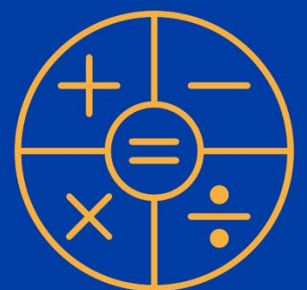
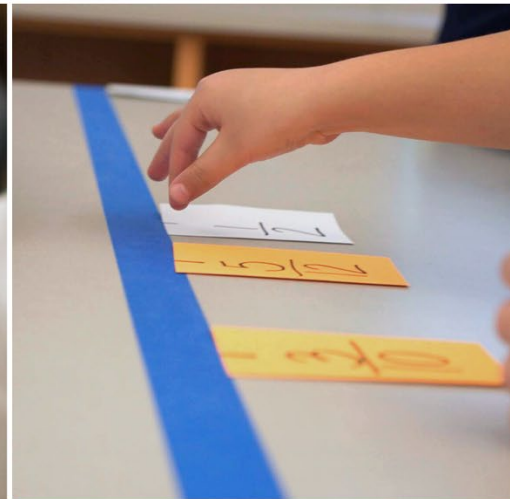
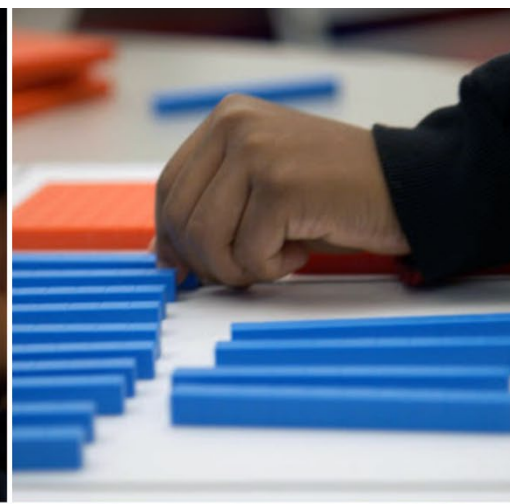
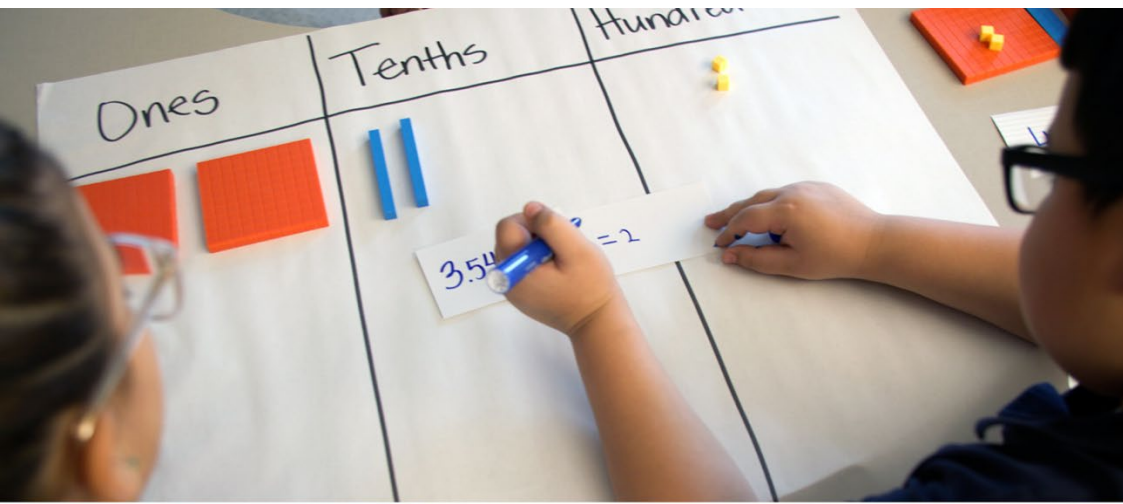


Mathematics Intervention Toolkit: Systematic Instruction Module

Participant Workbook

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Introduction to the Course

Welcome! This professional development (PD) course is designed to build participants' knowledge and practices for supporting students struggling with mathematics. It focuses on the evidence-based recommendations of the What Works Clearinghouse (WWC) Practice Guide *Assisting Students Struggling with Mathematics*.¹ These recommendations are based on a rigorous review and synthesis of research studies of effective intervention practices. The course is designed to connect this important research to participants' classroom practice.

The course has a series of **modules** to support in-depth professional learning. It starts with an Introductory Module and continues with five modules, each focusing on one recommendation (figure 1). The current module, Module 5, provides a deep dive into the recommendation for **systematic instruction**.

Figure 1. Course Sequence



The course is specifically designed for **teachers of mathematics intervention in grades 3–6**. This includes teachers in different roles, such as interventionists, Title I teachers, math specialists, general educators, and special educators. Participants will be able to apply the strategies in a variety of intervention settings, including separate intervention classes, intervention/enrichment blocks, and designated times for intervention during core mathematics classes. Similarly, the course will support participants who use a variety of intervention programs/curricula or who do not have a program.

The full course is intended to provide about **28 hours** of professional learning during one or two school years. It uses a **hybrid format** that combines online learning, Professional Learning Community (PLC) sessions, and opportunities for classroom implementation. The course focuses on key **number and operations** topics, such as **fractions**, that are a high priority for mathematics intervention.



¹ Fuchs, L.S., Newman-Gonchar, R., Schumacher, R., Dougherty, B., Bucka, N., Karp, K.S., Woodward, J., Clarke, B., Jordan, N. C., Gersten, R., Jayanthi, M., Keating, B., and Morgan, S. (2021). *Assisting students struggling with mathematics: Intervention in the elementary grades* (WWC 2021006). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance. Retrieved from <http://whatworks.ed.gov/>.

Module Overview

This module focuses on the WWC Guide’s recommendation for systematic instruction. The term “systematic” describes instruction that is intentional and strategically sequenced to build students’ knowledge toward identified learning outcomes. This approach is specifically designed to support students struggling with mathematics. Systematic instruction incorporates evidence-based strategies from prior modules, such as the use of visual and verbal supports. Focusing on systematic instruction in the final module provides opportunities to bring together and build upon ideas in the full PD course.

Recommendation for Systematic Instruction

Provide systematic instruction during intervention to develop student understanding of mathematical ideas.

Source: WWC Guide, p. 5



Professional Learning Goals

Participants will:

- Build knowledge of the WWC Guide’s recommendation for providing systematic instruction to support students struggling with mathematics.
- Build knowledge of evidence-based strategies and how to implement them effectively.
- Strengthen your ability to plan for and implement systematic instruction with your students.



Key Questions

Participants will explore these questions:

1. **What** is the WWC Guide’s recommendation for systematic instruction?
2. **Why** is systematic instruction important for student learning?
3. What are **strategies** for **how to** implement the recommendation?
4. What are **ways to apply** the recommended strategies with your students?
5. What are **potential challenges** and ways to address them?



Mathematics Content Focus

This module focuses on the high-priority topic of **fractions**. It includes information, examples, and activities on key topics: **fraction equivalence, comparison, addition, and subtraction**. The systematic instruction recommendation and strategies are also applicable to other mathematics topics.

Module Resources

The Systematic Instruction Module includes the following resources for participants:

- **Online Component:** Provides information about the module and links to resources.
- **Participant Workbook:** Provides all the handouts that participants will use at the PLC session. Three appendices provide additional resources: A) a copy of the WWC Guide’s recommendation for Systematic Instruction; B) resources for the card sorting activity; and C) an example of a completed Planner.
- **Glossary:** Provides relevant mathematical terms with definitions for the course.

The **Mathematics Intervention Toolkit**, which includes all the PD Course resources, is available for free at <https://ies.ed.gov/ncee/rel/math-support-grades-3-6>.

Online Component

This final module is intended to be about 1-week long, which is shorter than Modules 1–4. It includes **one** PLC session and does **not** have online activities to complete beforehand or a routine to use afterward. The module’s Online Component is organized by tabs, which are numbered to show the sequence of activities (figure 2).

Figure 2. Menu of Tabs



Description of tabs:

- **Tab 1. Intro:** Introduces the module’s goals, key questions, and sequence.
- **Tab 2. PLC Session:** Provides key resources and activities for use during and after the session.
- **Tab 3. Wrap-Up:** Provides short reflection activities for participants to do independently.
- **Resources:** Provides a hyperlinked list of module resources and optional additional resources.

