? What key ideas does the activity bring out?

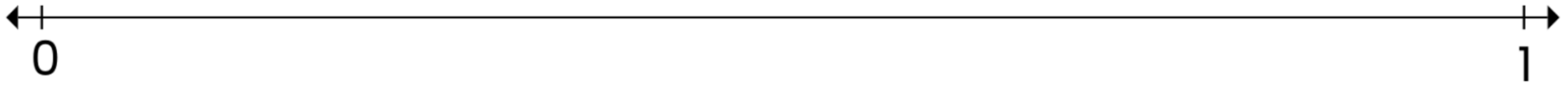
2. What are your suggestions for how to use this activity with students?

# Activity A. Use Fraction Tiles to Locate Fractions on a Number Line

1. Use fraction tiles to locate six fractions on the number line below. This number line is 8 inches long from 0 to 1, which is the same length as the fraction tiles for one whole.

   Place the tiles above the number line. Draw a point to show the fraction's location and write the fraction below it.

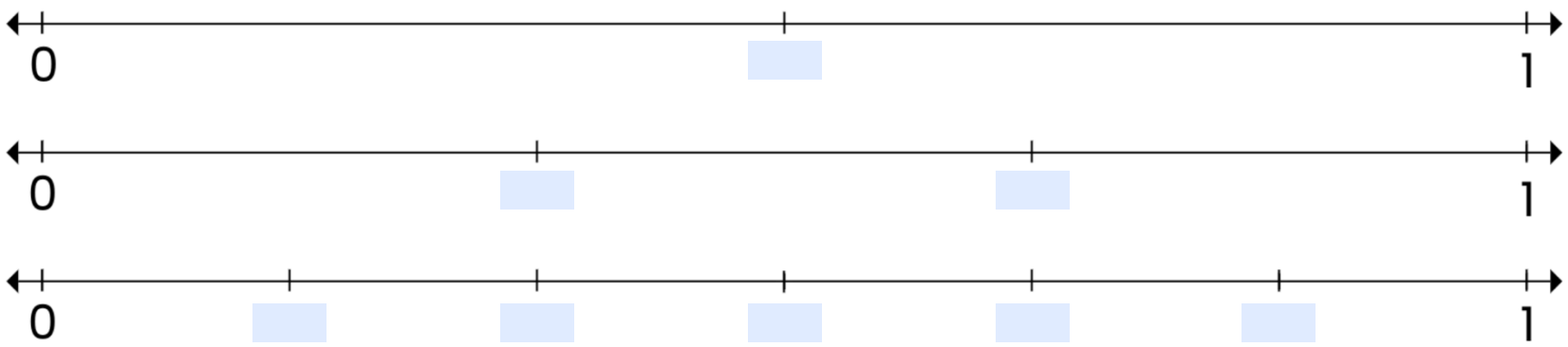
   a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. $\frac{3}{4}$ d. $\frac{5}{8}$ e. $\frac{3}{8}$ f. $\frac{4}{8}$




2. Use these clues to come up with three new fractions and write them below. Place each new fraction on the number line by using fraction tiles and label it.

| | |
|---|---|
| **Clue A.** Place a new fraction that is **equivalent to** $\frac{1}{2}$. | $\frac{\quad}{4}$ |
| **Clue B.** Place a new fraction that is **closer to 0 than to** $\frac{1}{4}$. | $\frac{\quad}{8}$ |
| **Clue C.** Place a new fraction that is **closer to 1 than to** $\frac{3}{4}$. | $\frac{\quad}{8}$ |

**3.** Label the tick marks on each number line with fractions. **Tip:** Place fraction tiles on the number lines to help you.




**4.** Use the number lines and fraction tiles to find equivalent fractions. Fill in the missing numerators and denominators.

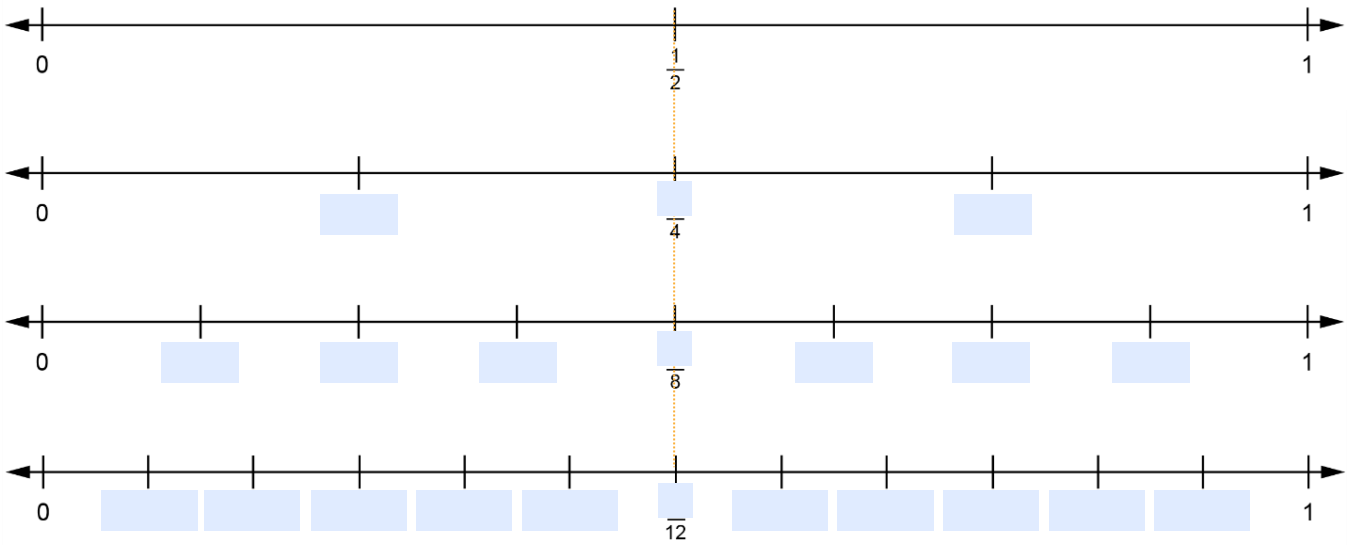
**a.** $\frac{1}{3} = \frac{\square}{6}$ **b.** $\frac{4}{6} = \frac{2}{\square}$ **c.** $\frac{\square}{2} = \frac{3}{6}$

**d.** $1 = \frac{\square}{3}$ **e.** $\frac{3}{\square} = \frac{1}{2}$ **f.** $\frac{6}{\square} = 1$

# Activity B. Use Multiple Number Lines to Find Equivalencies

These four number lines have 0 and 1 at the same locations, so they are helpful for finding **equivalent fractions**. An example dotted line shows fractions that are equivalent to $\frac{1}{2}$.

1. Label **all** the tick marks with fractions on each number line.




2. Use the number lines to find **two or more different fractions** that are **equivalent** to each fraction below. Write the equivalent fractions below.

   **Tip:** Place a ruler or straightedge vertically across the four number lines to see which fractions are at the same locations and are equivalent.

   a. $\frac{1}{4}$ is equivalent to: $\frac{\quad}{8}$ $\frac{\quad}{12}$

   b. $\frac{6}{8}$ is equivalent to: _____

   c. $\frac{1}{2}$ is equivalent to: _____

# Activity C. Fold Paper Strips to Partition Number Lines

*This handout provides teachers with information and examples for leading an activity.*

**Goal:** To use a linear concrete representation, folding paper strips into equal parts, to build understanding of partitioning number lines and locating fractions.

## Materials

The paper strips and the number lines need to be the **same length**. These directions use a length of 8″ long so that it is the same size as the fraction tile for 1 whole. You can choose a different length.

- One paper strip per student plus extras. Cut strips 8″ long and 1″ wide.
- One blank number line (8″ long) drawn on paper per student.
- For teacher: Draw one number line (8″ long) on chart paper or whiteboard.
- Sets of fraction tiles to use as needed; connect tiles to strips and number line.

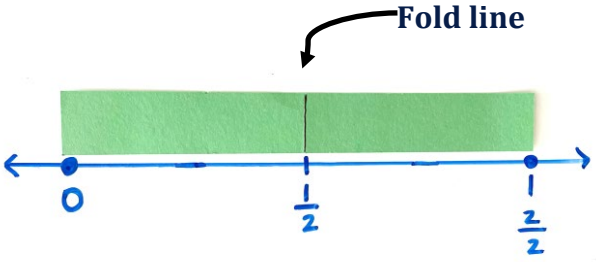
## Activity Sequence: Halves, Fourths, and Eighths

This sequence includes teacher questions and example answers in brackets and dark orange text.

1. Display on chart paper or a whiteboard a number line with arrowheads on both ends and a distance of exactly 8″ between the labeled points 0 to 1.
2. Introduce the task by explaining to students that we will be **folding paper strips** as a strategy for partitioning the number line. Provide each student with one paper strip measuring 8″ × 1″. They will use this same paper strip to fold halves, fourths, and eighths.

### *Halves*

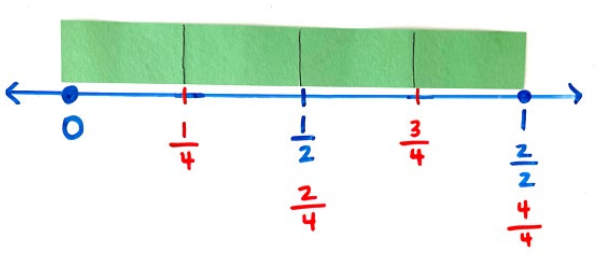
3. Ask students: How could we fold this paper strip in half? Model how to equally fold by lining up the short edges and corners.
4. Line up the folded strip with the number line on your board. Show students how to use the fold line to mark the location of $\frac{1}{2}$ on the number line, explaining that this fold line is exactly halfway between 0 and 1. The distance from 0 to $\frac{1}{2}$ is exactly the same as the distance from $\frac{1}{2}$ to 1. Ask:
   - Why does it make sense that this tick mark is labeled $\frac{1}{2}$? [This location is half the distance from 0 to 1.]
   - What does the denominator 2 represent? [The denominator 2 represents that the whole was divided into 2 equal parts.]




### *Fourths*

5. Ask students: How can we fold our strips into fourths? Model how to refold the strip into halves and then fold the strip in half *again* to make four equal parts. Mark the fold lines by drawing lines on them. Ask: What fraction does each part represent? [$\frac{1}{4}$ or fourths]

6. Next, line up the folded strip with the same number line. Label the tick marks as shown in the picture.

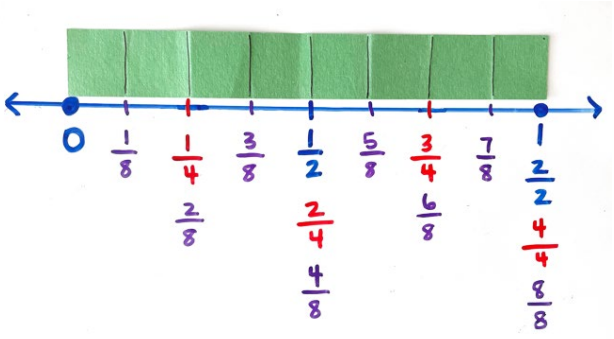



   - Why does it make sense that our number line shows fourths? [Our number line shows 4 equal parts, just like our folded strip. Each part is $\frac{1}{4}$.]
   - What does the denominator 4 represent? [The denominator 4 represents fourths. The whole was divided into 4 equal parts.]
   - Why is $\frac{2}{4}$ labeled at the same point as $\frac{1}{2}$? [Our folded strip shows that $\frac{1}{2}$ and $\frac{2}{4}$ are at the same location on the number line. These fractions are equivalent.]

**Tip:** Students may incorrectly think the number of folds should be the same as the number of parts. For example, they may think that the strip should have four fold lines (partitions) to get fourths. In actuality, the strip will be folded twice and have three fold lines that partition the strip into four equal parts for fourths.

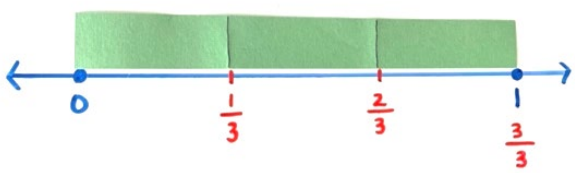
### *Eighths*

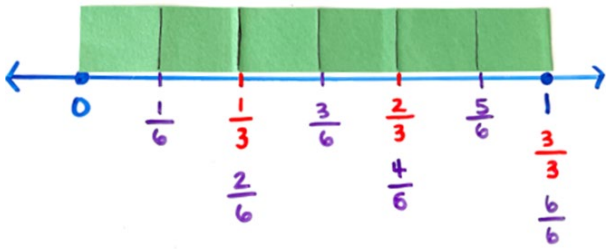
7. Repeat the folding process to create eighths. Use the strip to make partitions on the number line. Ask similar questions as the ones above for fourths.



## Optional: Fold Thirds and Sixths

To fold strips into **thirds** and **sixths**, provide each student with a new paper strip and a new number line and use the same process described above.



# H5. Explore Fraction Addition on the Number Line

## Overview

This PD activity for teachers builds on the Fraction Addition video (on the Explore-B tab).

## Materials

A set of fraction tiles, blank paper, and markers or colored pencils.

## Part 1. Review the Approaches from the Video

Review two approaches for the problem $\frac{1}{2} + \frac{3}{8}$. Then, you will apply them in part 2.

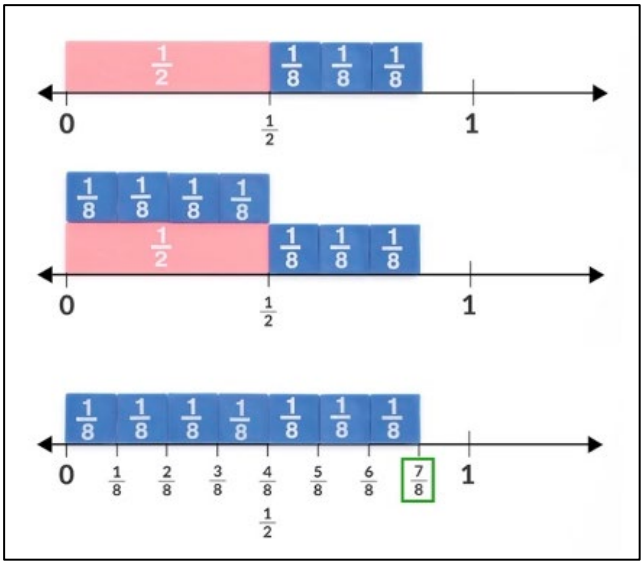
### Approach 1. Use fraction tiles on a number line.

$$\frac{1}{2} + \frac{3}{8} = ?$$

Place fraction tiles on the number line to build each addend: $\frac{1}{2}$ and $\frac{3}{8}$. The combined length shows the sum.

To determine the value of the sum as a fraction, find equivalent fractions so that all the pieces have the same denominator. The example shows that four of the $\frac{1}{8}$ tiles are the same length as the $\frac{1}{2}$ tile, which means that they are equivalent.

Use the tiles to partition the number line into eighths by drawing tick marks and labeling them. The sum of $\frac{1}{2}$ and $\frac{3}{8}$ is $\frac{7}{8}$.

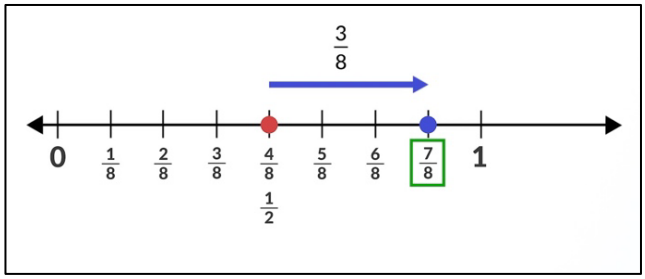



### Approach 2. Draw points and arrows to represent addends on a number line.

Draw a point for the location of the first addend, $\frac{1}{2}$. To represent the second addend, $\frac{3}{8}$, partition the number line into eighths.

Then, draw an arrow that starts at the point for $\frac{1}{2}$ or $\frac{4}{8}$, and count on units for a length of $\frac{3}{8}$.

The arrow goes to the **right** because we are adding a positive fraction, which will increase the distance from 0. The end of the arrow shows the sum of $\frac{7}{8}$.



## Part 2: Try Approaches with a New Problem

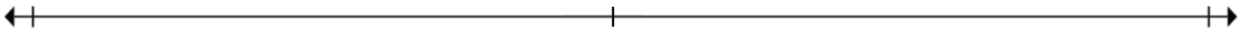
1. **Approach 1: Use fraction tiles on a number line to model and solve: $\frac{2}{6} + \frac{1}{2} =$**
   a. Draw an 8″ number line so it is the same length as the one whole fraction tile.

   b. Represent the first addend by placing tile(s) on the number line. Then, add on tiles for the second addend.

   c. Replace the tiles of one or both fractions to use a common denominator. Partition and label the number line to show the addends and the sum.

2. **Approach 2: Draw points and arrows on a number line to model and solve: $\frac{3}{8} + \frac{1}{4} =$**
   a. Partition the number line below for the fractions in the problem. Label the tick marks.

   b. Mark a point for the first addend and draw an arrow for the second addend. Label the sum.



## Part 3: Reflection Questions

1. What are benefits of using **fraction tiles** to model fraction addition on the number line?

2. What are benefits of **drawing points** and **arrows** to model fraction addition?

3. What are your **suggestions** and **questions** about using each approach with students?

# H6. Explore Fraction Subtraction on the Number Line

## Overview

This PD activity for teachers builds on the Fraction Subtraction video (on Explore-B tab).

## Materials

A set of fraction tiles, blank paper, and markers or colored pencils.

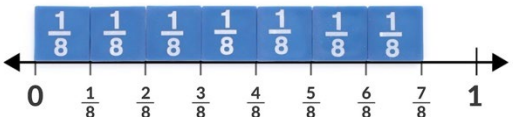
## Part 1: Take-Away Model for Subtraction

*Review the approaches from the video and then apply them.*

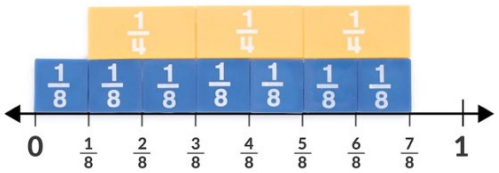
$$\frac{7}{8} - \frac{3}{4} = ?$$

### 1. Use fraction tiles

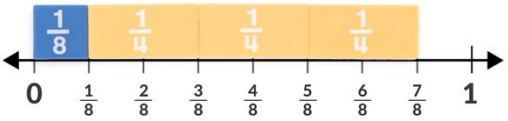
The first fraction (minuend) is modeled with seven $\frac{1}{8}$ tiles.




Then the second fraction (subtrahend) is taken away to find the difference. Place three $\frac{1}{4}$ tiles on top of $\frac{7}{8}$ to show the length that will be subtracted or taken away.




Replace the tiles for $\frac{6}{8}$ with the tiles for $\frac{3}{4}$ because they are equivalent. Then take away the tiles for $\frac{3}{4}$.




There is $\frac{1}{8}$ left. It is the difference.

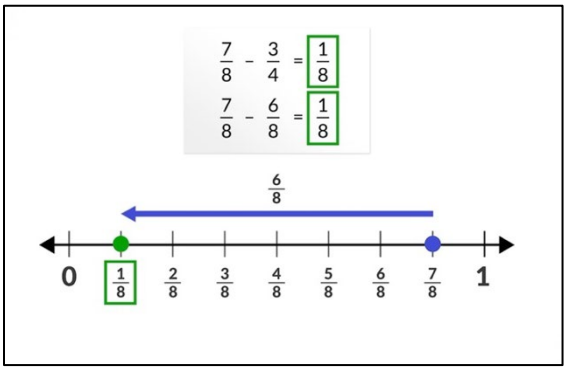

$$\frac{7}{8} - \frac{3}{4} = \frac{1}{8}$$

### 2. Draw points and arrows

Partition the number line into eighths to be able to represent both fractions.

The first fraction (minuend) is represented with a **point** on the number line.

Subtracting the second fraction (subtrahend) is shown with an **arrow that goes toward 0**. The arrow has a length of $\frac{3}{4}$, which is equivalent to $\frac{6}{8}$. Draw a **point** where the arrow ends to show the difference.



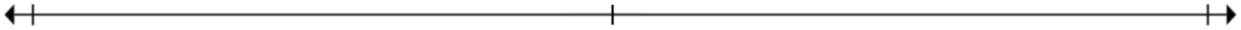
3. **Use the take-away model** for this problem. (Note: In this handout, the word problems are written in purple font so that they stand out.)

   **Problem.** Gavin has a piece of rope that is $\frac{5}{8}$ of a yard long.
   He cuts off a length of rope that is $\frac{1}{4}$ yard long and uses it for a project.
   How long is the rope that Gavin has left?

   Choose which method you will use to represent and solve the problem.

   ___ **a.** Use fraction tiles. Draw a number line 8" long so it is the same length as the tile for 1 whole.

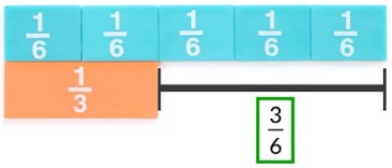
   ___ **b.** Make a number line and draw points and arrows to model the problem.



## Part 2: Comparison Model for Subtraction

*Review the approaches in the video and then apply them.*

**1. Use fraction tiles to model and solve $\frac{5}{6} - \frac{1}{3}$**

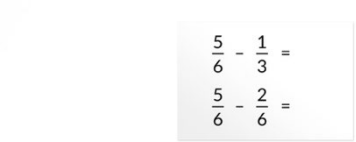



Use the fraction tiles to build each fraction, one above the other, to compare their lengths. Then think about what tile would fill the gap in the shorter length to make it equal in length to the other fraction. That is the difference. This example shows that the difference between $\frac{5}{6}$ and $\frac{1}{3}$ is $\frac{3}{6}$.

**2. Locate each fraction and determine the distance between them**

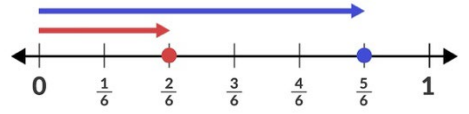
Partition the number line into eighths to be able to represent both fractions.

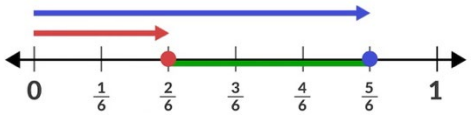
Represent each fraction, $\frac{5}{6}$ and $\frac{1}{3}$, as a **point** on the number line.




Determine the distance between the two points.

The difference between $\frac{5}{6}$ and $\frac{1}{3}$ is shown by the distance between them: $\frac{3}{6}$ (shown by the green line segment).



**3. Use the comparison model with tiles or by drawing points and arrows** for this problem:

> **Problem.** Tia ran $\frac{6}{10}$ of a mile. Jackie ran $\frac{1}{5}$ of a mile.
> How much farther did Tia run than Jackie?

Use one of the approaches to represent and solve the problem.

___ **a.** Use fraction tiles. Draw a number line 8" long so it is the same length as the tile for 1 whole. Represent each fraction with the tiles. Determine the difference between their lengths.

___ **b.** Draw points for each fraction on a number line and determine the distance between them.

## Part 3. Reflect on Your Experiences with the Approaches

**1.** Why is it **important** to use different approaches for modeling subtraction on the number line?

**2.** What are the **benefits** of using each approach? Write responses for the ones that you tried.

**a.** Take-Away Model: ___Use fraction tiles ___Draw points and arrows

**b.** Comparison Model: ___Use fraction tiles ___Find the distance between points

**3.** What are your **suggestions** and **questions** for using these approaches with students?

# H7. Challenges and Suggestions

## Directions

Read about potential challenges and the expert panel's advice.[4] Then, choose **one challenge** (A, B, or C) to focus on and add your responses to the questions.

**Challenge A**

"I used the number line for fraction multiplication and my students were confused."

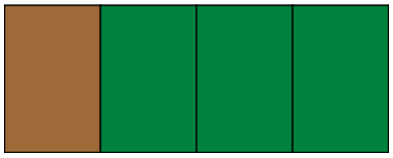
## Panel's Advice

Number lines are not always the best representation to help students understand all mathematical ideas. Multiplication and division with two fractions of less than 1 are not represented well on a number line, especially when fractions have large denominators. Instead, try using an area model for multiplication when the fractions are both less than 1 (see Example 1). More examples of the number line model for operations are in the handout How to Carry Out the Recommendation (**H2**).

**Example 1: Area model for multiplying $\frac{1}{3}$ and $\frac{1}{4}$.**

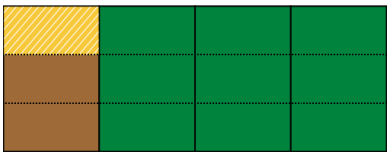
We are looking for $\frac{1}{3}$ of $\frac{1}{4}$.

Let's partition into the fourths by drawing vertical lines. Shade one part to show $\frac{1}{4}$ (in brown).



Next, let's partition into thirds by drawing horizonal lines.

Shade $\frac{1}{3}$ of $\frac{1}{4}$. This yellow shaded area shows $\frac{1}{3}$ of $\frac{1}{4}$. The yellow part shows the product: $\frac{1}{12}$.



$\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$ $\quad \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

## Add Your Ideas

1. Which operations with whole numbers or fractions do you find particularly helpful to model on the number line?

2. What are your suggestions for how to use an area model or other approaches for fraction multiplication of two fractions of less than 1?

[4] Excerpt from WWC Guide, page 39; Example 1 and questions from the Toolkit authors.

**Challenge B**

"My students don't want to use the number line and benchmark fractions when comparing fractions because cross-multiplying is easier and faster."

## Panel's Advice

Some teachers present cross-multiplying as a way to compare fractions, probably because it is fast. The panel believes, however, that cross-multiplying does not help students understand the comparison of fractions in a meaningful way. Help students see that using benchmarks and thinking about the relative magnitude of fractions will help them understand operations with fractions more deeply. Do not allow students to revert to cross-multiplying during intervention and continue to focus on fraction concepts and the understanding of procedures, which will be more helpful to students later on.

## Add Your Ideas

1. What kinds of activities are helpful for building students' abilities to use the number line to compare fractions?

2. What are your suggestions for helping students see the benefits of comparing fractions on a number line?

**Challenge C**

"My students don't seem to have a good grasp of the number line and what it represents."

## Panel's Advice

Concrete representations can be used at any grade level to support students' understanding of number lines. Use concrete representations with length models to help transition students toward understanding the number line. Show students how to build a number line with manipulatives that use consistent and equal length units. Fraction tiles and Cuisenaire Rods® can be linked to number lines, for example. Be sure to show students which edge of the tile or rod represents the length of the unit or the fraction. Demonstrate this by lining up the rods on the number line (starting from zero) to show how a fraction's length connects to its location on the number line.

## Add Your Ideas

**1.** What have you observed about your students' understandings and difficulties with number lines?

**2.** What are your suggestions for ways to use concrete representations, such as fraction tiles, or other strategies to build students' understanding of the number line representation?

# H8. Reflection for Online Session

**Directions:** Reflect on your learning by writing responses to these questions. Bring your completed handout to PLC Session-A so you can refer to the ideas you recorded.

1. What are **two important ideas** that you learned about the recommended strategies for using number lines?

2. What is **one instructional strategy** for number lines (from this online session) that you are interested in trying with students or that you currently use? How is this strategy helpful for students?

3. What is **one idea, strategy,** or **question** about using number lines with students that you want to discuss with colleagues at PLC Session-A?

# H9. Use Questioning Strategies

This handout lists example questions to provide a resource for teachers to use in planning and instruction. Feel free to put the questions into your own words and to add questions.

[Text in brackets] indicates words you can replace to fit your lesson.

## Example Questions

### Interpreting number lines

- Where is 1 whole located on the number line?
- What are the least and greatest numbers shown on the number line?
- What do the tick marks represent?
- How is the number line partitioned? Describe what you notice.
- What fractions are shown on the number line?

### Creating number lines

- What are the least and greatest numbers you need to show on your number line?
- How would you partition the number line to show the fraction ___?
- What strategies can you use to make equal partitions on your number line?
- What are important things to check to make sure your number line is correct?

### Locating and comparing fractions on number lines

- Where would you place the fraction ___ on the number line? Why?
- Is the fraction __ greater than or less than the benchmark number of $\frac{1}{2}$? Why?
- Is the fraction __ closer to 0, closer to $\frac{1}{2}$, or closer to 1? Why?
- Is the fraction __ greater than [less than; equivalent to] the fraction ___?
- What does the location of the fractions on the number line tell you about their size?
- How can you use a number line to compare two fractions?
- How can you use a number line to determine whether two fractions are equivalent?

## Adding and subtracting fractions on number lines

- How would you partition the number line to be able to show fractions in the problem?
- Estimate first: Will the sum [difference] be less than or greater than 1 [$\frac{1}{2}$]? Use the number line to help you estimate.
- Which direction will you move on the number line to model adding [subtracting] a fraction?
- What strategies did you use to add [subtract] fractions on the number line?
- How do you know that the sum is correct? Use the number line to explain why.
- How do you know that the difference is correct? Use the number line to explain why.

# H10. Card Sorting Routine: Number Lines

## Goal

Reinforce and solidify students' understanding of the number line representation.

## Materials

A card set with 11 examples of number lines. Use printed cards to sort or use a virtual version of the activity on the PLC-A tab (of the Online Component).

**Step 1. Sort cards and explain reasons.**

**a.** Teacher goes over the directions.

**b.** Students work in pairs. They take turns sorting the cards into two categories:

| **Accurate:** Number line has no errors. | **NOT Accurate:** Number line has one or more errors. |
|---|---|

- One partner takes a card and looks at the number line to decide which category to put it in.
- They place the card in a category and explain their reasons by using this sentence starter*.
  - **I placed the card in the __ category because …**
- Then the other partner will respond by using **one** sentence starter:

  **I agree because…** **I disagree because…**

**Step 2. Discuss and generalize.**

**a.** Teacher reveals the answers. Students have opportunity to ask questions.

**b.** Teacher facilitates a discussion of the cards in the **Accurate number lines category**:
- What do the cards showing accurate representations of the number lines have in common?
- What are some things to look for to see whether a number line is correct?

**c.** Teacher facilitates a discussion of the **Not Accurate category**:
- What errors do you see on card [letter]? How would you correct this number line?

**Step 3. Wrap up and reflect.**

**a.** Students complete an exit task to reflect on their learning.

**b.** Teacher summarizes the mathematics focus on the number line representation for fractions.

Appendix B has resources for this card sorting routine, including number line cards.

*In this Participant Workbook, the **sentence starters** are written in **blue font** so that they stand out.

# H11. Video Discussion at PLC Session

**Videos:** *Instructional Routine: Using a Benchmark Strategy to Place Fractions on a Number Line (Parts 1 and 2)*

## Directions

**1. Watch and discuss video, Part 1** (excerpt: 0:49–4:12 min.).

- Watch an excerpt of part 1 and write notes for the focus questions below.
- Discuss one focus question: **A**.

**2. Watch and discuss video, Part 2** (6:12 min.).

- Watch part 2 and write notes for the focus questions below.
- Discuss all three focus questions: **A, B,** and **C.**

## Focus Questions

*As you watch, focus on these questions, and write notes below.*

**a.** How does the teacher support students with 1) placing fractions on the number line; and 2) explaining their ideas?

**b.** What do you notice about the students' understanding of or difficulties with fractions and the number line representation?

**c.** What ideas from the videos do you want to apply when you use the routine with students?

# H12. Walk-Through of Routine: Script

**Routine: Using a Benchmark Strategy to Place Fractions on a Number Line**

Steps:

1. Introduce learning goals.
2. Use benchmark numbers, estimation, and reasoning.
3. Notice and discuss the placement of the fractions.
4. Create fractions to place on the number line.
5. Wrap up.

## Purpose

To get familiar with the routine by going through some of the steps in the roles of teachers and students. We will use a script to get a sense of what teachers and students might say and do. The script is designed for this PD activity and is **not** intended for teaching students. After we finish the walk-through, we will discuss ways to implement the routine with students.

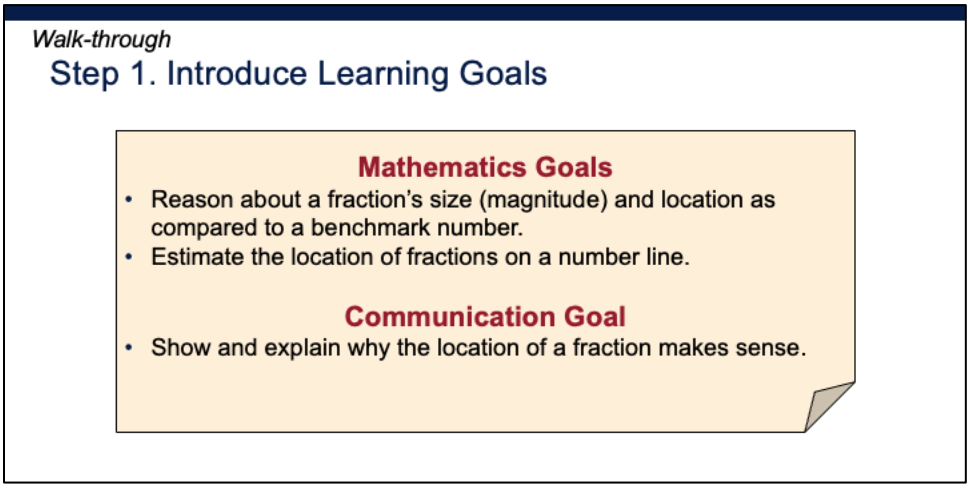
**The script** is set up as follows:

- The script is labeled with the roles of "Teacher" and "Student" to provide examples of what the teacher or students might **say**. You can read the text aloud or say it in your own words.
- *Text in italics* describes actions that the teacher and students will **do**.
- [Brackets] provide information for doing the walk-through.
- The script includes some **slide images** to orient you to what is happening in each step. We will not use slides during the walk-through and instead will just use the script handout.

**Roles:**

- **Teacher:** We will take turns in the role of teacher. The script provides suggestions about when to switch the teacher role.
- **Students:** Everyone will take on the role of students when they are not the teacher.

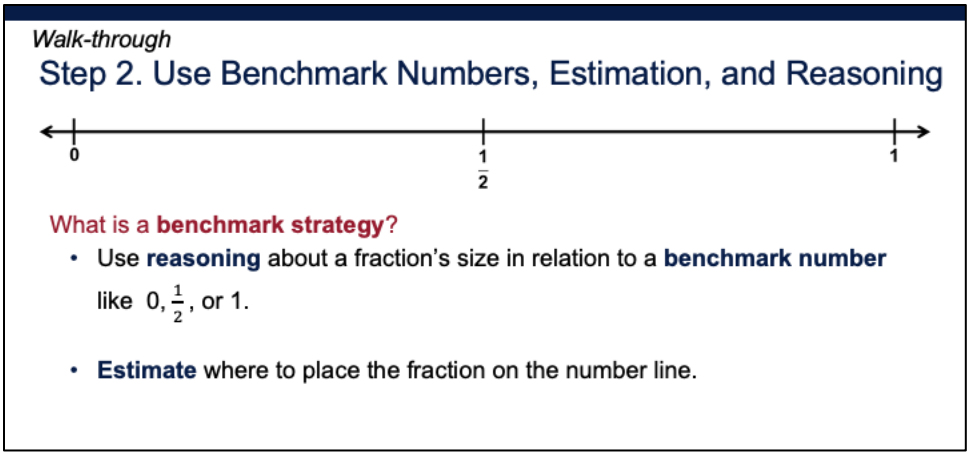
## Step 1. Introduce the Learning Goals




**Teacher:** Today we are going to place fractions on this large number line. We have two mathematical goals. This first is to reason about a fraction's size—this is also called the fraction's magnitude—and its location as compared with those of a benchmark number. We'll talk about what a benchmark number is in one moment. Our second mathematical goal is to be able to estimate the location of a fraction on the number line.