Mathematics Intervention Course Checklist

Use this checklist to keep track of your progress in the course.

	Introductory Module	Dates
<input type="checkbox"/>	Kick-Off Session	
<input type="checkbox"/>	Wrap-Up (Complete tab 4 of Online Component)	
	Module 1. Mathematical Language	
<input type="checkbox"/>	Online Session (Complete tabs 1–3 of Online Component)	
<input type="checkbox"/>	PLC Session-A	
<input type="checkbox"/>	Try It!: Use Routine with Students	
<input type="checkbox"/>	PLC Session-B	
<input type="checkbox"/>	Wrap-Up (Complete tab 7 of Online Component)	
	Module 2. Representations	
<input type="checkbox"/>	Online Session (Complete tabs 1–3 of Online Component)	
<input type="checkbox"/>	PLC Session-A	
<input type="checkbox"/>	Try It!: Use Routine with Students	
<input type="checkbox"/>	PLC Session-B	
<input type="checkbox"/>	Wrap-Up (Complete tab 7 of Online Component)	
	Module 3. Number Lines	
<input type="checkbox"/>	Online Session (Complete tabs 1–3 of Online Component)	
<input type="checkbox"/>	PLC Session-A	
<input type="checkbox"/>	Try It!: Use Routine with Students	
<input type="checkbox"/>	PLC Session-B	
<input type="checkbox"/>	Wrap-Up (Complete tab 7 of Online Component)	
	Module 4. Word Problems	
<input type="checkbox"/>	Online Session (Complete tabs 1–3 of Online Component)	
<input type="checkbox"/>	PLC Session-A	
<input type="checkbox"/>	Try It!: Use Routine with Students	
<input type="checkbox"/>	PLC Session-B	
<input type="checkbox"/>	Wrap-Up (Complete tab 7 of Online Component)	
	Module 5. Systematic Instruction	
<input type="checkbox"/>	PLC Session	
<input type="checkbox"/>	Wrap-Up (Complete tab 3 of Online Component)	

Course Website URL: <https://ies.ed.gov/ncee/rel/math-support-grades-3-6>.

H1. Reference Sheet: Systematic Instruction

This handout provides an overview and a place to write notes for future reference.

1. What is the WWC Guide's recommendation?²

Recommendation for Systematic Instruction

Provide systematic instruction during intervention to develop student understanding of mathematical ideas.

2. What is systematic instruction and why is it important for student learning?

According to the WWC Guide:

“Effective interventions for improving the mathematics achievement of students with mathematics difficulties share one key feature: the design of the curricular materials and the instruction provided are systematic. The term systematic indicates that instructional elements intentionally build students’ knowledge over time toward an identified learning outcome(s). Systematic intervention materials are designed to develop topics in an incremental and intentional way, and the instruction provided supports student learning. This approach specifically addresses the needs of students who are struggling.”

“Systematically designed interventions most often include a ‘bundle’ of practices used to build and support student learning strategically. These practices and design features appear in other recommendations.”

3. How do you carry out the recommendation? What are the implementation steps?

The WWC Guide provides five main implementation steps.

Implementation Steps*

- Sequence instruction so that the mathematics that students are learning builds incrementally.
- When introducing new concepts and procedures, use accessible numbers to support learning.
- Provide visual and verbal supports.
- Provide immediate, supportive feedback to students to address any misunderstandings.
- Review and integrate previously learned content throughout intervention to ensure that students maintain understanding of concepts and procedures.

*Listed in the order used in the module.

4. What are strategies for how to implement the recommendation with students?

During the module, use this chart to list strategies you want to remember and apply.

² The text for the systemic instruction recommendation and implementation steps are excerpted from the WWC Guide, pp. 5–10. The full text of the recommendation is in [Appendix A](#).

Recommended Strategies for Systematic Instruction**Carefully sequence instruction.**

- Collaborate with colleagues to identify high-priority topics to focus on in mathematics intervention.
-

Use accessible numbers.

- When starting to teach new concepts and processes, use simpler numbers in problems.
-

Provide visual and verbal supports.

- Use visual supports, such as manipulatives, number lines, and diagrams, to help students understand and solve problems and explain ideas.
-

Provide timely and specific feedback.

- Ask questions that give students opportunities to reflect and self-correct.
-

Review and integrate previously learned content.

- Help students see how new content connects to previously learned content.
-

H2. Suggestions for Choosing Accessible Numbers

This handout lists the suggestions from the PLC session slides and additional ideas.

General Suggestions

- Start with smaller numbers to help students understand and solve problems.
- Choose accessible numbers that **align** with the mathematics goals. Make sure that changing the numbers does *not* change the mathematics goals.
- Aim for a just-right level of challenge. If the numbers are too easy, the task may not seem motivating to do.
- Increase the variety of numbers over time to avoid overgeneralizations.
-

Fractions

- Start with unit fractions: $\frac{1}{4}$. Then move to non-unit fractions: $\frac{3}{8}$
- Use benchmark fractions and fractions equivalent to or close to benchmarks: $\frac{1}{2}$ $\frac{3}{6}$ $\frac{5}{8}$
- Choose even-number denominators—easier to draw and partition: $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{6}$ $\frac{1}{8}$
- Choose denominators with smaller numbers— fewer pieces to build and draw: $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{6}$
- Choose fractions that can be represented with fraction circles or tiles: $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{8}$ $\frac{1}{10}$ $\frac{1}{12}$
-

Representing Problems with Manipulatives or Drawings

- Use smaller numbers to facilitate building/drawing and reduce counting errors.
- For larger numbers, use base ten blocks or diagrams with written numbers.
-

Word Problems

- Start with simpler numbers to help students understand and solve word problems.
- Prioritize building understanding of word problems rather than doing complex calculations.
-

H3. Example Sequences for Fraction Problems

This resource summarizes suggestions for sequencing instruction and using accessible numbers (from the PLC Session). There are three example sequences for A) Fraction Comparison, B) Fraction Addition, and C) Fraction Subtraction.

Support student learning by selecting and sequencing problems to:

Provide more practice with similar problems.

- Move forward incrementally to more complex problems.
- Move back to more foundational problems.
- Gather formative information about students' understandings and difficulties.

A. Fraction Comparison: Example Sequence

This table provides guidance for creating sequences of fraction comparison problems that progress from simpler to more complex. *Optional:* Write more examples in the second column.

Pairs of Fractions to Compare	Example Problems: Which fraction is greater?
1. Same Denominators	$\frac{3}{8}$ $\frac{7}{8}$
2. Same Numerators	$\frac{3}{5}$ $\frac{3}{8}$
3. Different Denominators and Numerators	
One denominator is a multiple of the other.	$\frac{3}{4}$ $\frac{5}{8}$
Denominators are not multiples but have common factors.	$\frac{2}{6}$ $\frac{3}{8}$
Denominators are relatively prime (no common factors except 1).	$\frac{1}{4}$ $\frac{2}{7}$

B. Fraction Addition: Example Sequence

This table provides guidance for sequencing fraction addition problems to progress from simpler to more complex. *Optional:* Write more problems in the Examples column.

Types of Fractions to Add	Example Problems
1. Like Denominators	
Like denominators with sum less than 1.	$\frac{5}{8} + \frac{2}{8} =$
Like denominators with sum greater than 1.	$\frac{6}{8} + \frac{4}{8} =$
2. Unlike Denominators	
One denominator is a multiple of other.	$\frac{3}{4} + \frac{1}{8} =$
Denominators have a common factor.	$\frac{5}{6} + \frac{1}{8} =$
Denominators are relatively prime (no common factors except 1).	$\frac{2}{3} + \frac{4}{7} =$
3. Fractions Greater than 1	
Like denominators (follow sequence above).	$\frac{13}{8} + \frac{7}{4} =$
Unlike denominators (follow sequence above).	$\frac{5}{3} + \frac{9}{4} =$
4. Mixed Numbers	
No regrouping.	$1\frac{2}{3} + 2\frac{1}{6} =$
Regrouping.	$3\frac{5}{6} + 4\frac{2}{3} =$

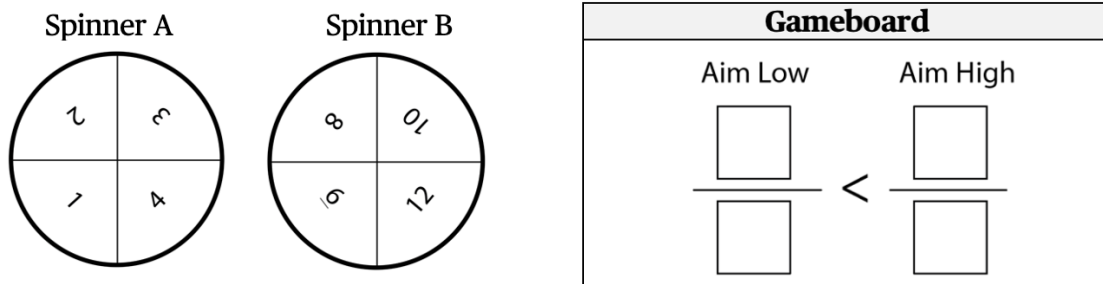
C. Fraction Subtraction: Example Sequence

This table provides guidance for sequencing fraction subtraction problems to progress from simpler to more complex. *Optional:* Add more problems to the Examples column.

Types of Fractions to Subtract	Example Problems
1. Like Denominators	
Like denominators with a difference less than 1.	$\frac{5}{8} - \frac{2}{8} =$
1 minus a fraction.	$1 - \frac{5}{8} =$
2. Unlike Denominators	
One denominator is a multiple of other.	$\frac{1}{3} - \frac{1}{6} =$
Denominators have a common factor.	$\frac{5}{6} - \frac{1}{4} =$
Denominators are relatively prime (no common factors except 1).	$\frac{2}{3} - \frac{1}{4} =$
3. Fractions Greater than 1	
Like denominators (follow sequence above).	$\frac{7}{4} - \frac{5}{4} =$
Unlike denominators (follow sequence above).	$\frac{9}{2} - \frac{5}{4} =$
4. Mixed Numbers	
No regrouping.	$3\frac{7}{8} - 2\frac{1}{4} =$
Regrouping.	$4\frac{1}{2} - 1\frac{3}{4} =$

H4. Aim Low, Aim High Game: Comparing Fractions

Goal: Make an Aim Low fraction that is **less than** the Aim High fraction.



Directions

The game is played in pairs. Each player has their own gameboard. Take turns. Use the spinners on handout H5.

1. Choose Spinner A or Spinner B. Spin to get a number.
2. Write the number in a box on your gameboard. Say where you put it.
 - It's okay to make fractions greater than 1.
 - After you write a number in a box, you **cannot** change it.
3. Take turns until each player has filled all four boxes on their gameboard.
4. Decide: Is your Aim Low fraction **less than** your Aim High fraction?

Tip: You can use fraction tiles, drawings, or other approaches to compare the two fractions.
5. Read your fractions out loud to the other player. Explain how the fractions compare.
6. **Score Points:** Answer Yes/No questions about your fractions. You get **1 point** for each **Yes**. The highest possible score for one round is 3 points.

Extension

Choose **one** of your finished gameboards and try to rearrange the numbers.

- Can you rearrange your numbers to **make a greater fraction** than the Aim High fraction you already have. What fraction can you make?
- Can you rearrange your numbers to **make a lesser fraction** than the Aim Low fraction you already have? What fraction can you make?

Gameboards for Aim Low, Aim High: Comparing Fractions

Round 1.

$$\begin{array}{ccc} \text{Aim Low} & & \text{Aim High} \\ \frac{\square}{\square} & < & \frac{\square}{\square} \end{array}$$

Goal: Make an Aim Low fraction that is less than your Aim High fraction.

On each turn, spin one spinner (A or B) to get a number. Write the number in a box.

After you fill in all four boxes, answer these questions:

- Is your Aim Low fraction **less than** your Aim High fraction? ___ **Yes** ___ **No**
- Is your Aim Low fraction **less than** $\frac{1}{2}$? ___ **Yes** ___ **No**
- Is your Aim High fraction **greater than** $\frac{1}{2}$? ___ **Yes** ___ **No**

Round 2.

$$\begin{array}{ccc} \text{Aim Low} & & \text{Aim High} \\ \frac{\square}{\square} & < & \frac{\square}{\square} \end{array}$$

Goal: Make an Aim Low fraction that is less than your Aim High fraction.

On each turn, spin one spinner (A or B) to get a number. Write the number in a box.

After you fill in all four boxes, answer these questions:

- Is your Aim Low fraction **less than** your Aim High fraction? ___ **Yes** ___ **No**
- Is your Aim Low fraction **less than** $\frac{1}{2}$? ___ **Yes** ___ **No**
- Is your Aim High fraction **greater than** $\frac{1}{2}$? ___ **Yes** ___ **No**

Score Points. Get **1 point** for each **Yes** you selected.

Your score: ___

Fraction Comparison Game: Information for Teachers

Here are some example questions to use for discussing the game with students. By focusing on strategies for placing numbers in numerators and denominators, the questions help to bring out key ideas about fraction magnitude and comparison.

Questions to support students in explaining their ideas:

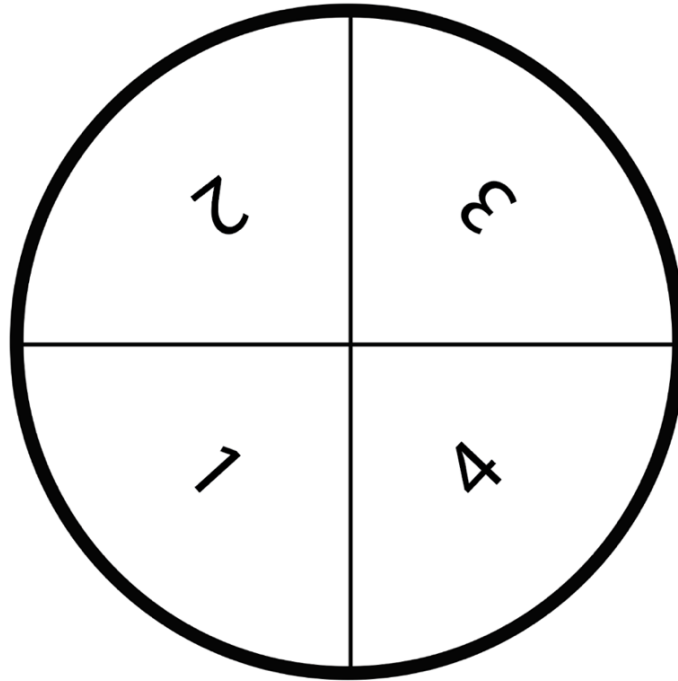
1. Where is a helpful spot to place 1? Why?
2. Where is a helpful spot to place 12? Why?
3. What strategies did you use to make a **greater** fraction (for the Aim High fraction)?
4. What strategies did you use to make a **lesser** fraction (for the Aim Low fraction)?
5. Why did you decide to write ___ [number] in this ___ [numerator/denominator]?
6. How did you decide when to use Spinner A or Spinner B?
7. How did you compare the fractions to determine which fraction is greater?

Suggestions:

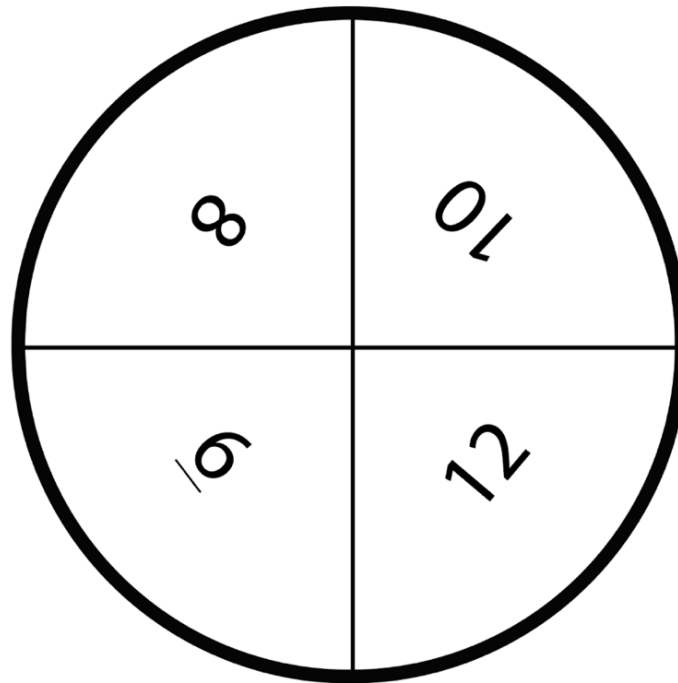
- After the discussion, have students play another round of the game so they can apply the strategies.
- Have fraction tiles available for students to use to show and explain their ideas.