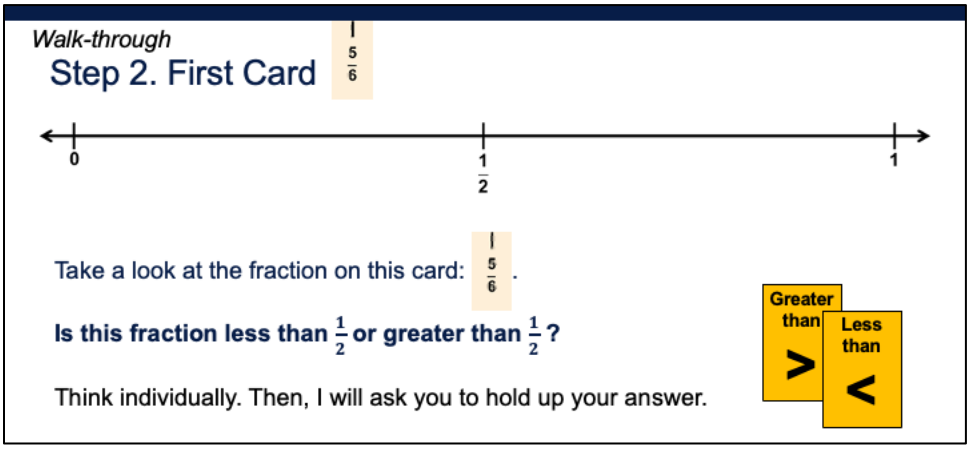
For our communication goal, we will work on showing and explaining why the location where we placed a fraction on the number line makes sense. At the end of the routine, we will revisit the goals to reflect on our learning.

## Step 2. Use Benchmark Numbers, Estimation, and Reasoning




**Teacher:** We have a set of fraction cards to place on this number line that extends from 0 to 1. We will use a **benchmark strategy** to decide where each fraction should be located. This means you'll think about how the fraction compares in size to the benchmark numbers: 0, $\frac{1}{2}$, or 1. Once you've reasoned about the size of the fraction, you'll use **estimation** to place it on the number line.

## Step 2. First Card




**[Switch to a new teacher.]**

**Teacher:** Let's try the first one together. Here's the first fraction.

*Hold up the card* $\frac{5}{6}$ *but do not read it.*

**Teacher:** Is this fraction greater than $\frac{1}{2}$ or less than $\frac{1}{2}$? Think individually and then I'll ask you to hold up your card.

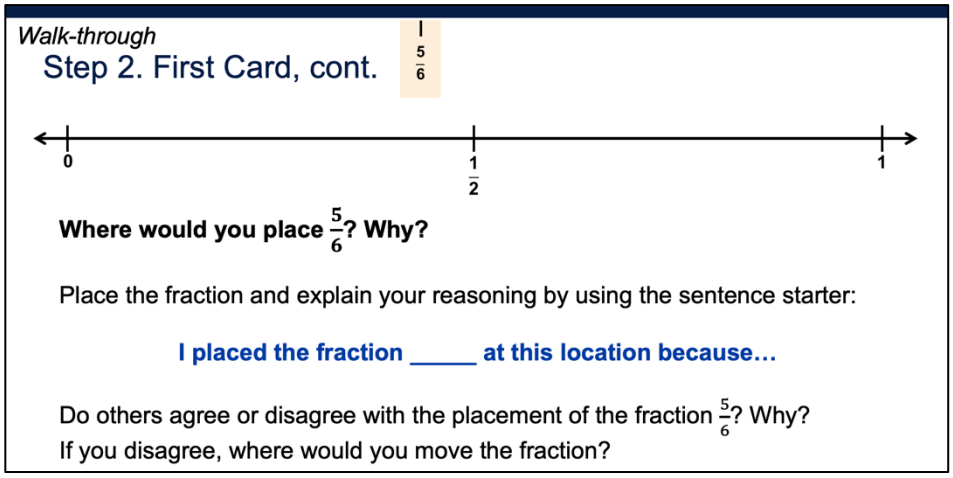
*Give students time to think.*

Please hold up your cards.

*Notice which card students are holding up. Choose a student to place the fraction.*

**[Continue with the script on the next page.]**

## Step 2. First Card, cont.




**Teacher:** [Student's name], where would you place the card on the number line and why? Please use this sentence starter to explain your reasoning:

**"I placed the fraction ____ at this location because…"**

**Student:** *Places the card and uses the sentence starter to share their reasoning.*

**Teacher:** *Ask the class:* Do you agree or disagree with this placement? Show a thumbs-up for agree or a thumbs-down for disagree. If you disagree, where do you think the fraction needs to be moved?

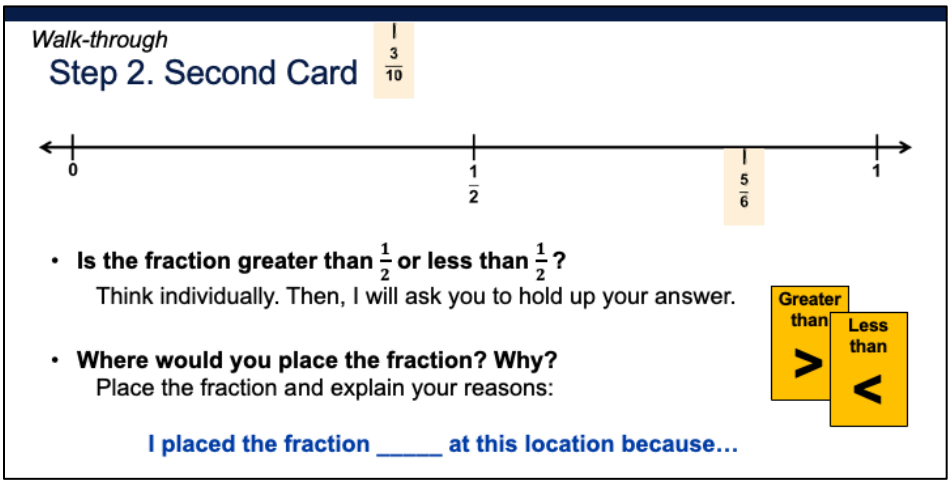
**Students:** *Agree or disagree and adjust the position of the fraction if needed.*

**Note to read aloud**:

If the fraction's position is incorrect, it's important to adjust the placement at this time so that it doesn't lead to errors in the placement of subsequent fractions. If you need to adjust the placement, say something like the following:

> **Teacher:** I'm going to adjust the placement of this fraction. I'm moving it here because [give reasons]. I'm adjusting it so that you can use this fraction's location, if you want, when you are placing the next fractions.

## Step 2. Second Card




**[Switch to a new teacher.]**

**Teacher:** Here's the next fraction.

*Hold up the card $\frac{3}{10}$ but do not read it aloud.*

Is this fraction greater than $\frac{1}{2}$ or less than $\frac{1}{2}$? Think individually and then I'll ask you to hold up a card to show your answer.

*Give students a little time to think.*

**Teacher:** Hold up your card to show your answer. Is the fraction greater or less than $\frac{1}{2}$?

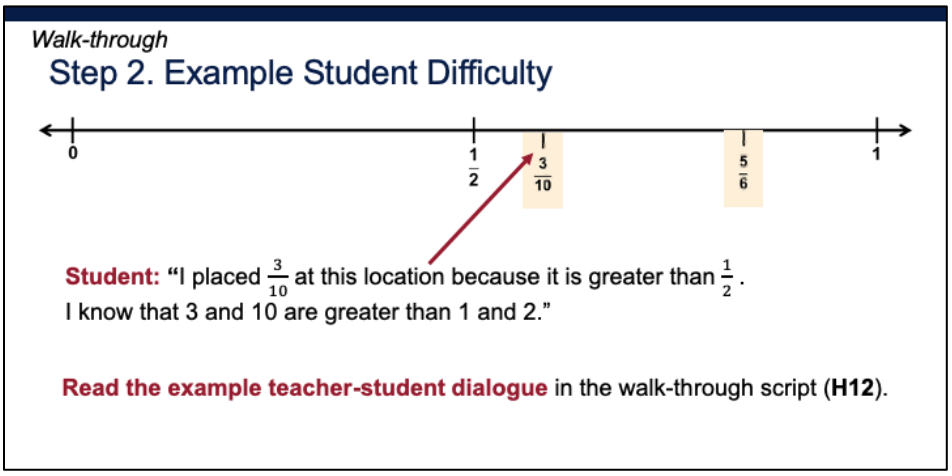
*Notice which cards students are holding up and choose someone to place the fraction.*

**Teacher:** [Student's name], where would you place this fraction on the number line? Place the fraction card and use the sentence starter to explain your reasoning:

**"I placed the fraction ___ at this location because…."**

**[Continue on the next page to read sample dialogue of students sharing their reasons.]**

## Step 2. Example Dialogue




**Teacher:** Where would you place this fraction? Why?

*Student places $\frac{3}{10}$ to the right of $\frac{1}{2}$.*

**Student 1:** "I think $\frac{3}{10}$ is greater than $\frac{1}{2}$ because 3 and 10 are greater than 1 and 2."

**Teacher:** Okay, so you are noticing that $\frac{3}{10}$ has a 3 in the numerator and a 10 in the denominator, and those numbers are greater in value than 1 and 2 in $\frac{1}{2}$. Let's take out the fraction tiles. Use the tiles to represent the two fractions ($\frac{1}{2}$ and $\frac{3}{10}$) to show how they compare in size.

**Student 1** *(Builds with tiles)*: Oh, wait—$\frac{3}{10}$ is actually less than $\frac{1}{2}$ because I would need more of the $\frac{1}{10}$ pieces to equal $\frac{1}{2}$. I would need $\frac{5}{10}$ to equal $\frac{1}{2}$. So, $\frac{3}{10}$ is less than $\frac{1}{2}$.

**Teacher:** You are correct: $\frac{3}{10}$ is less than $\frac{1}{2}$. Now that you have revised your thinking, where would you place $\frac{3}{10}$ and why? Use the sentence starter to share your reasoning.

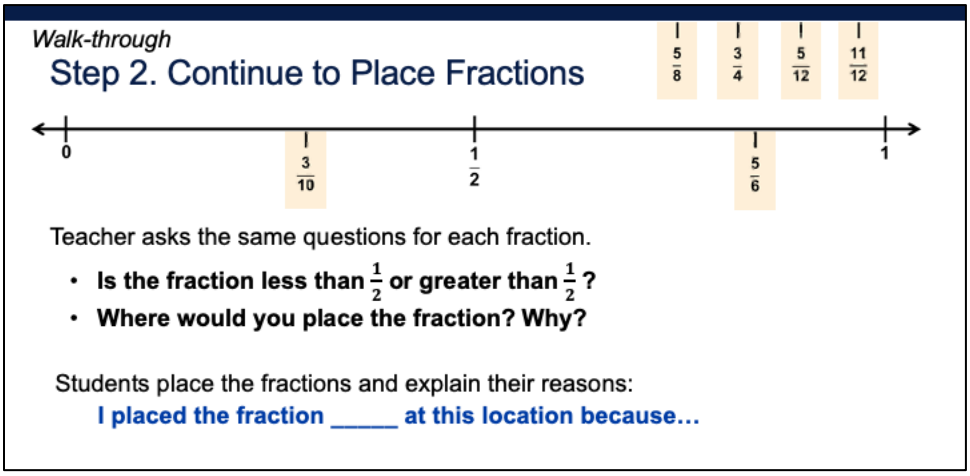
**Student 1:** *(Places $\frac{3}{10}$ to the left of $\frac{1}{2}$ and closer to $\frac{1}{2}$ than* to 0.*)*

I put the fraction $\frac{3}{10}$ at this location because $\frac{3}{10}$ is closer to $\frac{1}{2}$, which is equivalent to $\frac{5}{10}$, than to $\frac{0}{10}$.

**Teacher:** Do others agree with where this fraction is located on the number line?

**Student 2**: I agree with where the fraction is placed because $\frac{1}{2}$ is equivalent to $\frac{5}{10}$, so it makes sense that $\frac{3}{10}$ is there. Then $\frac{4}{10}$ would be here (*gestures*) and $\frac{5}{10}$ here.

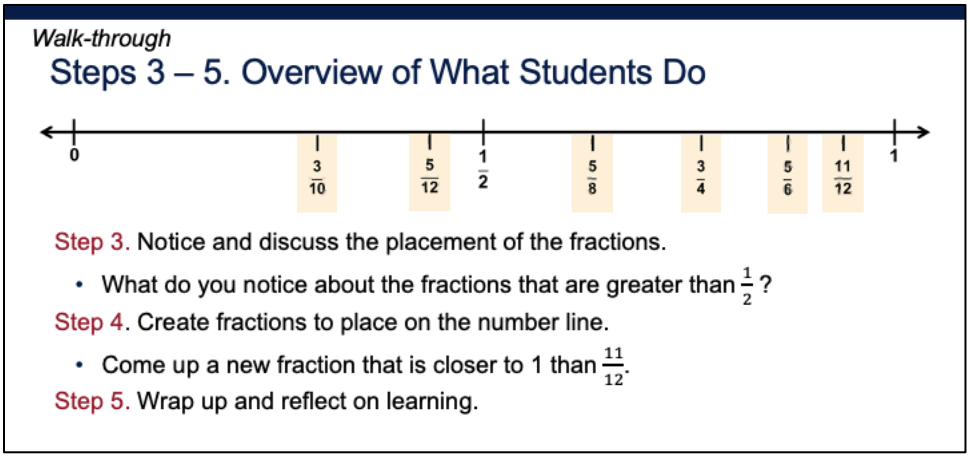
## Step 2. Example Dialogue, cont.




**Note to read aloud:** The routine continues with students placing more fractions, one at a time. The teacher continues to ask the same guiding questions and students use the sentence starter to explain their reasons.

To save time in the role play, we will skip acting out the placement of the remaining four fractions. With your students, you would follow the same process to place the next four fractions as we did the first two fractions. The step ends when all six fractions are placed.

### Overview of Steps 3–5




**Note to read aloud:** The routine continues with Step 3, in which students notice and discuss the placement of the fractions on the number line. Then, in Step 4, they create a new fraction that is closer to 1 than to $\frac{11}{12}$ and they order the fractions that they create from least to greatest. The routine ends with a wrap-up in Step 5 where they reflect on their learning by completing a sentence starter.

**The end of the walk-through!**

# H13. Debriefing Questions and Protocol

| Debriefing Questions |
| --- |
| 1. Description of how you implemented the routine. Who? When? What?<br>2. Show or describe one or two student work examples. What did you notice in students' work about their mathematics understandings and difficulties?<br>3. What worked well to support students' mathematics learning and ability to communicate their ideas?<br>4. What was challenging for students?<br>5. What would you do again or do differently in the future? Why? |

## Debriefing Protocol

### Part 1: Sharing Experiences

**1a. Each person takes a turn presenting their experiences using the routine.**

- *Presenter* describes experiences and answers the debriefing questions (~6 min.).
- *Group members* are active listeners during the presentation (avoid interruptions).
- *Timekeeper* gives a 1-minute warning and says when time is up.

**1b. After each presenter finishes, group members can:**

- Ask clarifying questions and note ideas they would like to discuss in Part 2. (~1–2 min.)

***Time**:* About 7–8 minutes per presenter; total time varies by number of presenters.

### Part 2: Group Discussion of Common Themes and Suggestions

**The group discusses the following questions.**

1. Think about the ideas shared by your colleagues. What's one idea that you want to use with this routine or apply in other mathematics activities?
2. Based on what you learned about students' understanding and challenges with using mathematical language, what next steps will you take?
3. What other strategies, challenges, or questions related to the routine would you like to discuss further?

***Time**:* About 6–10 minutes for group discussion.

# H14. Recap Strategies for Number Lines

This handout lists strategies to provide a resource for future use. **Star*** strategies that you want to try or to strengthen in your practice.

**1. What are recommended strategies for number lines?**[5]

***Implementation Step 1.*** **Represent** numbers on the number line and build understanding of their magnitude.

- **a.** **Use concrete linear representations**, such as fraction tiles, to build number lines and locate fractions.
- **b.** Have students **fold strips of paper** into equal parts to build understanding of partitioning. Use the folded strips to mark the location of fractions on a number line.
- **c.** Use **kinesthetic activities**, such as walking on the number line, to build understanding of directionality and that a number's magnitude or size is represented by its distance from 0.
- **d.** Use a **variety of number lines** that have different labeled endpoints, such as 0–3, to avoid having students overgeneralize that all fraction number lines are labeled 0–1.
- **e.** Provide **templates with partitioned number lines** to reduce the challenge of drawing equal partitions, particularly for students with fine motor difficulties.
- **f.** Start with **accessible numbers**, such as fractions with even denominators, to make it easier for students to partition number lines.
- **g.** Model how to use a **systematic approach to partition number lines**, such as first partitioning into halves, then partitioning the halves in half to make fourths, and then partitioning the fourths in half to make eighths.
- **h.** Teach students how to **further partition a number line** to be able to label points that are located between tick marks. (These types of problems tend to be challenging.) Making more partitions shows the flexibility of number lines to represent different numbers.

***Implementation Step 2.*** **Compare** numbers on the number line and determine their relative magnitudes.

- **i.** **Connect fraction tiles to number lines** to build understanding of equivalent fractions.
- **j.** Use **benchmark numbers**, **estimation**, and **reasoning** to locate, and compare fractions on a number line. The module's routine uses this approach.
- **k.** Use **kinesthetic activities to compare fractions**, such as having two students walk on a floor number line to different fractions and comparing their distance from 0.
- **l.** Provide **multiple number lines** that have the same distance from 0 to 1 and are partitioned to show different denominators. Have students use these number lines to compare fractions and find equivalencies.

[5] These strategies are from the WWC Guide with adaptations by the Toolkit authors. The lists are not exhaustive.

**m.** Use **questioning strategies** to focus students on key aspects of number lines and have them explain their ideas. Using a consistent set of questions can help students internalize questions to ask themselves.

***Implementation Step 3.*** **Model operations** on the number line to build understanding of their underlying concepts.

**n.** Place fraction tiles on the number line to **model fraction addition** and **subtraction,** and connect the concrete, semi-concrete, and numeric representations.

**o.** **Use multiple number lines to identify common denominators** and equivalent fractions for adding and subtracting fractions with unlike denominators.

**p.** **Draw points and arrows** on the number line to model adding and subtracting fractions.

**q.** Use word problems to **provide contexts** for operations to model on the number line.

**r.** Use the number line to represent and solve **two types of subtraction problems**: take-away and comparison.

**s.** Use number lines to **model multiplication and division for a whole number and a fraction**. Note that this model does not work well for modeling multiplication and division of two fractions that are both less than one.

**2. What are things to avoid? What should you do instead?**

- Always using just one type of number line, such as with labeled endpoints of 0 and 1.

  Instead, use a variety of number lines with different labeled endpoints to increase students' attention to using this information to interpret the number line.

- Starting activities on partitioning number lines with challenging fractions, such as ones with odd-numbered denominators or large numerators that require a lot of partitions.

  Instead, start by giving students fractions with even-number denominators and smaller denominators that are easier to partition on a number line.

- Always using a number line that is partitioned to match the fraction's denominator, because students may assume that they should not make more partitions on number lines.

  Instead, provide activities in which students need to further partition number lines to represent different fractions.

# H15. Strengthen Strategies

**Directions**: Use the prompts to reflect on current practices, brainstorm ideas, and plan actions.

## 1. Brainstorm Together

How will you strengthen your strategies for teaching number lines? Choose prompts and list ideas.

**a.** Try the strategy of . . .

**b.** Increase . . .

**c.** Decrease . . .

**d.** Make sure to . . .

**e.** Find out about students' understanding of ___ by . . .

**f.** Build on students' strengths with ___ by . . .

**g.** Address students' challenges with ___ by . . .

## 2. Set an Implementation Intention

Look over your ideas above and choose one or two to focus on. Work individually to write an implementation intention and plan specific actions to take.

| Implementation Intention |
| --- |
| Set a **goal** to strengthen your instructional strategies for teaching number lines. What will you focus on? |
| Plan **two actions** to take. What will you do? When?<br>• <br>• |

# H16. Self-Reflection Form: Number Lines

## Introduction

This form provides an opportunity to reflect on your learning and current understanding of the recommendation for number lines. This self-reflection can help you celebrate progress and guide you in identifying areas for continued growth in your professional learning. It is not intended to be evaluative and will not be submitted or shared with others unless you choose to do so.

The form has two parts. In **Part 1**, you are asked to self-assess your understanding of specific strategies from the module on a scale of 1 –3 (1: A little understanding; 2: Some understanding; and 3: A strong understanding). It is not expected that participants would have a "strong understanding" of every strategy at the end of a module. Please be assured that it's fine to select "some" or "a little" understanding. Your learning is evolving, and you will have opportunities to strengthen your understanding of the strategies in the subsequent modules and in your classroom. Continuing to use strategies with your students is a critical step in deepening your understanding and skills.

In **Part 2**, you are asked to reflect on the strategies and select one that you would like to improve. Focusing on one strategy is helpful for planning concrete next steps that are manageable to carry out. If you would like support, reach out to your facilitator and colleagues in the course. You may also want to revisit resources on specific strategies in the Participant Workbook and the online component.

This self-reflection form captures your thinking at one point in time. It's helpful to revisit the form later in the course to consider how your understanding has changed and to plan ways to continue your professional learning of the recommended strategies.

# Part 1. Reflect on your understanding of strategies.

**Directions:** The table has a list of recommended strategies for teaching number lines from the module. Read each strategy and self-assess your level of understanding by using this **rating scale**:

1. **A Little Understanding:** Have a vague sense of the strategy.
2. **Some Understanding:** Able to explain the strategy in general terms.
3. **A Strong Understanding:** Able to explain it in detail and give examples.

**N/A:** Have not learned about the strategy yet.

| **Strategies**<br>What is your current level of understanding for each strategy? | **Select Your Rating** |
|---|---|
| **a.** How to connect concrete representations (such as fraction tiles) to number lines to support students in representing fractions on number lines. | ◯ ◯ ◯ ◯<br>**1 2 3 NA** |
| **b.** How to support students in building understanding of partitioning by having them fold strips of paper into equal parts and then use the folded strips to partition number lines. | ◯ ◯ ◯ ◯<br>**1 2 3 NA** |
| **c.** How to build understanding that a number's magnitude is represented by its distance from zero on a number line. | ◯ ◯ ◯ ◯<br>**1 2 3 NA** |
| **d.** How to use number lines to build understanding of the location of unit fractions and their corresponding magnitude. | ◯ ◯ ◯ ◯<br>**1 2 3 NA** |
| **e.** How to support students in estimating where to place fractions on a number line. | ◯ ◯ ◯ ◯<br>**1 2 3 NA** |
| **f.** How to build students' understanding that equivalent fractions have the same location on a number line. | ◯ ◯ ◯ ◯<br>**1 2 3 NA** |
| **g.** How to use double or multiple number lines to help students find equivalent fractions or common denominators. | ◯ ◯ ◯ ◯<br>**1 2 3 NA** |
| **h.** How to use benchmark numbers to help students locate and compare fractions on the number line. | ◯ ◯ ◯ ◯<br>**1 2 3 NA** |
| **i.** How to use questioning strategies to support students in explaining their reasons for the placement of fractions on the number line. | ◯ ◯ ◯ ◯<br>**1 2 3 NA** |
| **j.** How to model fraction addition or subtraction on a number line, such as by placing fractions tiles on a number line or drawing points and arrows. | ◯ ◯ ◯ ◯<br>**1 2 3 NA** |

## Part 2. Reflect on your professional learning and use of strategies.

**Directions:** Look over the strategies on the prior page and then answer these questions.

1. What is one strategy that you find particularly helpful for building students' understanding and use of number lines? Why?

2. What is one strategy that you would like to improve in your practice? What are two actions you will take to strengthen your use of this strategy.

3. Reflect on your learning in this module on the recommendation for number lines. What are one or two ways that your knowledge or practices have changed?

# Routine Teaching Guide

This section has information and resources for teaching the routine *Using a Benchmark Strategy to Place Fractions on a Number Line.* Routine handouts are labeled with the letter R and a number.

## Contents



Appendix A has additional resources for using the routine with students, including reproducibles for card sets and sentence starters.

**Slide Decks for Routine**

- Debriefing Slides Template for Sharing Experiences
- Optional Slides for Teaching the Routine

# Introduction to the Routine

This Routine Teaching Guide provides information, suggestions, and resources to support teachers in planning and teaching the routine with students and then sharing their experiences. The routine uses multiple strategies from the WWC Guide's recommendations for number lines. It has a consistent sequence of steps to allow for ease of implementation by teachers and students.

In this routine, the teacher works with students to build their abilities to locate fractions on a large number line by using a benchmark strategy, estimation, and reasoning. The routine engages and supports students in comparing fractions with benchmark numbers, estimating where to place each fraction on the number line, and explaining their reasoning. The fraction cards are strategically sequenced to elicit information about students' understandings and difficulties.

At PLC Session-A, you and your colleagues will learn how to use the instructional routine by watching and discussing a classroom video example, doing a walk-through, and discussing ways to use the routine with students. PLC group members are expected to use the routine with students one or more times and then to share experiences at PLC Session-B.

## Frequently Asked Questions (FAQs) about the Routine

### 1. What recommended strategies does the instructional routine incorporate?

The routine focuses on the WWC Guide recommendation for using number lines implementation Step 2: "Compare numbers and determine their relative magnitude using a number line to help students understand quantity." It also includes strategies from the mathematical language recommendation. The routine uses these strategies:

- A large number line to provide a kinesthetic experience.
- A benchmark strategy, estimation, and reasoning to place fractions.
- Card sets with a **strategic sequence of fractions** to gather information about students' understandings and difficulties.
- **Focused questions** to support placement of fractions and communication.
- **Response cards** (greater than/less than cards) to support communication and gather formative information on students' understandings.
- **Sentence starters** to support students in communicating their reasons for placing fractions and specific locations.
- **Fraction tiles** (concrete representation) for building and comparing fractions, to use as needed.

### 2. What mathematics content does the routine focus on?

Knowing how to represent and compare fractions on a number line are important standards for the elementary grades. The routine focuses on locating and comparing fractions on the number line. Students use a benchmark strategy and estimation to place the fractions. There is a choice of three card sets that

have different fractions: 1) unit fractions, 2) non-unit fractions in simplified form, and 3) non-unit fractions, including equivalent fractions.

## 3. How do you integrate the routine with your intervention program?

The routine is designed to reinforce and deepen understanding of fractions and the number line representation. It also provides helpful formative information about students' understandings and difficulties with these concepts. To plan when to use the routine in your sequence of instruction, consider these prerequisites. Students should have prior experiences with fractions using linear models, such as fraction tiles or paper strips, and with using the number line.

You have a choice of three card sets that have different types of fractions (described above). Select a set that is a good fit for your students and the instructional sequence in your program. If you are unsure, begin with the set of unit fractions cards to gather information on students' foundational understanding of fractions and the number line. Then, you can adjust the level of challenge as needed for subsequent card sets.

The instructional routine will take about 30–45 minutes during one intervention class session. You can adjust the routine to fit your class time. If you have shorter class periods, you have the option of focusing on the first three steps of the five-step routine.

# Using the Routine with Students

## Expectations

- Use the routine with students **at least one time** *before* PLC Session-B.
- It's okay to adjust the routine to fit your teaching situation. The five-step routine takes about 30–45 minutes. If you have shorter intervention classes, you can use Steps 1–3 and skip Steps 4 and 5 or use two class sessions to do the full routine.
- After you use the routine, prepare to share experiences at PLC Session-B by using the **Debriefing Slides Template** to create five slides.

## Suggestions

- The routine is likely to take more time the first time you use it. That's okay!
- Use the Two-Page Overview **(R1)** when you teach. It outlines each step. Feel free to put the routine into your own words.
- It's helpful to use a routine more than one time. Some teachers said they felt unsure the first time and that they felt more comfortable when they used the routine again. Consider using the routine more than once, such as with different intervention sections.
- Keep in mind that the routine is also new for your students. Encourage and support their participation.

# R1. Two-Page Overview of Routine

## Routine: Using a Benchmark Strategy to Place Fractions on a Number Line

Total time: 30–45 min.

**Step 1. Introduce learning goals. (Whole Group) 2 min.**

**a.** Display the learning goals and introduce them to students.

**Step 2. Use benchmark numbers, estimation, and reasoning. (Whole Group) 12-20 min.**

**Important:** Use the fraction cards in the given sequence.

**a.** Explain what a benchmark strategy is and orient students to the number line.

**b.** Hold up one fraction card, but do **not** read it.

- Ask: Is this fraction greater than $\frac{1}{2}$ or less than $\frac{1}{2}$?
- Give students a moment to think individually, and then give a cue for them to hold up a **greater than** or **less than card** to show their answer.

**c.** Have one student place the fraction and explain their reasons by using this sentence starter:

- **I placed the fraction_____ at this location because…**
- Ask the other students: "Do you agree or disagree with the placement of this fraction? Why? If you disagree, where would you move the fraction?"

Repeat Steps 2b–c for each fraction in the set. Adjust the positions of fractions, if needed, before moving to the next fraction.

**Tips:** If students have difficulty placing fractions, consider these suggestions:

- Use fraction tiles to build and compare fractions.
- Ask questions about which benchmark a fraction is closer to, such as 0 or $\frac{1}{2}$.
- Have students imagine walking on a number line to the fraction—is it before or after $\frac{1}{2}$?
- Use gestures to show dividing the number line into parts for the given denominator.

**Step 3. Notice and discuss the placement of the fractions. (Pairs, Whole Group). ~5 min.**

**a.** Introduce the discussion questions. Have students first talk with a partner:

- What do you notice about the position of the fractions on the number line? Are there any patterns?

**b.** Whole group: Have students share ideas about the discussion questions:

- What do you notice about the position of the fractions on the number line?
- What do you notice about the fractions that are closer to 0? $\frac{1}{2}$? 1?
- Optional: Which fraction is closest to 0? $\frac{1}{2}$? 1?

Note: The Answer Key **(R7)** has completed number lines and sample responses.

**Step 4. Create fractions to place on the number line. (Individual, Pairs, Group) 10 min.**

**a.** Ask students to create a new fraction by using the prompt for your card set (below). Give students independent think time to choose a fraction and then write it on a card.

- Set 1. Come up with a unit fraction that is even **closer to 0 than $\frac{1}{12}$**.
- Set 2. Come up with a non-unit fraction that is even **closer to 1 than $\frac{11}{12}$**.
- Set 3. Come up with a non-unit fraction that is even **closer to $\frac{1}{2}$ than $\frac{2}{5}$**.

**b.** Move into pairs. Have students compare their fractions and discuss the question:

- Set 1. Which of our fractions is closer to 0? Why?
- Set 2. Which of our fractions is closer to 1? Why?
- Set 3. Which of our fractions is closer to $\frac{1}{2}$? Why?

**c.** With the whole group, use the activity for your chosen fraction set.

- Set 1. Have students line up their unit fractions in order, from the closest to 0 to the farthest from 0. They could stand in order holding up their fractions or order their cards on a table.
- Sets 2. Ask pairs to share how they determined which of their fractions was closer to 1.
- Sets 3. Ask pairs to share how they determined which of their fractions was closer to $\frac{1}{2}$.

**Optional for Sets 2 & 3:** Ask the group: Which new fraction is the closest to 1? $\frac{1}{2}$? Why?

**Step 5. Wrap up. (Whole Group) 5 min.**

**a.** Revisit the goals.

**b.** Ask students to reflect on their learning by doing an exit task. They choose one of the two sentence starters below to complete and share with a partner:

- **One thing I learned about placing [unit, non-unit] fractions on a number line is …**
- **One thing I learned about using a benchmark strategy is …**

## Suggestions Related to Time

If your intervention time is less than 30 minutes, consider these options:

- Use Steps 1–3 and skip Steps 4–5.
- Use two class sessions: Complete Steps 1–2 in the first session and Steps 3–5 in the next session. You will need to use the completed number line in the second session so students can discuss the fractions and create new fractions. One option is to take a photograph so you can recreate it for the second session.

# R2. Planner for Routine

## 1. List Ideas to Apply for Teaching the Routine

**a.** What ideas from today's session do you want to apply with your students? What do you want to make sure to do when you use this routine?

## 2. Make Specific Plans

**Suggested Prerequistes for the Routine**

Students should have prior experiences with using number lines and with representing fractions using linear models, such as fraction tiles or paper strips. You can use the routine in different ways, such as a formative assessment to gather information about students' understanding, as a content-building activity, or as a review.

**a.** **When** will you use the routine with your students (before PLC Session-B)? The routine will take about 30–45 minutes.

**b.** **How will you set up the large number line?** Here are several options:

___On a table: Use masking tape to set up a number line.

___On a whiteboard: Draw a long number line.

___On the floor: Use masking tape to set up the number line.

___On a clothesline: Hang a clothesline and use clothespins to hang the fractions.

*Note:* The number line needs to be 4 feet long for card set 1 and 3 feet long for card sets 2 and 3. Check to make sure that there is enough room for the cards on the number line and adjust accordingly.

**c.** **Which fraction card set(s)** do you want to use with students? The choices of three card sets are below and on the routine handout **(R5):**

**Card Set 1: Unit fractions**
Benchmarks: 0, $\frac{1}{2}$, and 1

Fractions: $\frac{1}{10}$ $\frac{1}{12}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{6}$ $\frac{1}{1}$

**Card Set 2: Non-unit fractions** (in simplified form)
Benchmarks: 0, $\frac{1}{2}$, and 1

Fractions: $\frac{5}{6}$ $\frac{3}{10}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{5}{12}$ $\frac{11}{12}$

**Card Set 3: Non-unit fractions** (includes equivalent fractions)
Benchmarks: 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1.

Fractions: $\frac{9}{10}$ $\frac{3}{12}$ $\frac{2}{5}$ $\frac{4}{4}$ $\frac{7}{8}$ $\frac{2}{3}$

**Tip:** If you are unsure which card set to use, we recommend starting with the unit fractions (Set 1). Set 1 is a good choice for formatively assessing your students' understanding of fractions and the number line representation.

## 3. Consider Potential Challenges and Ways to Address Them

**a.** How will you provide additional support if students have difficulty placing fractions on the number line or doing other parts of the routine? (See handout **R3** for suggestions.)