H5. Spinners for Aim Low, Aim High Games

Spinner A



Spinner B



H6. Fraction Addition Games

Aim High Game: Directions

Materials: Each player gets their own Aim High gameboard. Use the spinners on handout H5.

Goal: Try to make a **greater sum**.

- On your turn, spin **one** spinner (A or B). Write the number in a box on your gameboard.
 - It's okay to make fractions that are greater than 1.
 - Important:** After you write the number in a box, you cannot change it.

2. Keep taking turns until each player has filled their **four boxes** to make two fractions.

3. Determine your **sum**. Read your fractions and sum out loud.

Tip: You can use fraction tiles, calculations, or other strategies to find and compare the sums.

4. Answer these questions about your sum:

- Is your sum greater than 1? ___ Yes ___ No
- Is your sum greater than 3? ___ Yes ___ No
- Is your sum greater than the other player's sum? ___ Yes ___ No

5. **Score points.** You get 1 point for each Yes.

Aim Low Game: Directions

Materials: Each player gets their own Aim Low gameboard. Use the spinners on handout H5.

Goal: Try to make a **lesser sum**.

1. On your turn, spin **one** spinner (A or B) to get a number. Write the number in a box on your gameboard.

- Important:** After you write the number in a box, you cannot change it.

2. Keep taking turns until each player has filled their **four boxes** to make two fractions.

3. Determine your **sum**. Read your fractions and sum out loud.

4. Answer these questions about your sum:

- Is your sum less than 2? ___ Yes ___ No
- Is your sum less than 1? ___ Yes ___ No
- Is your sum less than the other player's sum? ___ Yes ___ No

5. **Score points.** You get 1 point for each Yes.

Gameboards for Aim High Fraction Addition Game

Round 1. Aim High

$$\frac{\square}{\square} + \frac{\square}{\square} = ?$$

Goal: Try to make a *greater* sum.

On each turn, spin one spinner (A or B) to get a number. Write the number in a box.

After you fill the four boxes, determine your sum. Answer these questions:

- Is your sum **greater than 1**? Yes No
- Is your sum **greater than 2**? Yes No
- Is your sum **greater than the other player's sum**? Yes No

Score points. You get 1 point for each Yes.

Your points: ___

Round 2. Aim High

$$\frac{\square}{\square} + \frac{\square}{\square} = ?$$

Goal: Try to make a *greater* sum.

On each turn, spin one spinner (A or B) to get a number. Write the number in a box.

After you fill the four boxes, determine your sum. Answer these questions:

- Is your sum **greater than 1**? Yes No
- Is your sum **greater than 3**? Yes No
- Is your sum **greater than the other player's sum**? Yes No

Score points. You get 1 point for each Yes.

Your points: ___

Extension

Try to rearrange the numbers on one of your gameboards to make a **greater sum** than the one you got. What sum can you make?

Gameboards for Aim Low Fraction Addition Game

Round 1. Aim Low

$$\frac{\square}{\square} + \frac{\square}{\square} = ?$$

Goal: Place the numbers to try to make a *lesser* sum.

After you fill the four boxes, determine your sum. Answer these questions:

- Is your sum **less than 2**? Yes No
- Is your sum **less than 1**? Yes No
- Is your sum **less than the other player's sum**? Yes No

Score points. You get 1 point for each Yes.

Your points:

Round 2. Aim Low

$$\frac{\square}{\square} + \frac{\square}{\square} = ?$$

Goal: Place the numbers to try to make a lesser sum.

After you fill the four boxes, determine your sum. Answer these questions:

- Is your sum **less than 2**? Yes No
- Is your sum **less than 1**? Yes No
- Is your sum **less than the other player's sum**? Yes No

Score points. You get 1 point for each Yes.

Your points:

Extension

Try to rearrange the numbers on one of your gameboards to make a **lesser sum** than the one you got. What sum can you make?

Fraction Addition Games: Information for Teachers

Here are some example questions to use for discussing the fraction addition games with students. By focusing on strategies for placing numbers in numerators and denominators, the questions help to bring out ideas about fraction magnitude and addition. The Aim High game is helpful for focusing on fractions that are greater than 1. In the Aim Low game, students explore ways to make lesser fractions.

Aim High Game: Questions

Use these questions to support students in explaining their strategies.

1. What strategies did you use to make a **greater sum**?
2. Where is a helpful spot to place 1? Why?
3. Where is a helpful spot to place 12? Why?
4. Why did you decide to write ___ [number] in this ___ [numerator/denominator]?
5. How did you decide when to use Spinner A (lower numbers) or Spinner B (higher numbers)?
6. **Extension:** With the numbers you have on **one** finished gameboard, could you rearrange the numbers to **make a greater sum** than the one you have? What sum can you make?

Aim Low Game: Questions

Use these questions to support students in explaining their strategies.

1. What strategies did you use to make a **lesser sum**?
2. Where is a helpful spot to place 1? Why?
3. Where is a helpful spot to place 12? Why?
4. Why did you decide to write ___ [number] in this ___ [numerator/denominator]?
5. How did you decide when to use Spinner A (lower numbers) or Spinner B (higher numbers)?
6. **Extension:** With the numbers you have on **one** finished gameboard, could you rearrange the numbers to **make a lesser sum** than the one you have? What sum can you make?

Suggestion for Both Games: After the discussion, have students play another round of the game so they can apply the strategies.

H7. Provide Feedback to Students

Mathematics intervention provides opportunities for teachers to give students timely, specific, and actionable feedback, and for students to respond to the feedback.

Directions. Look over the suggestions. Star* ideas that stand out for you and add more ideas.

1. What are the purposes of providing feedback to students in mathematics intervention?

- Move learning forward.
- Help students to self-correct.
- Build on strengths.
- Address misunderstandings.
- Help students to reflect and self-assess.
-

2. What are ways to find out about students' understandings and difficulties so that you can provide specific and actionable feedback?

- Provide verbal and visual supports for students to communicate their ideas.
- Ask follow-up questions when students are correct and incorrect.
- Use wait time.
- Use formative assessment approaches, such as exit tasks.
- Ask probing questions to help identify strengths and misconceptions.
- Identify and build on students' strengths to address misunderstandings.
-

3. What are suggestions for providing feedback to students struggling with mathematics?

- Align feedback with learning goals to prioritize what to emphasize.
- Provide feedback in different ways: Questions, comments, gestures, etc.
- Use questions to help students move forward without giving too much away.
- Give students opportunities to identify errors and self-correct.
-

H8. Card Sorting Routine: Fraction Equivalence

Routine: Sort, Explain, and Generalize

Steps:

1. Sort cards and explain reasons.
2. Discuss and generalize.
3. Wrap up and reflect.

Step 1. Sort cards and explain reasons.

- a. Teacher goes over the directions. Each card has two fractions. The shaded parts show the fractions. The goal is to sort the cards into two categories:

Equivalent Fractions

NOT Equivalent Fractions

- b. Students work in pairs and take turns placing the cards.
 - One partner picks a card and decides whether the fractions are equivalent.
 - The student places the card in a category and explains why by using **one** sentence starter.

The fractions are equivalent because...

The fractions are not equivalent because...

- Then the other partner responds by saying whether they agree or disagree and why.

I agree because... I disagree because...

Step 2. Discuss and generalize.

- a. Teacher reveals the answers to each category. Students check their work and can ask questions.
- b. Teacher facilitates a discussion about the Equivalent Fractions category:
 - What do you notice about the pairs of equivalent fractions?
 - For the visual models, how do you know that the pairs of fractions are equivalent?
 - For the numeric fractions, how do you know that the pairs of fractions are equivalent?
- c. Teacher leads discussion about the Not Equivalent Fractions category by using questions like 2b.

Step 3. Wrap up and reflect.

- a. Students complete an exit task to reflect on their understanding of equivalent fractions.
- b. They share responses in pairs or with the whole group.

[Appendix B](#) has resources for the routine, including fraction cards.

Information for Teachers

Summary of Recommended Strategies in the Routine

The routine uses strategies from the systematic instruction recommendation and other recommendations from the WWC Guide.

- The routine incorporates these strategies:
 - Review** previously learned content on fraction equivalence.
 - Use semi-concrete representations** that accurately represent fractions.
 - Connect semi-concrete and abstract representations** of fractions.
 - Use representations to explain ideas** about fraction equivalence.
 - Use sentence starters** to support students in explaining reasons.
 - Pose questions** to support communication and probe for understanding.
- Table** of strategies with the corresponding recommendations in the WWC Guide.

Strategies in Routine	WWC Guide Recommendations
Review previously learned content and reinforce understanding of fraction equivalence.	Systematic Instruction (review content)
Use semi-concrete representations that accurately represent fractions, such as area models and number lines, to determine equivalence.	Systematic Instruction (visual supports) Representations Number Lines
Connect semi-concrete and abstract numeric representations of fractions.	Systematic Instruction (visual supports) Representations
Use representations to explain ideas , such as having students point to pictures of fractions to explain why they are equivalent.	Systematic Instruction (visual and verbal supports) Representations Mathematical Language
Use sentence starters to support students in explaining their reasons for choosing the equivalent or not-equivalent category and why they agree or disagree with a placement.	Systematic Instruction (verbal supports) Mathematical Language
Pose questions to support students in communicating their ideas and to find out about their understanding of equivalence.	Systematic Instruction (verbal supports and feedback) Mathematical Language

H9. Planner for Using Card Sorts to Review Content

Use this Planner to plan ways to use card sorts to review specific mathematics content.

Planning Questions

1. **What? Who?** Choose a topic to review with the students in one or more intervention classes.

- Mathematics topic to review:
- Intervention class(es):

2. **When?** Plan when to use the review activity.

- When did students get instruction on this topic?
- When would be a good time to provide a review activity?

3. **Which cards do you want students to sort?**

Look over the card deck(s) for your chosen sort. Some decks have a variety of representations.

- Do you want to use the full deck or a subset of cards to focus on specific representations?
- Do you want to add or change cards? If yes, what will you do?

4. **Prepare to check for understanding** during the activity. What will you look for, listen for, and ask students to find out about their understanding of fraction equivalence?

- Look for...
- Listen for...
- Ask these questions...
- Anticipate potential challenges for students:

Note: It's helpful to identify specific cards that might cause challenges so you can check for understanding.

Reflection Questions (After Using the Review Activity)

1. What did you notice about students' understanding and difficulties with the content?
 - Understanding:

 - Difficulties:

2. What **next steps** will you take to support student learning? (Check all that apply.)
 - a.** Provide more instruction. What will you focus on specifically?

 - b.** Provide more practice. What activities/tasks will you use?

 - c.** Focus on a different topic. Students show good understanding of this content.

 - d.** *Add other ideas.*

3. As part of planning systematic instruction, do you plan to revisit this content again in the future?
 - Yes, revisit the content on _____ (approximate date).
 - No

Overview of Card Sorts in the Course

Use the information below to select card sorts to use for review activities. Add ideas for when and how you might use the activity in your sequence of instruction.

Card Sorts: Mathematics Focus	Types of Representations on the Cards	When might you use this sort as a review activity? How? List ideas.
Fractions		
Comparing fractions to benchmark numbers. Module: Introductory	Fraction circles, bars, and numeric fractions.	
Unit fractions. Module: Mathematical Language	Fraction circles, bars, number lines, and numeric fractions.	
Number line representation for fractions. Module: Number Lines	Number lines labeled with different endpoints and fractions.	
Equivalent fractions. Module: Systematic Instruction	Fraction circles, bars, number lines, and numeric fractions.	
Other Topics		
Word problems: Sort Change and Compare problem types. Module: Word Problems	Examples of Change and Compare word problems. The cards have problems that use whole numbers and fractions.	

H10. Recap Strategies for Systematic Instruction

A. What are recommended strategies for implementing systematic instruction?

Review the list³ and add examples to the first four strategies.

- a. When introducing new concepts/procedures, use accessible numbers, such as...
- b. Provide visual supports, such as...
- c. Provide verbal supports, such as...
- d. Provide periodic reviews of content by using engaging activities, such as...
- e. Use an intentional sequence of instruction to build concepts strategically.
- f. Use a mix of previously and newly learned content.
- g. Provide opportunities for students to explain and discuss their ideas.
- h. Provide students with timely and specific feedback.
- i. Ask questions to give students opportunities to identify errors and self-correct.

Add other strategies.

B. What things should you avoid?

Review the list. Then, add ideas for which strategies to use instead for **two** of the items below.

- Avoid teaching lesson topics in isolation. Instead...

- Avoid moving to more complex concepts too quickly. Instead...

- Avoid providing opportunities for discussion only at the end of lessons. Instead...

³ Strategies adapted from WWC Guide by Toolkit authors. These lists are not exhaustive.

H11. Self-Reflection Form: Systematic Instruction

Introduction

This form provides an opportunity to reflect on your learning and current understanding of the recommendation for systematic instruction. This self-reflection can help you celebrate progress and guide you in identifying areas for continued growth in your professional learning. It is not intended to be evaluative and will not be submitted or shared with others unless you choose to do so.

The form has two parts. In **Part 1**, you are asked to self-assess your understanding of specific strategies from the module on a scale of 1 –3 (1: A little understanding; 2: Some understanding; and 3: A strong understanding). Participants are not expected to have a “strong understanding” of every strategy at the end of a module. Please be assured that it’s fine to select “some” or “a little” understanding. Your learning is evolving, and you will have opportunities to strengthen your understanding of the strategies during and after the course. Continuing to use strategies with your students is a critical step in deepening your understanding and skills.

In **Part 2**, you are asked to reflect on the strategies and select one that you would like to improve. Focusing on one strategy is helpful for planning concrete next steps that are manageable to carry out. If you would like support, reach out to your facilitator and colleagues in the course. You may also want to revisit resources on specific strategies in the Participant Workbook and the online component.

This self-reflection form captures your thinking at one point in time. It’s helpful to revisit the form after the course to consider how your understanding has changed and to plan ways to continue your professional learning of the recommended strategies.

Part 1. Reflect on your understanding of strategies.

Directions: The table has a list of recommended strategies for implementing systematic instruction. Read each strategy and self-assess your level of understanding by using this **rating scale**:

- 1. A Little Understanding:** Have a vague sense of the strategy.
- 2. Some Understanding:** Able to explain the strategy in general terms.
- 3. A Strong Understanding:** Able to explain it in detail and give examples.
- N/A:** Have not learned about the strategy yet.

Strategies	Select Your Rating
What is your current level of understanding for each strategy?	
a. How to use an intentional sequence of instruction to reach specific learning goals.	1 2 3 NA
b. How to choose accessible numbers.	1 2 3 NA
c. How to sequence fraction comparison problems to move from simpler to more complex problems.	1 2 3 NA
d. How to provide visual supports for students.	1 2 3 NA
e. How to provide verbal supports for students.	1 2 3 NA
f. How to use mathematics games with supports to build and reinforce mathematics understanding.	1 2 3 NA
g. How to provide timely and specific feedback to students struggling with mathematics.	1 2 3 NA
h. How to use questioning strategies to give students opportunities to reflect and self-correct	1 2 3 NA
i. How to review mathematics content by using engaging activities, such as card sorts.	1 2 3 NA

Appendix A: Systematic Instruction Recommendation

This resource is an excerpt from the WWC Guide.⁴

Provide systematic instruction during intervention to develop student understanding of mathematical ideas.

Introduction

Effective interventions for improving the mathematics achievement of students with mathematics difficulties share one key feature: The design of the curricular materials and the instruction provided are systematic. The term systematic indicates that instructional elements intentionally build students' knowledge over time toward an identified learning outcome(s). Systematic intervention materials are designed to develop topics in an incremental and intentional way, and the instruction provided supports student learning. This approach specifically addresses the needs of students who are struggling.

Systematically designed interventions most often include a “bundle” of practices used to build and support student learning strategically. These practices and design features appear in other recommendations. For example, reviewing and integrating previously and newly learned content is highlighted in the word problems recommendation; incrementally building knowledge is illustrated in recommendations for representations, number lines, and word problems; and providing adequate supports for students to learn and understand new concepts and procedures is highlighted in the recommendations for mathematical language and representations. Regardless of the intervention's focus, these aspects of instructional design are critical for supporting student learning.