# R3. Suggestions for Addressing Potential Challenges

**Directions:** Look over the suggestions. Star* ideas you want to try and add ideas.

1. **If students have difficulty placing fractions on the number line**, here are suggestions for providing additional support and example questions/prompts:

**a.** Ask questions about which benchmark number the fraction is closer to.

- Is the fraction closer to 0 or to $\frac{1}{2}$?
- Is the fraction closer to $\frac{1}{2}$ or to 1?

**b.** Have students use fraction tiles to compare the fraction with a benchmark number or another fraction.

- Is $\frac{1}{3}$ closer to $\frac{1}{2}$ or to $\frac{1}{4}$? Use fraction tiles to build and compare the fractions.

  [The fraction $\frac{1}{3}$ is closer in length to $\frac{1}{4}$, which may be surprising to students.]

**c.** Use gestures to show dividing the number line into equal parts to help students visualize where a fraction would be located.

- Imagine the number line broken into three equal parts. Where would $\frac{2}{3}$ be located?

**d.** Have students imagine walking on the number line to the location of the fraction.

- Imagine you are starting at 0 and will keep walking until you get to the fraction. Will you stop before or after $\frac{1}{2}$? How close or far from 1 whole will you stop?

2. **Suggestions for using the fraction card sets effectively:**

**a.** Use the fractions in the designated order. The fractions were sequenced strategically to learn about students' understandings and difficulties.

**b.** The card sets have six or seven fractions so that there will be enough fractions on the number line to look for patterns and discuss them in Step 3. Aim to have students place all the fractions. If time is short, use one of these options:

- Have students place at least five fractions on the number line and then have the discussion.
- Have the students place four or five fractions and then add the remaining fractions yourself to have more fractions shown for the discussion.

# R4. Preparation and Materials Checklist

Use this list to prepare for using the routine with students.

☐ **1. Choose which card set you will use and how you will display the number line**. The handout **R5: Choice of Fraction Sets** provides three card sets to choose from. If you are unsure, start with unit fractions (Set 1).

☐ **2. Plan how you will set up the number line.** You could draw a number line on the whiteboard, or you can put masking tape on a table or on the floor. Alternatively, you could draw a number line on the whiteboard.

- The number line should be a minimum of 4' long for Card Set 1 and 3' long for Card Sets 2 and 3.
- Test the number line to make sure there's enough space to fit all the fractions. You may need to make the number line longer or make the cards smaller.

☐ **3. Prepare the fraction cards.** A reproducible handout of the card sets is in **Appendix A**. Cut the cards to a size that will work for the size of your number line. The cards are labeled with letters to show the sequence. Make sure to use the cards in the designated sequence.

☐ **4. Prepare goals to display.** Use the example goals (**Appendix A**) or write your own mathematics learning goal and communication goal. Decide how you will display the goals for students, such as on chart paper or on a whiteboard.

☐ **5. Prepare response cards that say "greater than" and "less than.** Print and cut out copies of the cards (**Appendix A**) to provide one set for each student. Another option is to write the words and symbols on index cards.

☐ **6. Prepare sentence starters.** Students will use a sentence starter to explain where they placed the fraction. Print and cut out the sentence starters (**Appendix A**) or write them on chart paper/whiteboard for students to reference.

# R5: Choice of Fraction Sets

You have a choice of three card sets. All the sets use denominators that can be represented with fraction tiles. If you are unsure which set to use, the recommendation is to start with card set 1. The card sets are listed below. Appendix A has a reproducible handout with fraction cards that you can print and cut out. These cards are labeled to show the order for placing them on the number line.

## Card Set 1: Unit fractions

**Fractions:** All cards have unit fractions.

Benchmark cards: 0, $\frac{1}{2}$, and 1.

Fraction cards: $\frac{1}{10}$ $\frac{1}{12}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{6}$ $\frac{1}{1}$

## Card Set 2: Non-unit fractions

**Fractions:** All cards are non-unit fractions in simplified form. There are no equivalent fractions.

Benchmark cards: 0, $\frac{1}{2}$, and 1.

Fraction cards: $\frac{5}{6}$ $\frac{3}{10}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{5}{12}$ $\frac{11}{12}$

## Card Set 3: Non-unit fractions, including equivalent fractions

**Fractions:** This set includes equivalent fractions for $\frac{1}{4}$ and 1.

Benchmark cards: 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1.

Fraction cards: $\frac{9}{10}$ $\frac{3}{12}$ $\frac{2}{5}$ $\frac{4}{4}$ $\frac{7}{8}$ $\frac{2}{3}$

## Suggestions

- **Use the cards in the given sequence.** The fractions in each card set are carefully sequenced to support students' learning and to elicit information about their understandings and misconceptions. In each set, use the cards in the order in which they are labeled.
- **Length of the number line.** For Card Set 1, make the number line a *minimum* of 4' long so that there is enough room to place the fractions. For Card Sets 2 and 3, the number line can be a *minimum* of 3' long.

# R6. Detailed Teaching Notes for the Routine

This section provides notes for teaching the routine. It includes pictures of the slides, but it is not necessary to use the slide deck. You can present the information to students by holding up the fraction cards and displaying the goals and the sentence starter on chart paper or a whiteboard. The notes are organized as follows:

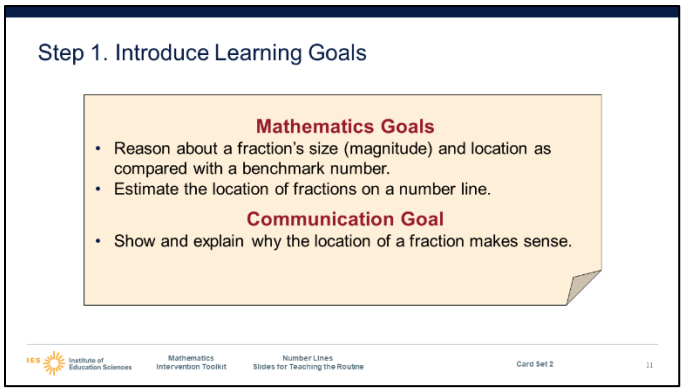
- **SAY:** Provides information about what to say and do. It is *not* meant to be a script. Use these examples to communicate the ideas in your own way.
    - Regular text indicates things to **say**.
    - *Italicized text* indicates things for the teacher or students to **do.**
- **NOTES:** Provides directions and clarifying information for teachers. Some slides do not have any notes.
- **TIPS:** Provides suggestions for implementing specific steps. Some slides do not have any tips.

The examples below have slides for **Card Set 1: Unit Fractions**. The slide deck also has slides for the **Card Set 2** (Slides 11-19) and **Card Set 3** (Slides 20–28)

**Slides 0 and 1** provide information for teachers. The routine starts with Slide 2.

## Step 1 (Slide 2)

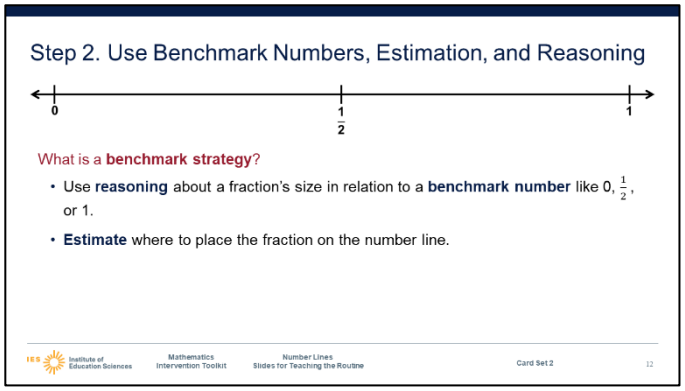
**SAY:** Today, we are going to place fraction cards on this large number line. We have two mathematical goals. This first is to reason about a fraction's size—this is also called magnitude—and the fraction's location as compared with a benchmark number. We'll talk about what a benchmark number is in one moment. Our second mathematical goal is to be able to estimate the location of a fraction on the number line.




For our communication goal, we will work on showing and explaining why the location where we placed a fraction on the number line makes sense. At the end of the routine, we will revisit the goals to reflect on our learning.

## Step 2a (Slide 3)

**SAY:** We have a set of fraction cards to place on this number line that extends from 0 to 1. We will use a **benchmark strategy** to decide where each fraction card should be located. This means you'll think about how the fraction compares in size with benchmark numbers: 0, $\frac{1}{2}$, or 1. Once you've reasoned about the size of the fraction, you'll use estimation to place it on the number line.

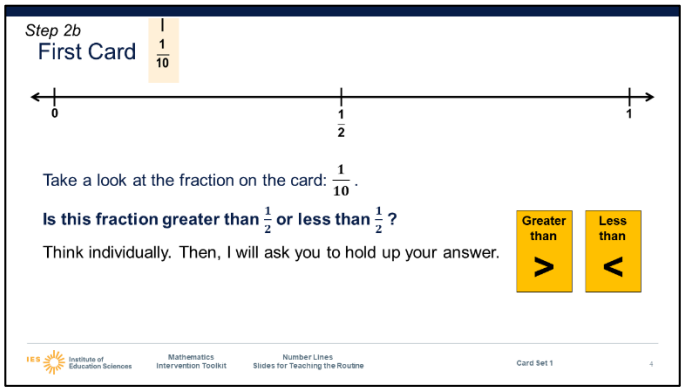



## Step 2b (Slide 4)

**NOTES:** Use the first guiding question, which is to compare the fraction with the benchmark of $\frac{1}{2}$. When you hold up a fraction card initially, do not read the fraction aloud because that may provide clues to students. Have students look at the card and read it silently to themselves.

**SAY:** Let's start with the first fraction.

*Hold up the card $\frac{1}{10}$ but do not read it aloud. Ask the students:*



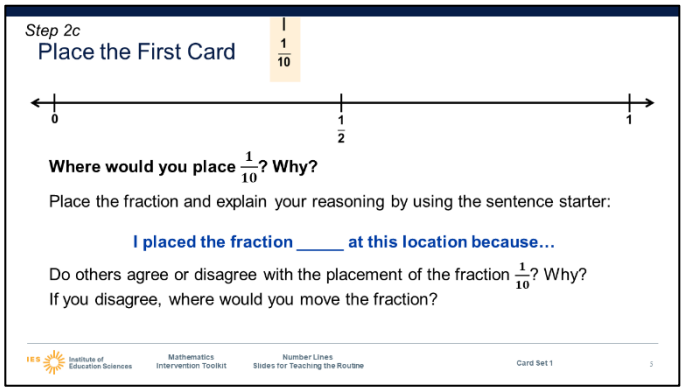

Is this fraction greater than $\frac{1}{2}$ or less than $\frac{1}{2}$? Think individually and then I'll ask you to hold up your card.

*Give students time to think.*

Please hold up your cards.

*Notice which cards students are holding up.*

## Step 2c (Slide 5)




**SAY:** Next we need to decide where to put the fraction on the number line. Where would you place the fraction $\frac{1}{10}$ on the number line and why? Who would like to place the fraction? Explain your reasons by using this sentence starter:

**I placed the fraction ___ at this location because…**

*Choose a student to place the fraction. Tip: Choose someone who held up a less than card.*

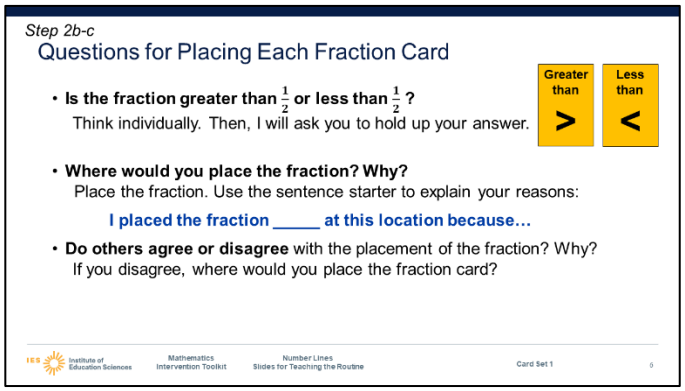
*Student places card and explains their reasons using the sentence starter.*

*Ask follow-up questions to hear more about the student's thinking.*

Do others agree or disagree with the placement of $\frac{1}{10}$? Hold up a thumbs up or thumbs down. If you disagree, where would you move the fraction?

*Students respond. If they disagree, ask them to suggest where to move the fraction.*

## Step 2b–c (Slide 6)




**NOTES:** Repeat the process for each of the remaining fractions, one at a time, until all seven fractions are placed. This slide has the key questions and sentence starter on it.

Place the fractions in the designated sequence (see list below).

**SAY:** Here's the next fraction.

*Hold up the card but do not read it.*

Is this fraction greater than $\frac{1}{2}$ or less than $\frac{1}{2}$? Think individually and then I'll ask you to hold up a card to show your answer.

*Give students time to think.*

Hold up your card to show your answer. Is the fraction greater than or less than $\frac{1}{2}$?

*Notice which cards students are holding up and choose someone to place the fraction.*

[Student's name] where would you place this fraction? Please use the sentence starter to explain your reasoning: **I placed the fraction ___ at this location because…**

*Student places fractions and explains reasons using the sentence starter.*

Do others agree or disagree with this placement? If you disagree, where would you move the fraction?

*Students agree or disagree and adjust the placement of the fraction if needed.*

**Important note:** If the fraction's position is incorrect, adjust the placement at this time so that it doesn't lead to errors in the placement of subsequent fractions. If you need to adjust the placement, say something like the following:

> **SAY:** Okay. I'm going to adjust the placement of this fraction. I'm moving it here because [give reasons]. I'm adjusting it so that you can use this fraction's location, if you want, when you are placing the next fractions.

**Sequence for placing fraction cards** (from left to right)

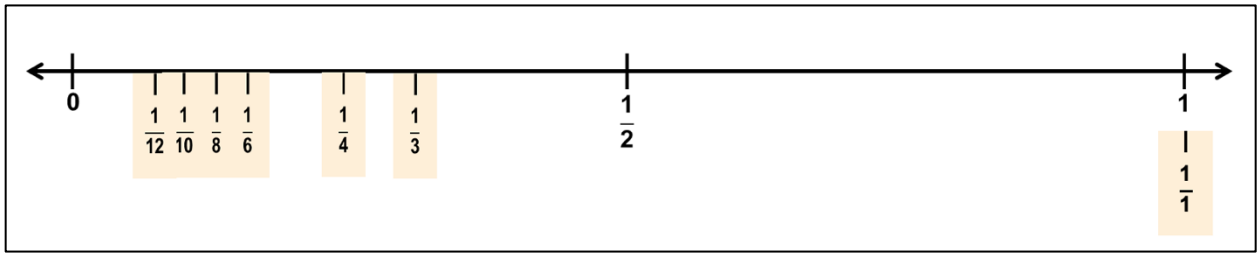
Card Set 1: $\frac{1}{10}$ $\frac{1}{12}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{6}$ $\frac{1}{1}$

Card Set 2: $\frac{5}{6}$ $\frac{3}{10}$ $\frac{5}{8}$ $\frac{3}{4}$ $\frac{5}{12}$ $\frac{11}{12}$

Card Set 3: $\frac{9}{10}$ $\frac{3}{12}$ $\frac{2}{5}$ $\frac{4}{4}$ $\frac{7}{8}$ $\frac{2}{3}$

**Answer Key for Card Set 1:** Answers are shown in brackets and dark orange font.

[This number line shows the placement of the unit fractions.]



**Tips for Step 2**

When students have difficulty placing a fraction, consider these suggestions:

a. Ask questions about which benchmark number the fraction is closer to.

- Is the fraction closer to $\frac{1}{2}$ or to 1? (Use two different benchmarks.)

b. Have students use fraction tiles to compare the fraction with a benchmark number or another fraction.

- Is $\frac{1}{3}$ closer to $\frac{1}{2}$ or to $\frac{1}{4}$? Use fraction tiles to build and compare the fractions. [Its length is closer to $\frac{1}{4}$, which may be surprising to students.]

c. Use gestures to show dividing the number line into equal parts to help students visualize where a fraction would be located.

- Imagine the number line broken into 3 equal parts. Where would $\frac{2}{3}$ be located?

d. Have students imagine walking on the number line to get to the location of the fraction.

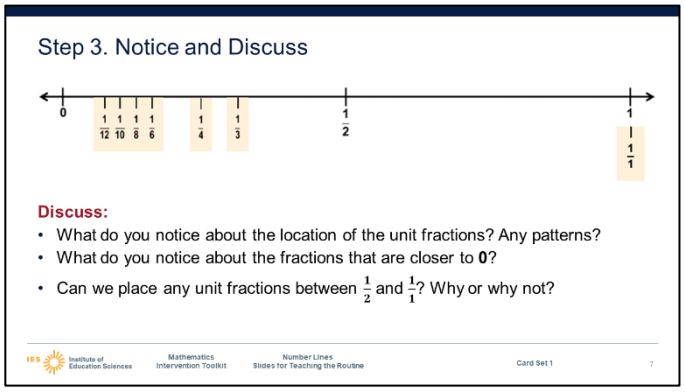
- Imagine you are starting at 0 and will keep walking until you get to the fraction. Will you stop before or after $\frac{1}{2}$? How close or far from 1 whole will you stop?

## Step 3 (Slide 7)

**NOTES:** Pose one discussion question at a time. Example responses are listed below.

**SAY:** Now that we have used benchmark numbers, estimation, and reasoning to place all of the unit fractions, let's discuss what we see.

First, you'll talk with a partner about this question:




**1. What do you notice about the position of the fractions? Do you see any patterns?**

*Students turn and talk with a partner and then return to the whole group to share their ideas. Here are some things that students might notice. Example answers are shown in brackets and dark orange font.*

- [All the unit fractions have a numerator of 1.]
- [Only one unit fraction, $\frac{1}{1}$, is greater than $\frac{1}{2}$.]
- [The denominators go in size order, with the greatest denominator closest to 0 and the least denominator closest to $\frac{1}{2}$.]
- [Each unit fraction has its own unique location on the number line.]