The WWC and the expert panel assigned a strong level of evidence to this recommendation based on 43 studies of the effectiveness of systematic intervention design features and systematic instruction. Thirty-two of the studies meet WWC group design standards without reservations, and 11 studies meet WWC group design standards with reservations.

This recommendation highlights an overarching set of instructional features that form the backbone of effective, systematic interventions. This section describes strategies, examples, and tools that instructors can use to implement effective systematic interventions.

⁴ Excerpt from [WWC Guide](#), pp. 5–10 with minor adaptations by the Toolkit authors.

How to carry out the recommendation

1. Review and integrate previously learned content throughout intervention to ensure that students maintain understanding of concepts and procedures.

The panel recommends that interventions include systematic review of content by including a mix of *previously* and *newly learned* material within and across lessons. Review relevant previously taught material before introducing new, related content within and across lessons. Help students understand the link between the previous content and the new content they are learning. Present students with opportunities to think about or solve familiar types of problems. During this review, students can explain what they know about a topic, or they can solve problems. Review can be completed individually or while working with a small group or a partner (see recommendation on mathematical language).

Regularly present a variety of problems that require students to discriminate among problem types (see recommendations on word problems). This practice avoids having students overgeneralize new concepts or procedures to previously learned material. For example, once regrouping has been introduced with double-digit subtraction, include some problems that do not require regrouping so students do not overgeneralize regrouping to all problems. When fraction multiplication and division are introduced, continue to include previously covered fraction addition and subtraction problems throughout intervention. In doing this, students learn to discriminate when they do not need to find an equivalent fraction as in the case of fraction multiplication, from when they do, as in the case of addition, subtraction, or division of fractions when the denominators are not the same.

Mathematical ideas are complex, and virtually all learners need to use, discuss, and explain the ideas multiple times over a long period of time to understand them. Provide students with opportunities to use and explain previously or newly learned mathematics concepts or procedures throughout the allotted time for intervention. So that students remain actively engaged during all parts of intervention, avoid saving these opportunities for discussion until the end of each lesson.

2. When introducing new concepts and procedures, use accessible numbers to support learning.

Use numbers that are easy for students to understand and work with during initial instruction. When teaching a new concept or procedure, use single-digit or easy-to-understand numbers so that students can focus on the new concept or procedure rather than on difficult calculations. For example, when teaching students to find equivalent fractions, first work on equivalencies to unit fractions. Start with fractions equivalent to one half, one third, and one fourth that are familiar and accessible to students. Use a concrete representation or a number line (see recommendations on representations and number lines for more details). A concrete representation or a number line can help students visualize the equivalence as they compare two fourths to one half, two sixths to one third, and two eighths to one fourth. When students have a grasp of the concept, systematically add other fractions to prevent students from overgeneralizing that equivalencies are only applicable to unit fractions.

3. Sequence instruction so that the mathematics students are learning builds incrementally.

Present mathematics concepts in a cohesive and logical way. Introduce concepts strategically so that the new learning relates to concepts taught previously. In the opinion of the panel, lessons taught on topics in isolation should be avoided during intervention. Instead, lessons should be connected day to day and across units of study to build toward specific learning outcomes. This carefully planned, intentional sequence of instruction capitalizes on prior learning and ensures that students have the knowledge necessary to learn new content effectively.

Focus lessons on smaller tasks needed to solve complex problems before pulling it all together. This may apply to highly procedural multi-digit computation problems or when teaching students to understand and solve word problems. In the view of the panel, the key to building knowledge in this incremental way is to help students become comfortable with smaller subtasks of problem solving so they can eventually connect them to solve complex problems.

One way to do this is to use worked-out examples as a way to focus on smaller tasks. Strategically exclude steps in a worked-out example and ask students to provide those steps until they become more comfortable with the procedures in solving problems. Additional, specific ways to build knowledge incrementally in word problem solving is provided in the recommendation for word problems.

4. Provide visual and verbal supports.

At the heart of all interventions for students who struggle with mathematics are efforts to support student learning. This can be done using visual and verbal supports. Verbal supports may include teacher prompting or questioning to help students attend to and remember the connections between prior learning and the new mathematics they are doing. These verbal supports may be accompanied by a visual, which could include a gesture or a concrete or semi-concrete representation. A visual may also include a picture or diagram to be used as a “hint” for a next step or as a reminder to think about a certain concept.

Each recommendation in this guide offers specific approaches for supporting student learning visually and verbally. For example, the word wall offered in the mathematical language recommendation supports students in providing explanations of the mathematics. Recommendations for representations and number lines offer detailed explanations of how to help students visualize the mathematics concepts they are learning. The word problems recommendation describes the use of strategic prompt cards that support students in completing complex problems.

5. Provide immediate, supportive feedback to students to address any misunderstandings.

If students are not able to explain their understanding of key mathematics concepts or do not execute procedures correctly, provide them with immediate feedback. When students are solving problems, encourage them to articulate their thinking so that you can identify their strengths. Ask probing questions to identify any misconceptions and build on their strengths to correct those misunderstandings. Structure questions in such a way as to help students self-identify where their thinking went wrong. It might be helpful for students to use representations (see representations recommendation) to help them articulate what they are thinking. Correcting misunderstandings early can prevent the confusion from becoming an enduring problem. Tailor feedback to individual students, unless more than one student in a small group setting is struggling with the same misunderstanding.

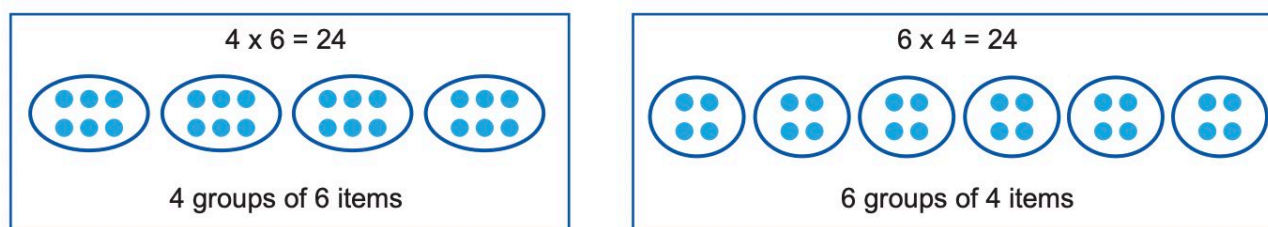
Example 1 depicts one way to address all the steps in this recommendation. Terms in the example that demonstrate steps in this recommendation are **bolded**.

In this example, the learning outcome that has been identified for the small group of 3–6 students is *multi-digit division*. The equal-sized groups model of multiplication and division has been built incrementally and intentionally over multiple lessons using correct mathematical language and visual representations. Students have demonstrated understanding of the equal-sized groups model and how it relates to both multiplication and division. Now, the students are ready to learn how to apply this model to solve multi-digit division problems. The teacher plans to launch her lesson by reviewing students' prior knowledge of multiplication and division concepts to lead into the lesson on multi-digit division.

Example 1. Putting together the steps of Recommendation 1.

The teacher **reviews what the students have previously learned** about the equal-sized groups model (also known as the “groups of” or repeated subtraction model) and how it can be used to solve multiplication and simple division problems. The teacher reminds students of fact families and the inverse relationship between multiplication and division.

After **explicitly reviewing** what students know, the teacher **asks a student to explain** how the equal-sized group model can be used to solve the problem 4×6 . Because the student has **recently practiced** solving multiplication and division problems **with a visual representation**, the student draws 4 circles with 6 dots in each and explains how she created 4 groups of 6 and skip counted to solve the multiplication problem. If needed, the teacher is poised to **prompt the student** if she misses a key point and provide **corrective feedback**. The teacher asks another student to solve and explain the problem 6×4 .



The teacher asks another student to explain how the equal-sized group model can be used to solve the division problem 24 divided by 6. The student draws 24 dots and puts them in equal groups of 6 dots. The teacher **helps the student explain how the problem** can be solved by repeatedly subtracting groups of 6

from 24 to find out how many equal groups of 6 are in 24. Notice that they record the number of groups subtracted on the right side.

$$24 \div 6 = 4$$

$$\begin{array}{r} 4 \\ \hline 6 \overline{) 24} \\ \underline{-6} \quad 1 \\ 18 \\ \underline{-6} \quad 1 \\ 12 \\ \underline{-6} \quad 1 \\ 6 \\ \underline{-6} \quad 1 \\ 0 \end{array}$$

} 4 groups of 6

The teacher **presents students with a variety of problems to solve** with multiplication and division fact families. The students practice solving the problems individually or with a partner. As students share their solutions with the group, the teacher **provides corrective feedback**. When the teacher hears or observes a student missing a key point, the teacher **asks guiding questions to support the students with their explanations and support students' use of mathematically accurate language**.

The teacher **builds on students' prior knowledge** by explaining that the same equal-sized group model used to solve multiplication and related division problems can be used to solve division problems with two-digit divisors. The teacher **reminds the students that they are already familiar** with the equal-sized group model that they use to solve division problems with single-digit divisors. The teacher **presents a worked-out example** of a multi-digit division problem that has been solved using the equal-sized group model using repeated subtraction. The problem uses groups of 12 in the solution strategy that are chosen according to known multiplication facts, **numbers that are familiar and accessible to the students**. The teacher explains each step of the solution strategy and the reasoning behind the steps taken.

$$\begin{array}{r}
 27 \\
 \hline
 12 \overline{) 324} \\
 \underline{- 120} \rightarrow 10 \text{ groups of } 12 \\
 204 \\
 \underline{- 120} \rightarrow 10 \text{ groups of } 12 \\
 84 \\
 \underline{- 60} \rightarrow 5 \text{ groups of } 12 \\
 24 \\
 \underline{- 24} \rightarrow 2 \text{ groups of } 12 \\
 0
 \end{array}$$



$$10 + 10 + 5 + 2 = 27 \text{ groups of } 12$$

Here's what it looks like when students understand the process and record the number of groups subtracted.

$$\begin{array}{r}
 27 \\
 \hline
 12 \overline{) 324} \\
 \underline{- 120} \quad 10 \\
 204 \\
 \underline{- 120} \quad 10 \\
 84 \\
 \underline{- 60} \quad 5 \\
 24 \\
 \underline{- 24} \quad + 2 \\
 0 \quad 27
 \end{array}$$

For the **next worked example**, the teacher **asks students to help explain each step of the solution strategy**. The teacher leads several examples, **asking students to help solve the problem** and discuss their thinking process with the rest of the group. The teacher **prompts students or asks guiding questions** to help the students engage in the problem solving for each problem they solve together. **Students are asked to share the reasoning** for the strategies they are using and the answers they give.

The teacher asks students to solve multi-digit division problems using accessible numbers with a partner. The teacher listens to each discussion and observes the recording of the process, **providing corrective feedback and prompts as necessary**.

The teacher includes **more difficult problems in subsequent lessons as the students gain confidence and competence** in the equal-sized group model to solve multi-digit division problems through repeated subtraction.

Potential Obstacles and the Panel's Advice

OBSTACLE:

"I don't have access to an intervention curriculum in my school. Are you saying I should create my own materials or locate free materials? How do I know if the resources I create or find are systematic?"

EXPERT PANEL'S ADVICE:

The panel is not suggesting teachers create materials that align with the steps in this recommendation. Instead, the panel suggests using these steps as guidelines for evaluating curricula to adopt. Finding materials on your own may be difficult. Work with a team (such as a mathematics coach and special educator) to look for materials that come with a scope and sequence of instruction which build from one lesson to the next to a learning outcome. Evaluate the lesson scope and sequence to determine if there are clear procedures for introducing new content, ample opportunities for students to respond, and built-in feedback procedures.

OBSTACLE:

"I feel like there is so much to cover at every grade level that choosing topics for more intensive instruction or slowing down instruction means I cannot cover all the grade-level material. This feels like I am doing my students a disservice."

EXPERT PANEL'S ADVICE:

Intervention is an opportunity for students to build understanding in the most critical grade-level topics. Students are receiving intervention because they need more time and more frequent work with an adult to learn grade-level mathematics. Structure the pace and topics in intervention in such a way that promotes learning the mathematics more deeply; this often means taking more time.

By collaborating, intervention teachers and general mathematics teachers can ensure that the intervention complements grade-level mathematics instruction. In particular, teachers can identify together what the students in intervention need to work on and understand in order to access grade-level content. Fractions in grades 3 and 4, for example, can be difficult for students and are critical for students to understand for virtually all new mathematics learning through middle and high school. For students with Individualized Education Programs (IEPs), the panel recommends that teachers make sure to look at students' specific goals to guide instructional decisions.

Appendix B: Resources for Card Sorting Routine

This appendix provides reproducible handouts for using the card sorting routine on equivalent fractions.

One-Page Overview of Routine: Fraction Equivalence	37
Equivalent Fractions Card Sets	38
Category Cards	42
Exit Task for Equivalent Fractions	43
Answer Key for Equivalent Fractions Card Sets	44

One-Page Overview of Routine: Fraction Equivalence

Mathematics Focus: **Fraction Equivalence**

Total Time: 25–35 min.

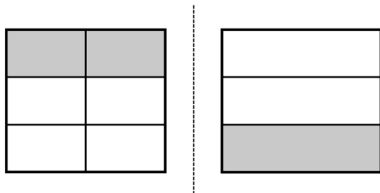
Step 1. Sort cards and explain reasons.	(Whole Group and Pairs) 10–15 min.
<p>a. With the whole group, go over the directions and place one card together.</p> <p>b. Working in pairs, students take turns to sort the cards and explain reasons.</p> <ul style="list-style-type: none"> • One partner picks a card, determines whether the two fractions are equivalent, and places it in the appropriate category: Equivalent or Not Equivalent. • They explain their reasons by using one sentence starter: The fractions are equivalent because... The fractions are not equivalent because... • Then, the partner will respond by using one sentence starter: I agree because... or I disagree because... <p>As pairs work, circulate to ask questions and provide additional support as needed.</p>	
Step 2. Discuss and generalize.	(Whole Group) 10–12 min.
<p>a. Show the answers (p. 44) by revealing one category at a time.</p> <ul style="list-style-type: none"> • Have students check their own work and make corrections as needed. • Ask students which cards they have questions about and discuss them. <p>b. Discuss the examples of equivalent fractions. Ask questions:</p> <ul style="list-style-type: none"> • What do you notice about the pairs of fractions that are equivalent? • For the visual models, how do you know that the fractions are equivalent? • For the numeric fractions, how do you know that the fractions are equivalent? <p>c. Discuss the examples of fractions that are not equivalent. Ask similar questions to 2b.</p>	
Step 3. Wrap up and reflect.	(Individual and Whole Group) 5–8 min.
<p>a. Give students an exit task to complete. Have students share their responses with partners and the whole group if time allows.</p> <p>b. Close the activity by summarizing the mathematics focus on equivalent fractions.</p>	

Equivalent Fractions Card Sets

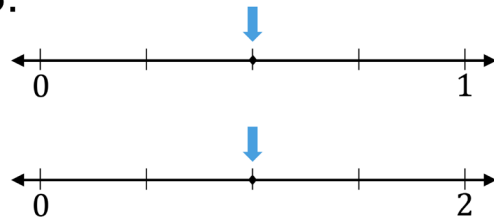
There are two card sets that each have 10 cards. Each set includes four representations of fractions: circles, rectangles, number lines, and numbers. You can choose to use a full deck or select a subset of cards that focus on specific representation(s). Prepare one card deck per pair of students.

Set 1: 10 cards labeled A-J

A.



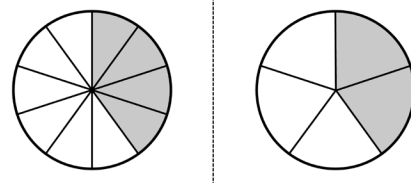
B.



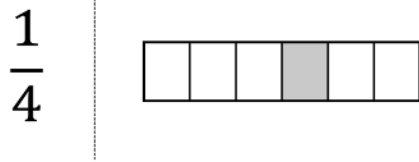
C.

$$\frac{1}{2} \quad \Bigg| \quad \frac{6}{12}$$

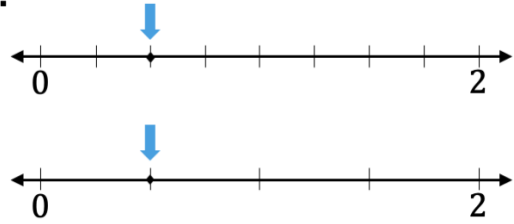
D.