*Bring the whole group back together. Have pairs share ideas for question 1. Then have a whole group discussion about questions 2 and 3.*

**2. What do you notice about the fractions that are closer to 0?**

*Students share their ideas. Look for understanding of these key mathematics ideas:*

- [The unit fractions that are closer to zero have larger denominators than the ones that are closer to $\frac{1}{2}$.]
- [The larger the denominator, the smaller the pieces. For a unit fraction with a greater denominator, like twelfths, you have only 1 of the small pieces, so that unit fraction will be closer to 0 than a unit fraction with a lower denominator like thirds.]

**3. Can any unit fractions go between $\frac{1}{2}$ and 1? Why or why not?**

*Students share their ideas. Ask follow-up questions. Support students in understanding these key mathematics ideas:*

- [A unit fraction is 1 piece, and the size of that piece is determined by the denominator.]
- [It is not possible for a unit fraction to be between $\frac{1}{2}$ and 1 because one over one is the whole and $\frac{1}{2}$ is one of 2 equal pieces. You cannot partition 1 whole to make a piece that is greater than $\frac{1}{2}$ and still have equal pieces.]

## Card Sets 2 & 3. Example Responses to Discussion Questions for Step 3

Here are some things that students might notice about the non-unit fractions:

- [None of the non-unit fractions have a 1 in the numerator.]
- [In fractions that are closer to 1, such as $\frac{11}{12}$ or $\frac{9}{10}$, the numerator is close to the denominator. These fractions are missing 1 part to make a whole.]
- [In fractions that are closer to 0, there is larger difference between the numerator and denominator than in the fractions that are closer to 1.]
- [The fractions that are close to $\frac{1}{2}$ have a numerator that is equal to or about half the value of the denominator.]
- [Set #2: There are no equivalent fractions in the set. Each of the fractions in this set has its own unique location on the number line.]
- [Set #3: There are equivalent fractions. We can tell that the fractions are equivalent because they are at the same location on the number line.]

**Note:** See the Answer Key (**R7**) for more examples of responses for each card set.

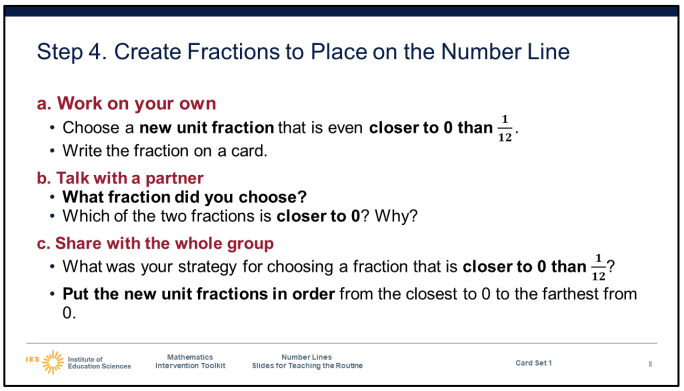
# Step 4 (Slide 8)

**SAY:** Work individually to come up with a fraction that is closer to 0 than to $\frac{1}{12}$. Write your fraction on a card.

*Give students time to choose their fractions and write them down.*

Now you'll work with a partner to compare your two fractions. Which fraction is closer to 0? Why?

*Students work together to compare your fractions.*




Now let's share our fractions with the whole group. What was your strategy for coming up with your fractions? How do you know that your fractions are closer to 0 than to $\frac{1}{12}$?

*Students share their reasoning and ideas.*

Let's line up the fractions you chose, from least to greatest. Stand up, holding your fraction cards, and put yourselves in order. The fractions at the left will be the ones that are closest to 0.
*Students sequence the fractions.*

How can we check that the fractions are in order?

*Students share their ideas and check the order.*

**Tips for Step 4**

Step 4 is a little different for each card set. Read the suggestions below for your card set.

**Card Set 1: Unit fractions**

There probably won't be enough room on the number line for them to place their new unit fractions. Students can sequence unit fractions by lining up the cards in order from left to right, with the lesser fractions to the left of the greater fractions. Here are some options:

- Line up the new fraction cards in order on a table.
- Tape the new fraction cards to a whiteboard.
- Have students stand up, holding their new fraction cards, and put themselves in order.

**Card Sets 2 & 3: Non-unit fractions**

Each card set has different directions for what fractions students should create:

- Set 2. Come up with a fraction that is even **closer to 1 than** $\frac{11}{12}$.
- Set 3. Come up with a fraction that is even **closer to** $\frac{1}{2}$ **than** $\frac{2}{5}$.

Students first write fractions individually. Then they work in pairs to compare their fractions:

- Set 2. Which of your fractions is **closer to 1**? **Why?**
- Set 3. Which of your fractions is **closer to** $\frac{1}{2}$? **Why?**

Bring the whole group together and have pairs share how they decided which fraction was closer.

**Note:** Students are **not** asked to put all the new non-unit fractions in order, because sequencing non-unit fractions can be challenging if students create fractions with a variety of numerators and denominators. Rather than having students put all the fractions in order, we suggest having pairs of students discuss which of their two fractions is closest to 1 and explain why.

## Step 5 (Slide 9)

**SAY:** Here are the goals we have worked on together today. You reasoned about a fraction's size or magnitude and location compared with a benchmark number. You also estimated the location of the fractions when you placed them on the number line. For the communication goal, you used the sentence starter to explain why the location of the fraction makes sense.


Next, to reflect and share something you learned, you'll pick one sentence starter and think individually. Then, when I cue you, you will share with your partner one thing that you learned about placing fractions on the number line OR one thing you learned about using a benchmark strategy.

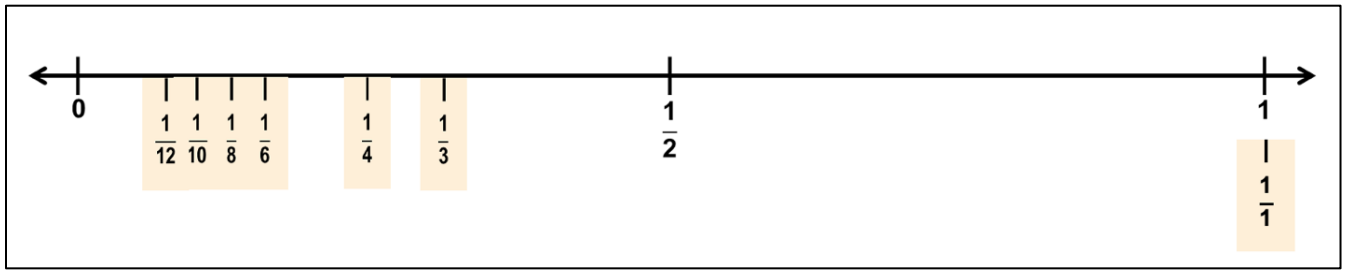
*Students complete a sentence starter to reflect on and share their learning.*

# R7. Answer Key for Each Card Set

## Example Number Line for Each Card Set

The pictures below show the placement of the fractions for each card set. Each picture shows the correct order of the fractions from 0 to 1.

## CARD SET 1




### Example Responses and Key Mathematical Ideas

Examples are shown in brackets and dark orange font.

**1.** What do you notice about the position of the fractions? Do you see any patterns?

- [There are more cards closer to 0 than to $\frac{1}{2}$ or 1.]
- [There is only one unit fraction that is greater than $\frac{1}{2}$.]
- [The denominators closer to 0 are higher numbers than those closer to $\frac{1}{2}$.]
- [All the unit fractions have a numerator of 1.]
- [All the unit fractions have different denominators from each other, and most have a denominator that is different from the numerator.]
- [Almost all of the unit fractions are less than $\frac{1}{2}$.]
- [Only one unit fraction, $\frac{1}{1}$, is greater than $\frac{1}{2}$.]
- [The denominators go in order, with the greatest denominator closest to 0 and the least denominator closest to $\frac{1}{2}$.]
- [Each unit fraction, except fraction $\frac{1}{1}$, has its own unique location on the number line.]

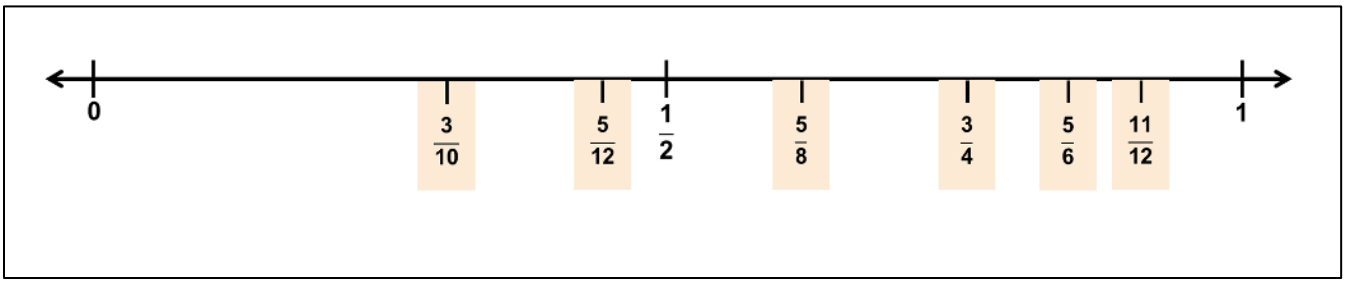
**2.** What do you notice about the fractions that are closer to 0?

- [The fractions closer to 0 have denominators that are greater than the fractions that are closer to $\frac{1}{2}$.]

**3.** Can any unit fractions go between $\frac{1}{2}$ and 1? Why or why not?

- [No unit fractions can go between 0 and $\frac{1}{2}$ because there isn't any other unit fraction between $\frac{1}{2}$ and $\frac{1}{1}$. But there are many fractions (an infinite number) between $\frac{1}{2}$ and 1, but none of those are unit fractions.]

## CARD SET 2




### Example Responses and Key Mathematical Ideas

**1.** What do you notice about the position of the fractions? Do you see any patterns?

- [None of the non-unit fractions have a 1 in the numerator.]
- [Only two of the non-unit fractions are between 0 and $\frac{1}{2}$.]
- [In this card set, more fractions are between $\frac{1}{2}$ and 0 than are between 0 and $\frac{1}{2}$.]
- [There are no equivalent fractions in the set. Each of the fractions in this set has its own unique location on the number line.]

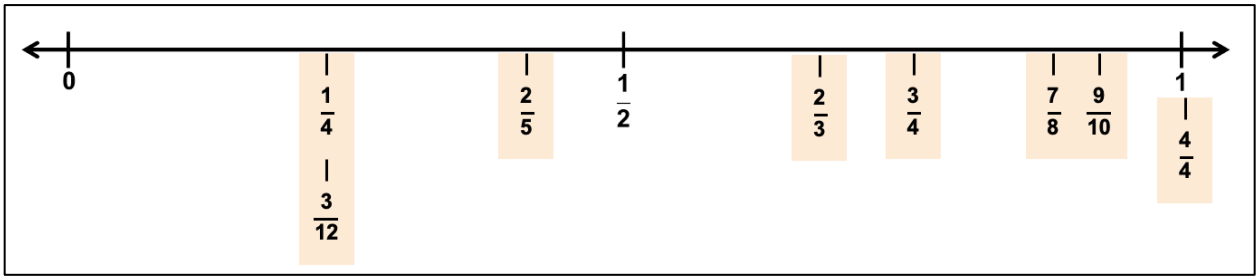
**2.** What do you notice about the fractions that are closer to 1? 0? $\frac{1}{2}$?

- [The cards closer to 1 have the numerator and denominator close to one another.]
- [In fractions that are closer to 1, such as $\frac{11}{12}$, the numerator is close to the denominator. These fractions are missing 1 part to make a whole.]
- [In fractions that are closer to 0, there can be a larger difference between the numerator and denominator than in the fractions that are closer to 1.]
- [The fractions that are close to $\frac{1}{2}$ have a numerator that is equal to or about half the value of the denominator.]

**3.** What do you notice about the fractions that are closer to $\frac{1}{2}$?

- [The cards closer to $\frac{1}{2}$ have a numerator that is about half the denominator. The numerator 5 in $\frac{5}{12}$ is close to 6, and $\frac{6}{12}$ is equivalent to $\frac{1}{2}$.]

## CARD SET 3




### Example Responses and Key Mathematical Ideas

1. What do you notice about the position of the fractions? Do you see any patterns?
   - [There are equivalent fractions. We can tell that the fractions are equivalent because they are positioned at the same location on the number line.]
2. What do you notice about the fractions that are closer to 1?
   - [In fractions that are closer to 1, such as $\frac{9}{10}$, the numerator is close to the denominator. These fractions are missing 1 part to make a whole.]
   - [In fractions that are closer to 0, there can be a larger difference between the numerator and denominator than in the fractions that are closer to 1.]
   - [The fractions that are close to $\frac{1}{2}$ have a numerator that is equal to or about half the value of the denominator.]
3. What do you notice about the equivalent fractions?
   - [The equivalent fractions are located at the same position on the number line.]

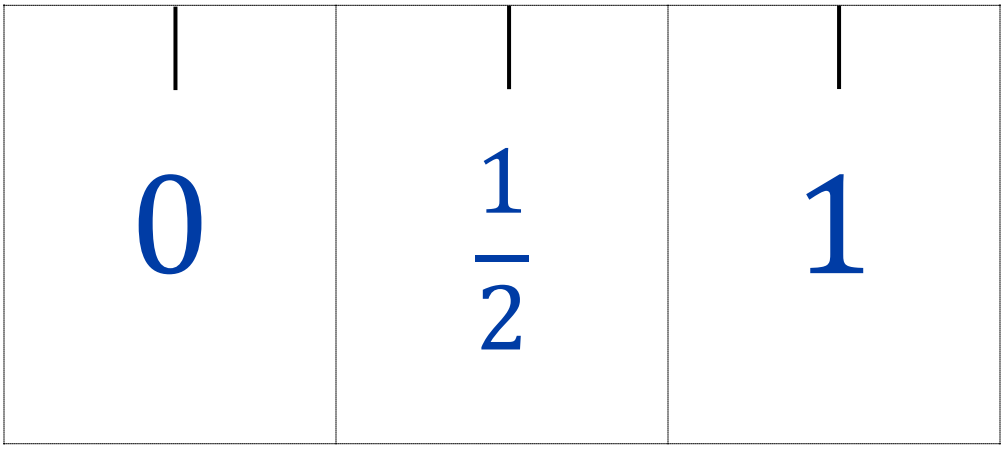
# Appendix A: Routine Resources for Use with Students

This appendix has reproducible handouts for using the number lines routine.

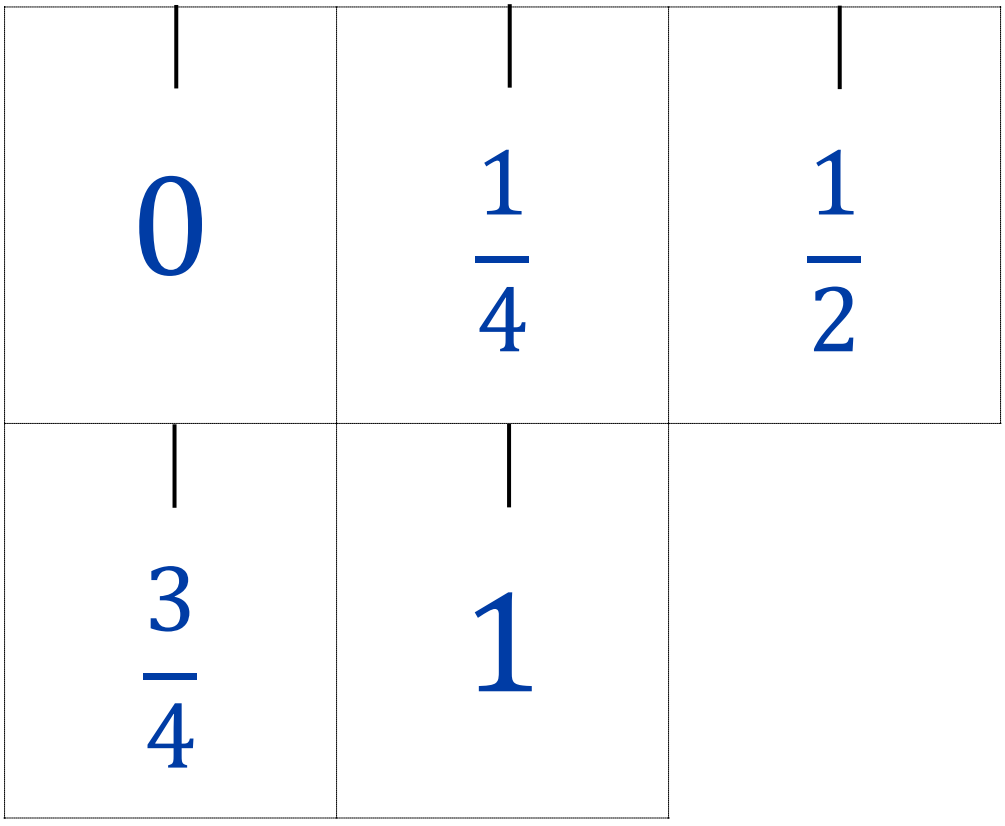
# Fraction Cards for Routine

## Cards with Benchmark Numbers

Use these benchmark cards with **Card Sets 1 and 2.**




Use these benchmark cards with **Card Set 3.**



# Card Set 1: Unit fractions

**Types of fractions:** All the cards have unit fractions.

**Number line:** The number line should be a minimum of **4' long** so that there is enough space to place six of the unit fractions between 0 and $\frac{1}{2}$.

**Benchmark numbers to place on the number line:** Mark the benchmarks of 0, $\frac{1}{2}$, and 1.

| | | |
|---|---|---|
| 1A. $\frac{1}{10}$ | 1B. $\frac{1}{12}$ | 1C. $\frac{1}{3}$ |
| 1D. $\frac{1}{4}$ | 1E. $\frac{1}{8}$ | 1F. $\frac{1}{6}$ |
| 1G. $\frac{1}{1}$ | | |

## Card Set 2: Non-unit fractions

**Fractions:** All cards are non-unit fractions in simplified form. There are no equivalent fractions.

**Number line:** The number line should be a **minimum of 3' long** so that there is enough space to place six fractions between 0 and 1. Use the benchmark cards 0, $\frac{1}{2}$, and 1.

| | | |
|---|---|---|
| 2A. $\frac{5}{6}$ | 2B. $\frac{3}{10}$ | 2C. $\frac{5}{8}$ |
| 2D. $\frac{3}{4}$ | 2E. $\frac{5}{12}$ | 2F. $\frac{11}{12}$ |

# Card Set 3: Non-unit fractions, including equivalent fractions

**Fractions:** This set has simplified and unsimplified fractions and equivalent fractions for $\frac{1}{4}$ and 1.

**Number line:** The number line should be a **minimum of 3' long**. Use the benchmark cards 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1.

| | | |
|---|---|---|
| 3A. $\frac{9}{10}$ | 3B. $\frac{3}{12}$ | 3C. $\frac{2}{5}$ |
| 3D. $\frac{4}{4}$ | 3E. $\frac{7}{8}$ | 3F. $\frac{2}{3}$ |

# Example Learning Goals for Routine

In Step 1 of the routine, the teacher shares the learning goals with students. This resource has example mathematics and communications goals for each card set. You can use these goals as is or adapt them.

## Card Set 1 (Unit fractions)

### Mathematics Goals

- Reason about a unit fraction's size (magnitude) and location as compared with a benchmark number.
- Estimate the location of a unit fraction on a number line.

### Communication Goal

- Show and explain why the location of a unit fraction on a number line makes sense.

## Card Set 2 (Non-unit fractions)

### Mathematics Goals

- Reason about a fraction's size (magnitude) and location as compared with a benchmark number.
- Estimate the location of a non-unit fraction on a number line.

### Communication Goal

- Show and explain why the location of a fraction on a number line makes sense.

## Card Set 3 (Non-unit fractions and equivalent fractions)

### Mathematics Goals

- Reason about a fraction's size (magnitude) and location as compared with a benchmark number.
- Estimate the location of a non-unit fraction on a number line.
- Use reasoning to determine the location of an equivalent fraction.

### Communication Goal

- Show and explain why the location of a fraction on a number line makes sense.

# Response Cards with Greater Than and Less Than

Print and cut out these cards. Provide a set to each student for use during the routine. Another option is to write the words and symbols on Index Cards.

| |
|---|
| Greater than<br>> |
| Less than<br>< |
| Greater than<br>> |
| Less than<br>< |

# Sentence Starters

This handout has 4 copies of the sentence starter. Provide a copy to each pair or individual student.

I placed the fraction_____ at this location because . . .

I placed the fraction_____ at this location because . . .

I placed the fraction_____ at this location because . . .

I placed the fraction_____ at this location because . . .

# Exit Task

This handout has two copies of the exit task. Provide a copy to each pair or individual student.

Name:_____ Date:_____

Write your response to **one** sentence starter. Choose A or B.

A. One thing I learned about placing fractions on a number line is . . .

B. One thing I learned about using a benchmark strategy is . . .

---

Name:_____ Date:_____

Write your response to **one** sentence starter. Choose A or B.

A. One thing I learned about placing fractions on a number line is . . .

B. One thing I learned about using a benchmark strategy is . . .

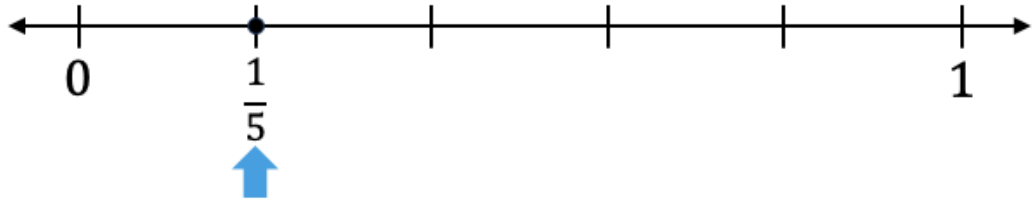
# Appendix B: Card Sorting Routine Resources

This appendix has resources for using the card sorting routine for number lines.

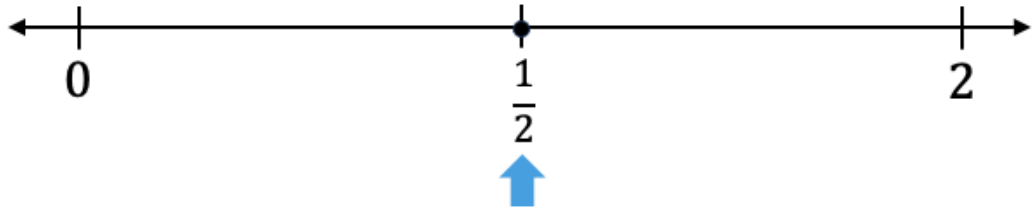
# Number Line Cards

Make one set of 11 cards per pair of students.

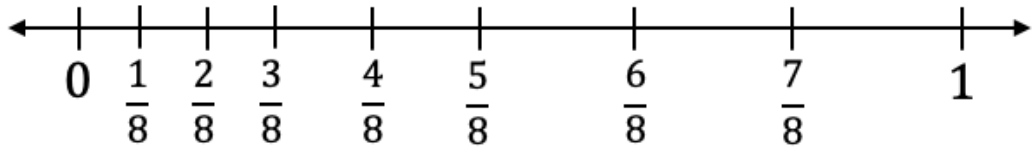
A.



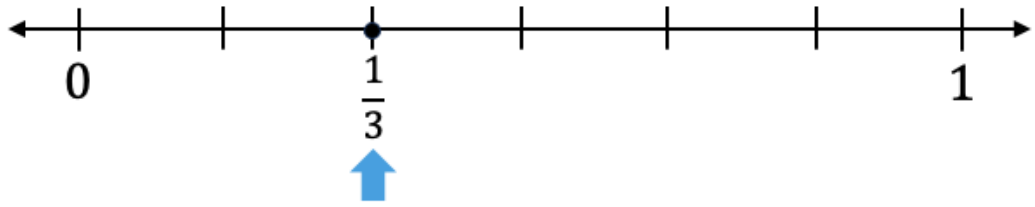

B.



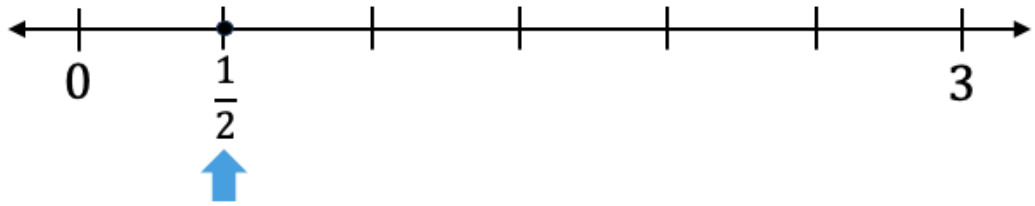

C.



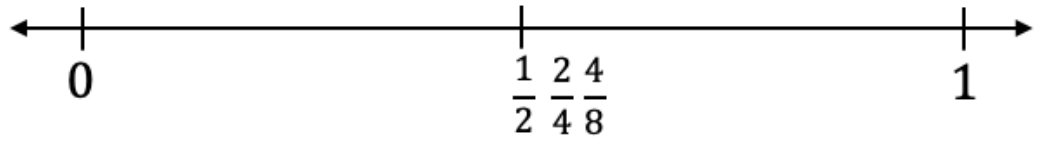

D.



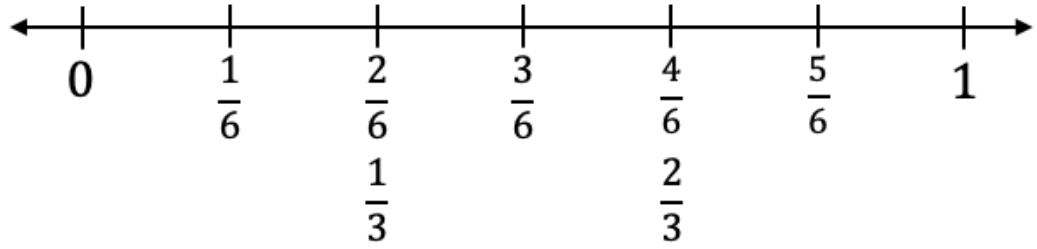
E.



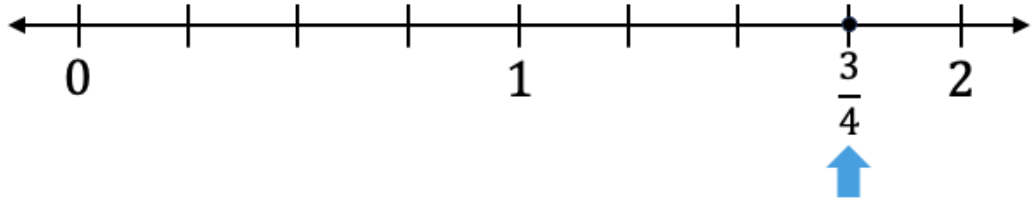

F.



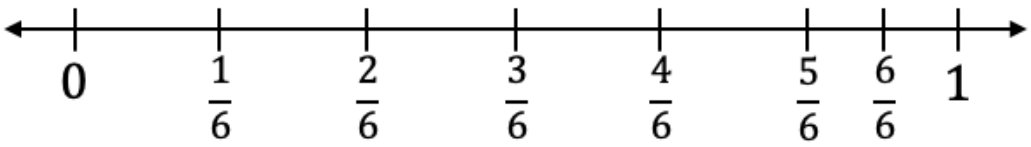

G.



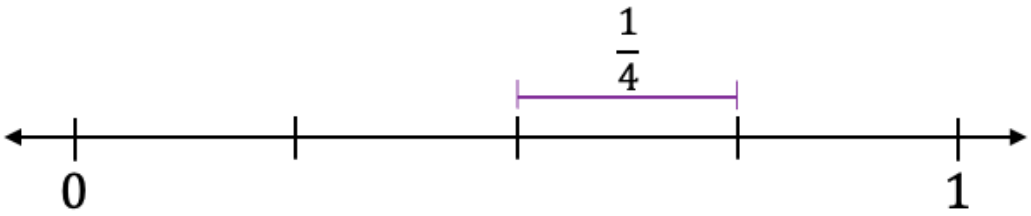

H.



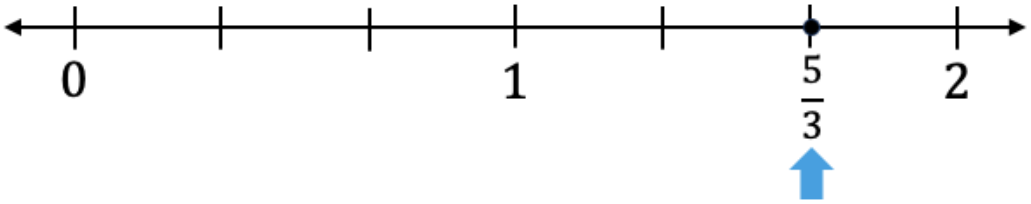
I.




J.




K.



## Category Cards

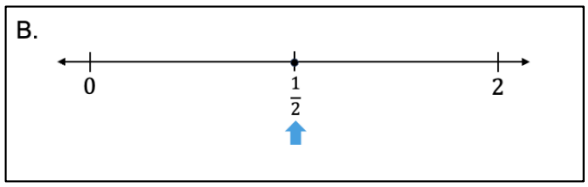
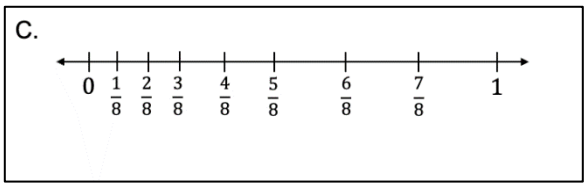
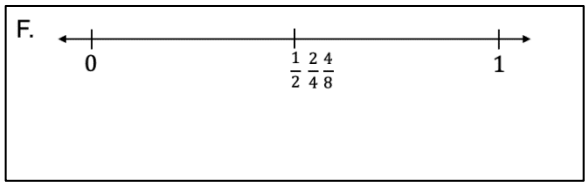
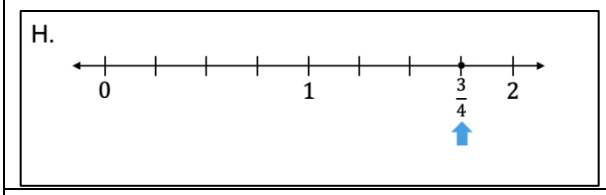
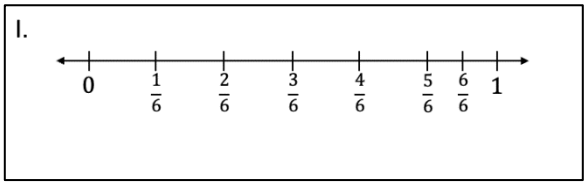
Make one set of 2 category cards per pair of students.

| | |
|---|---|
| **Accurate:**<br>Number line has<br>no errors | **NOT Accurate:**<br>Number Line has<br>one or more errors |
| **Accurate:**<br>Number line has<br>no errors | **NOT Accurate:**<br>Number Line has<br>one or more errors |
| **Accurate:**<br>Number line has<br>no errors | **NOT Accurate:**<br>Number Line has<br>one or more errors |
| **Accurate:**<br>Number line has<br>no errors | **NOT Accurate:**<br>Number Line has<br>one or more errors |

# Answer Key

**Accurate: Number line has no errors.** A, D, E, G, J, K. (6 cards)

**NOT Accurate: Number line has one or more errors**. B, C, F, H, and I. (5 cards)

| NOT Accurate Category | Example Explanation |
|---|---|
| B.   | [The number line is labeled with endpoints of 0 and 2, so the midpoint should be labeled 1.] |
| C.   | [The number line is not equally partitioned into eighths.] |
| F.   | [The equivalent fractions are not labeled properly. The fractions $\frac{2}{4}$ and $\frac{4}{8}$ should be written *below* $\frac{1}{2}$ because they all have the same location on the number line.] |
| H.   | [The label of $\frac{3}{4}$ is not correct. The point should be labeled $1\frac{3}{4}$ because it is located between 1 and 2.] |
| I.   | [The fraction $\frac{6}{6}$ is at an incorrect location. It is equivalent to 1 whole so it should be labeled below the 1.] |

# Appendix C: Answer Keys for Handouts

Answer Keys are provided for the following handouts. Example solutions are shown in brackets and dark orange text.

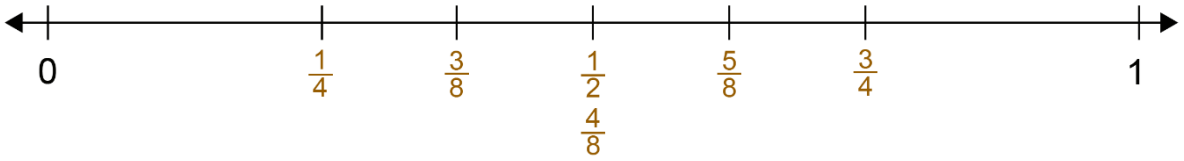
# H3A. Answer Key

## Use Fraction Tiles to Locate Fractions on a Number Line

**1.** Use fraction tiles to locate six fractions on the number line below. Place the tiles above the number line. Draw a point to show the fraction's location and write the fraction below it.

**a.** $\frac{1}{2}$ **b.** $\frac{1}{4}$ **c.** $\frac{3}{4}$ **d.** $\frac{5}{8}$ **e.** $\frac{3}{8}$ **f.** $\frac{4}{8}$

**[The example below shows the location of all the fractions for question 1.]**

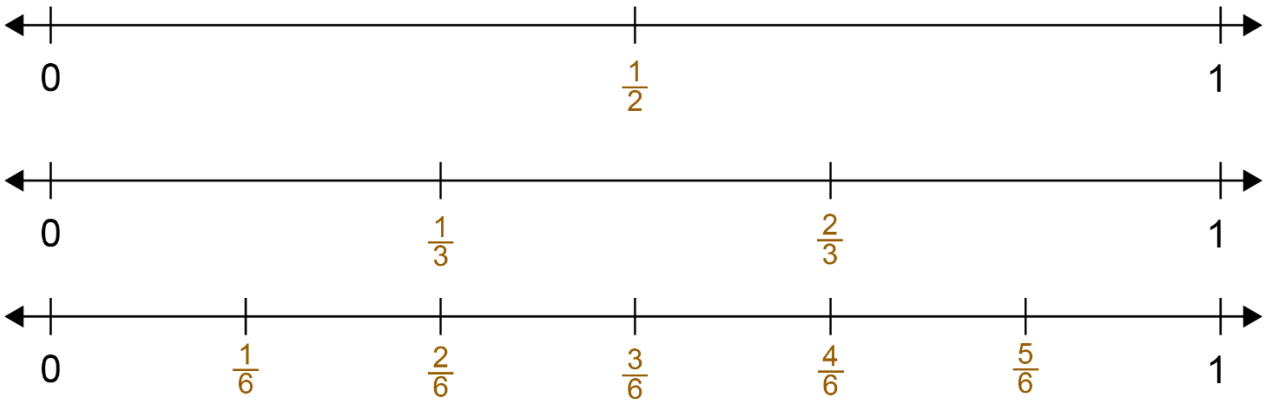



**2.** Use these clues to find three new fractions and write them below. Then, use the fraction tiles to locate each new fraction on the number line, draw a point, and label it with the fraction.