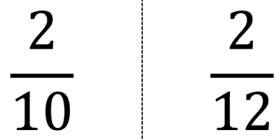
E.



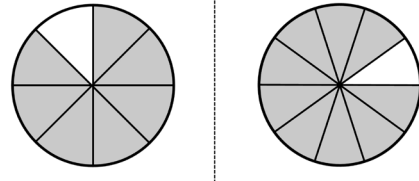
F.



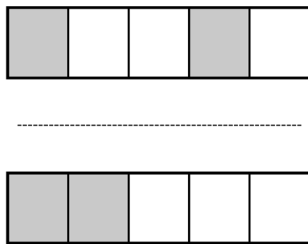
G.



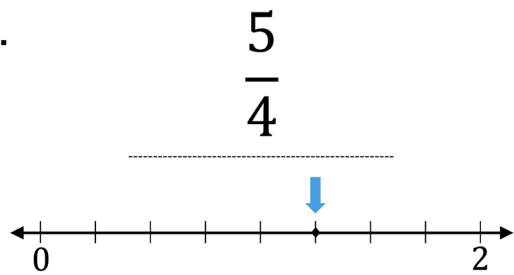
H.



I.

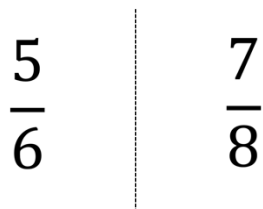


J.

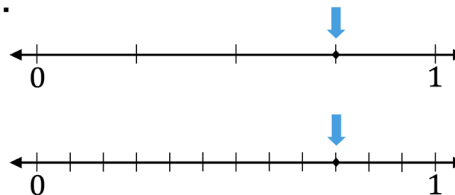


Set 2: 10 cards labeled K-T

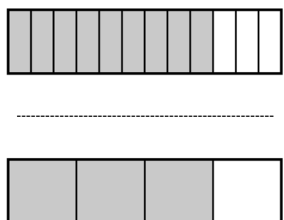
K.



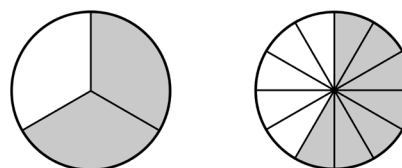
L.



M.



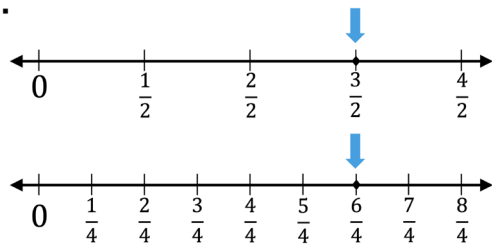
N.



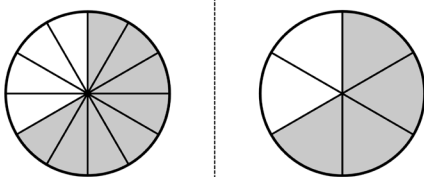
O.

$$\frac{1}{3} \quad \Bigg| \quad \frac{4}{12}$$

P.



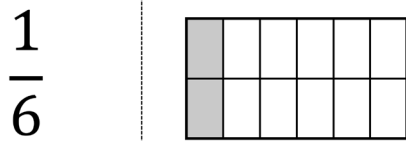
Q.



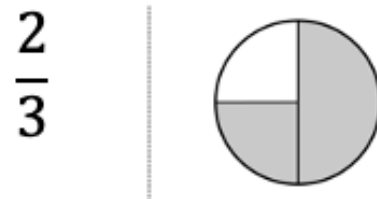
R.



S.



T.



Category Cards

Print and cut out category cards. Prepare two category cards for each pair of students. Another option is to write the words on index cards.

Equivalent	NOT Equivalent
Equivalent	NOT Equivalent
Equivalent	NOT Equivalent
Equivalent	NOT Equivalent
Equivalent	NOT Equivalent
Equivalent	NOT Equivalent
Equivalent	NOT Equivalent

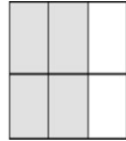
Name: _____

Date: _____

Exit Task for Equivalent Fractions

1. Are these fractions equivalent?

$$\frac{2}{3}$$



The shaded parts
show the fraction.

Select your answer. Explain why in words and/or by making drawings.

- The fractions are ____ equivalent ____ not equivalent.
- I chose this answer because...

2. Are these fractions equivalent?

$$\frac{3}{4}$$

$$\frac{5}{6}$$

Select your answer. Explain why in words and/or by making drawings.

- The fractions are ____ equivalent ____ not equivalent.
- I chose this answer because...

Answer Key for Equivalent Fractions Card Sets

Set 1

Equivalent Fractions (6 cards)	NOT Equivalent Fractions (4 cards)
A	B
C	E
D	G
F	H
I	
J	

Set 2

Equivalent Fractions (6 cards)	NOT Equivalent Fractions (4 cards)
L	K
M	N
O	R
P	T
Q	
S	

Appendix C: Example of a Completed Planner

H10. Planner for Using Card Sorts as Review Activities

Example responses are provided in brackets and dark orange font.

Planning Questions

1. **What? Who?** Plan a topic to review with the students in one or more intervention classes.

- **Mathematics Topic to Review:** [Fraction Equivalence]
- **Intervention class(es):** [Grade 4 intervention class]

2. **When?** Plan when to use the review activity.

- When did students get instruction on this topic? [During the fractions unit in October.]
- When would be a good time to provide a review activity? [In early December.]

3. **Which cards do you want students to sort?**

Look over the card deck(s) for your chosen sort. Some decks have a variety of representations.

- Do you want to use the full deck or focus on specific representations? [Use full Card Set 1.]
- Do you want to add or adapt cards? If yes, what do you want to add or change?

[Add cards L and P from Set 2. We have been focusing on number lines in recent classes, and they were an area of challenge for students, so I want more cards on this topic.]

4. **Prepare to check for understanding during the activity.** What will you look for, listen for, and ask students to find out about their understanding of fraction equivalence?

[Answers will vary. Here are examples:]

- **Look for...**
 - [How students count the fraction bar pieces, the fraction circle pieces, and the number line partitions.]
 - [If students notice when the end points on a number line vary (0 to 1 or 0 to 2).]
- **Listen for...**
 - [Students' understanding about the size of the denominators (e.g., the larger the denominator, the smaller the pieces or partitions are in size).]
 - [Students' reasoning when comparing numerators.]
 - [Students' reasoning when comparing denominators.]

- **Ask these questions...**
 - [What fraction is represented here? *Point to a specific representation on the card.*]
 - [How do you know that these two fractions ___ and ___ are equivalent?]

Reflection Questions (After Using the Review Activity)

1. What did you notice about students' understandings and difficulties with the content?

[Answers will vary. Here are examples:]

- Understandings:
 - [The relationship of $\frac{1}{2}$ to other equivalent fractions (e.g., $\frac{6}{12}$).]
 - [Paying attention to how the size of the pieces in two fractions compare when comparing two visual representations (e.g., just because one piece was missing from a fraction circle or bar, students compared the size of the pieces and didn't overgeneralize that just because one piece was missing, the pieces were equivalent).]
- Difficulties:
 - [When the endpoint of the number line changed from 0 to 1 and 0 to 2, some students did not recognize that the partitions were not eighths but instead fourths.]
 - [Students overgeneralizing by just looking at the location of points on the number line rather than paying attention to the labeling too. It presented a challenge when the point/arrow appeared to be at a different tick mark on the number line because the interval size was different. However, the fractions were equivalent.]

2. What **next steps** will you take to support student learning? (Check all that apply.)

a. Provide more instruction. What will you focus on? What activities/tasks will you use?

- [Varying the endpoints on a number line (not just focusing on 0 to 1).]
- [Identifying fractions on a number line when endpoints are 0 to 2, 0 to 3.]
- [Comparing fractions on number lines.]

[**Activity:** Use the benchmark number line routine with a large number line (made with painter's tape) and fractions on index cards. The interval for this number line will be from 0 to 2. I will include equivalent fractions—both numeric and fraction bars and circles— on cards.]

b. Provide more practice. What activities/tasks will you use?

c. Focus on a different topic. Students show a good understanding of this content.

3. As part of planning systematic instruction, do you want to revisit this content in the future?

Yes, revisit the content in [two weeks, after providing more instruction.]

No