**[Example answers:** Fractions that can be built with the fraction tiles are shown below. There are an infinite number of other possibilities of fractions to meet the criteria, but only a limited number of options when you use the fraction tiles.]

| | |
|---|---|
| Clue A. Place a new fraction that is **equivalent to $\frac{1}{2}$.** | [$\frac{3}{6}$ $\frac{5}{10}$ $\frac{6}{12}$] |
| Clue B. Place a new fraction that is **closer to 0 than $\frac{1}{4}$.** | [$\frac{1}{12}$ $\frac{1}{10}$ $\frac{1}{8}$ $\frac{2}{12}$ $\frac{2}{10}$] |
| Clue C. Place a new fraction that is **closer to 1 than $\frac{3}{4}$.** | [$\frac{4}{5}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{8}{10}$ $\frac{9}{10}$ $\frac{10}{12}$ $\frac{11}{12}$] |

**3.** Label the tick marks on each number line with fractions. Tip: Place fraction tiles on the number line to help you label the tick marks.




**4.** Use the number lines and fraction tiles to find equivalent fractions. Fill in the missing numerators and denominators.

**a.** $\frac{1}{3} = \frac{[2]}{6}$ **b.** $\frac{4}{6} = \frac{2}{[3]}$ **c.** $\frac{[1]}{2} = \frac{3}{6}$

**d.** $1 = \frac{[3]}{3}$ **e.** $\frac{3}{[6]} = \frac{1}{2}$ **f.** $\frac{6}{[6]} = 1$

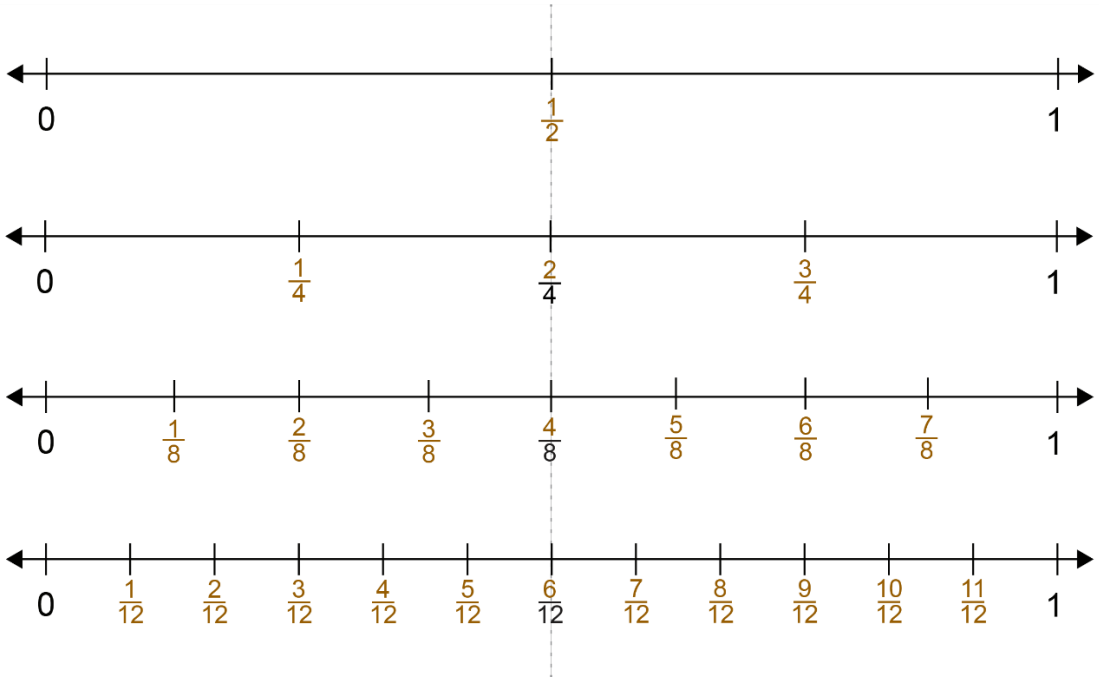
# H3B. Answer Key

## Example B. Use Multiple Number Lines to Find Equivalent Fractions

These number lines have 0 and 1 at the same locations, so they are helpful for finding **equivalent fractions.** An example dotted line shows the fractions that are equivalent to $\frac{1}{2}$.

**1.** Label **all** the tick marks with fractions on each number line.

**[Example answers:** The number lines below are correctly labeled with fractions.]




**2.** Use the number lines to find two or more fractions that are equivalent to each fraction below. Write the equivalent fractions below.

**Tip:** Line a ruler or straight edge up vertically to the number line at a fraction to make it easier to see which fractions are at the same locations and are equivalent.

**[Answers provided below:]**

$\frac{1}{4}$ is equivalent to: $\frac{[2]}{8}$ $\frac{[3]}{12}$

$\frac{6}{8}$ is equivalent to: $[\frac{3}{4}]$ $[\frac{9}{12}]$

$\frac{1}{2}$ is equivalent to: $[\frac{2}{4}]$ $[\frac{4}{8}]$ $[\frac{6}{12}]$

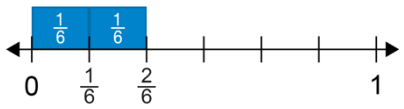
# H5. Answer Key: Explore Fraction Addition on the Number Line

## Part 2: Try Approaches with a New Problem

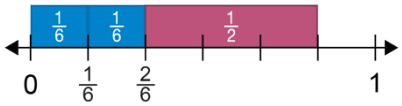
**1.** Try Approach 1 to model and solve: $\frac{2}{6} + \frac{1}{2} =$

**[Use Fraction Tiles. Here is an example solution** that shows the steps involved.

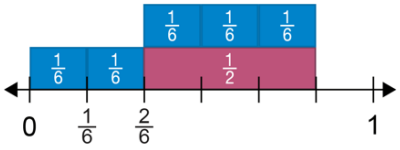
a. Build $\frac{2}{6}$ using two $\frac{1}{6}$ tiles. Label $\frac{1}{6}$ and $\frac{2}{6}$ on the number line.



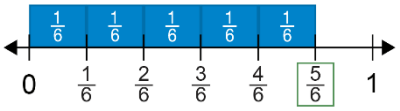

b. Then, add on $\frac{1}{2}$ by lining up one $\frac{1}{2}$ tile at the edge of the two $\frac{1}{6}$ tiles.




c. Replace the tiles for one fraction so that all the pieces will have the same denominator. This example uses sixths as a common denominator. The picture shows that three of the $\frac{1}{6}$ pieces are equivalent to the $\frac{1}{2}$ tile because they are the same length.




Replace the $\frac{1}{2}$ tile with three $\frac{1}{6}$ tiles. Then, use the $\frac{1}{6}$ tiles to partition the number line into sixths by drawing tick marks and labeling them. The sum is $\frac{5}{6}$.]




**2.** Try Approach 2 to model and solve: $\frac{3}{8} + \frac{1}{4} =$

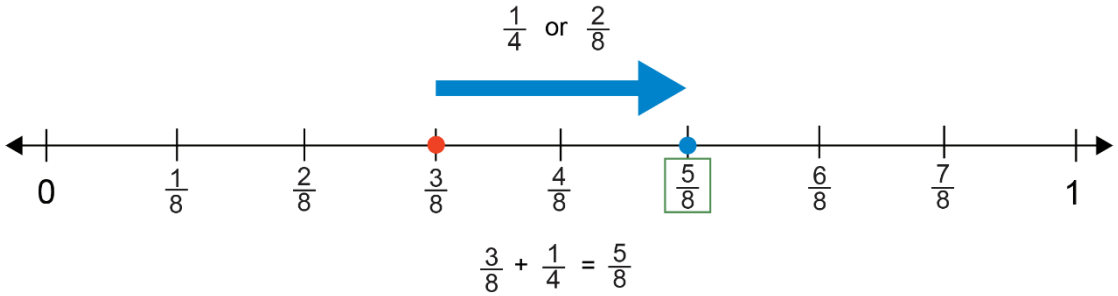
**Draw points and arrows on a number line** to find the sum.
**[Example solution:**

a. To represent both addends on the same number line, we can use the common denominator of eighths. We can rewrite the problem as shown below because $\frac{1}{4}$ is equivalent to $\frac{2}{8}$.

$$\frac{3}{8} + \frac{2}{8} =$$

b. Partition the number line into eighths. Draw a point for the location of the first addend, $\frac{3}{8}$.

c. Draw an arrow that starts at the point for $\frac{3}{8}$ and has a length of $\frac{2}{8}$ or $\frac{1}{4}$. The arrow goes to the **right** because we are adding a positive fraction, which will increase the distance from 0. The end of the arrow shows that the **sum** is $\frac{5}{8}$.]




## Part 3: Reflection Questions

**[Example Responses]**

**1.** What are benefits of using Approach 1, using fraction tiles, to model fraction addition on the number line?

[Using fraction tiles allows students to connect what they know about modeling addition with concrete materials to modeling addition on the number line. The tiles help build understanding of equivalent fractions and the need for a common denominator to express a sum.]

**2.** What are benefits of using Approach 2, drawing points and arrows, to model fraction addition?

[Drawing points and arrows helps to show how to "add on" an addend to find the total sum. It shows that when you add two positive fractions, the distance from 0 increases. It also helps build understanding that fractions can be represented on a number line in two ways: as a point with a precise location and as a length.]

**3.** What are your suggestions and questions about using each approach with students?

[Answers will vary.]

# H6. Answer Key: Explore Fraction Subtraction on the Number Line

## Part 2. Try the Approaches with a New Problem

### 1. Take-Away Model

**Problem**. Gavin has a piece of rope that is $\frac{5}{8}$ of a yard long.

He cuts off a length of rope that is $\frac{1}{4}$ yard long and uses it for a project.

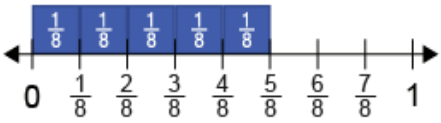
How long is the rope that Gavin has left?

Use one of the approaches to represent and solve the problem below.

___ Use fraction tiles ___Draw points and arrow.

**[Example solution: Use fraction tiles**

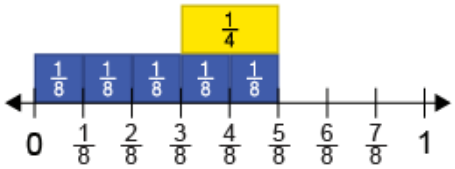
a. We can model this problem $\frac{5}{8} - \frac{1}{4} = ?$ by representing the first fraction, $\frac{5}{8}$, with five $\frac{1}{8}$ tiles.

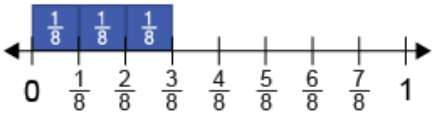



b. To subtract $\frac{1}{4}$, we need to use equivalent fractions. Since $\frac{1}{4}$ is equivalent to $\frac{2}{8}$, we can take away $\frac{2}{8}$ and are left with $\frac{3}{8}$.

So, $\frac{5}{8} - \frac{1}{4} = \frac{3}{8}$.

Gavin's remaining rope is $\frac{3}{8}$ of a yard.]

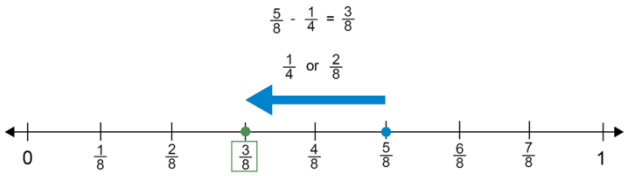



**[Example solution: Draw points and arrows**

a. Partition the number line into eighths to be able to represent both fractions.

b. Represent the first fraction, $\frac{5}{8}$ (minuend), with a **point** on the number line.




c. Show subtracting the second fraction (subtrahend), $\frac{1}{4}$, with an arrow that extends toward 0. The arrow extends a length of $\frac{1}{4}$, which is equivalent to $\frac{2}{8}$. Draw a point where the arrow ends to show the difference. The difference is $\frac{3}{8}$.]

## 2. Comparison Model

**Problem.** Tia ran $\frac{6}{10}$ of a mile. Jackie ran $\frac{1}{5}$ of a mile.

How much farther did Tia run than Jackie?

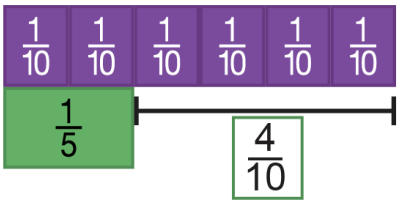
Use one of the approaches to represent and solve the problem below.

___ Use fraction tiles ____ Locate each fraction and determine the distance between them.

**[Example solution: Use fraction tiles**

a. We can represent this problem as what is the difference between the distance that Tia and Jackie ran using the equation $\frac{6}{10} - \frac{1}{5} = ?$

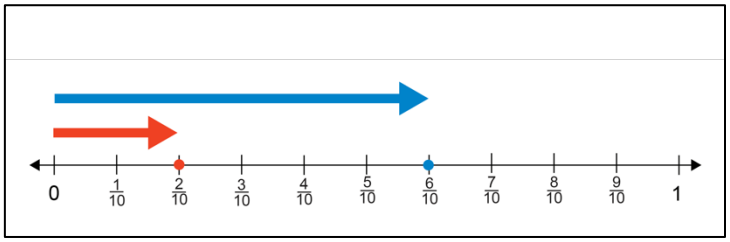
b. Place tiles for $\frac{6}{10}$ to represent the distance Tia ran. Then, put a $\frac{1}{5}$ tile to represent the distance that Jackie ran.




c. Line up the pieces to compare the distances. The gap is the **difference** between the lengths. The gap is the length of four $\frac{1}{10}$ tiles. This model shows that $\frac{6}{10} - \frac{1}{5} = \frac{4}{10}$.]

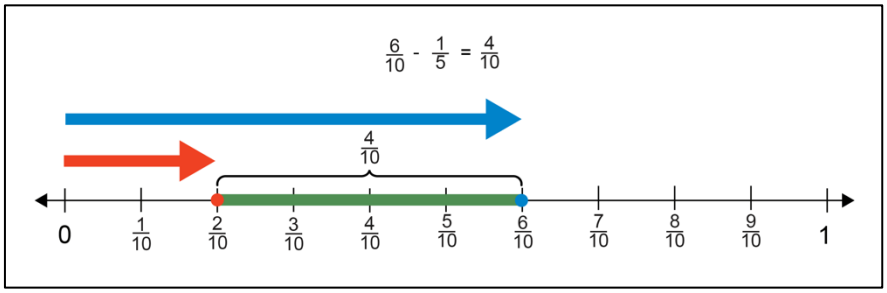
**[Example solution: Locate each fraction and determine the distance between them**

a. To solve this problem, $\frac{6}{10} - \frac{1}{5} = ?$, we can use a common denominator of tenths. Partition the number line into tenths.




b. The fraction $\frac{1}{5}$ is equivalent to $\frac{2}{10}$, so we can rewrite our equation as $\frac{6}{10} - \frac{2}{10} = ?$

c. We can represent $\frac{6}{10}$ by starting at 0 and drawing an arrow to $\frac{6}{10}$ and marking it with a point. Next, we can draw an arrow to show $\frac{2}{10}$ and mark it with a point.




d. The *difference* is the distance between the two points. The green line segment shows that the distance between $\frac{6}{10}$ and $\frac{2}{10}$ is $\frac{4}{10}$.]

# Part 3. Reflect on your experiences with the approaches.

**[Answers will vary. Example responses provided below.]**

**1. Why is it important to use different approaches to model subtraction on the number line?**

[It is important to use different approaches because there are two different ways to think about subtraction: As taking away/from or as comparing two quantities. For take-away problems, using fraction tiles or drawing an arrow to show taking away a quantity makes sense. We can also think about subtraction as comparing two lengths. We can think about the difference as the tiles needed to fill the gap or the distance between two points on the number line. Comparing and taking away are two different actions, so it makes sense to model them differently.]

**2. What are the benefits of using each approach?**

*Take Away/From Model of Subtraction*

- Use fraction tiles:
  [Using fraction tiles is beneficial because students can connect what they know about solving subtraction problems with fraction tiles to modeling subtraction on a number line. The tiles can also be helpful for building understanding of the importance of equivalent fractions for determining common denominators. Lining up tiles on a number line and then removing a quantity helps students understand how to represent a take-away model of subtraction.]
- Draw points and arrows:
  [When students are ready to move beyond using fraction tiles on the number line, they can use the number line on its own to represent and solve problems. Drawing points and arrows is beneficial because it helps students represent and solve a variety of subtraction problems.]

*Comparison Model for Subtraction*

- Use fraction tiles:
  [The benefit of using fraction tiles is that students can visually see the size (length) of each fraction and compare the difference between the fractions' lengths. This concrete model supports students in determining the difference between the two lengths because they can use the pieces to find what will fill the gap exactly.]
- Find the distance between points:
  [Using the number line to determine the difference between fractions is beneficial because students can draw arrows to show the length of each fraction and find the distance between them. This approach builds on the approach of comparing the fraction tiles to support students in moving to using the number line on its own to determine the difference between the fractions.]

**3. What are your suggestions and questions about using these approaches with students?**

- [When drawing arrows or points, I think it would be helpful to have students use colored pencils to visually show the arrows and points, which would help them calculate the distance.]
- [When using fraction tiles for the Comparison Model, I think it would be helpful to have students build fraction tiles on top of a mini whiteboard so that they could label the gap between the lengths by writing a fraction.]