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Supporting Mathematical Problem Solving at Home

Regional Educational Laboratory Northeast & Islands

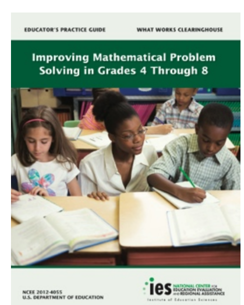
From the National Center for Education Evaluation at IES

Three tips for supporting mathematical problem solving at home

1. Help children get started and reflect on word problems
2. Help children use visual representations
3. Encourage using multiple problem-solving approaches

This guide helps families and caregivers carry out recommended practices described in the What Works Clearinghouse educator's practice guide, *Improving Mathematical Problem Solving in Grades 4 Through 8.*[1]

Download a free copy of the practice guide at https://ies.ed.gov/ncee/wwcPracticeGuide/16



1 The WWC develops Practice Guides with an expert panel, combining the panel's expertise with the findings of rigorous research to produce specific recommendations. The evidence supporting the practice guide recommendations comes from studies examining interventions in school-based settings. Therefore, although the practices described in the tips might be effective in home settings, the studies did not involve parents or caregivers using these practices at home.

Woodward, J., Beckmann, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., Koedinger, K. R., & Ogbuehi, P. (2012). *Improving mathematical problem solving in grades 4 through 8: A practice guide* (NCEE 2012-4055). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education.

All children can learn to be mathematical problem solvers, and children who can tackle problem solving in mathematics are better prepared for more advanced mathematics. Problem solving includes word problems, visual-spatial puzzles, games, and logic problems. Supporting your child in problem solving can enrich their learning of mathematics concepts and help them develop as mathematicians.

The following resources to support children in upper elementary and middle grades to engage productively in mathematical problem solving are based on an educator's practice guide funded by the Institute of Education Sciences at the U.S. Department of Education. The *Improving Mathematical Problem Solving in Grades 4 Through 8* practice guide was developed by bringing together the best evidence and expertise to review research and make recommendations to support mathematics learning and problem solving. The three strategies[2] described below are drawn from the five recommendations (see Appendix 1) made by the panel of mathematics learning experts and researchers. You may also find it helpful to read through the entire practice guide before working with your child.

1. Help Children Get Started and Reflect on Word Problems

Children need a wide variety of mathematical experiences and engaging in problem solving should be a key activity during any time spent on mathematics. Word problems can offer an opportunity for children to think about mathematics in context and consider how mathematics relates to their own lives. However, in order to see success, children need to understand the problem and its context before attempting to solve it.

Children may struggle when they read a math problem the first time. To begin the process of solving the problem, help your child focus on what the math problem is about instead of trying to move too quickly to using an equation or mathematical procedure.

- Children can often solve problems more successfully when they are familiar with the context or when the situation in the problem has been explained to the child.
- Clarify any unfamiliar contexts and draw on children's own experiences as needed.
- Children also learn mathematics better when they connect to the problem, think about what they are doing, and reflect on their problem-solving steps.
    - Talk about what is happening in the problem, not what math operation to use. For example, is the problem about baking cupcakes, counting chickens, or measuring a garden? When children make connections to the context of a task, they can create a picture in their mind and connect to their background knowledge.
    - Making connections also supports children in building upon the mathematical skills and concepts they already know. When children can explain the context, they may be more interested in reasoning through the mathematics of a problem.

Children will see more success learning mathematics as they reflect on their reasoning–thinking about what they are doing and why they are doing it. The more that children reflect on their problem-solving processes, the more that they will be able to apply the same reasoning to new problems. To support your children, prompt them about what they are doing. The *Sample question list* from the problem solving practice guide (below) can help children better understand the problem, determine a way to solve it, and evaluate their solution. Encourage children to explain and justify their response to each prompt in the *Sample question list*, either orally or in writing. As children become more comfortable with reflecting on their problem solving, they can use these prompts independently.

[2] The three strategies are drawn from Recommendation 2, 3, and 4 of the *Improving Mathematical Problem Solving in Grades 4 Through 8* practice guide respectively.

Sample question list[3]

- What is the story in this problem about?
- What is the problem asking?
- What do I know about the problem so far? What information is given to me? How can this help me?
- Which information in the problem is relevant?
- In what way is this problem similar to problems I have previously solved?
- What are the various ways I might approach the problem?
- Is my approach working? If I am stuck, is there another way I can think about solving this problem?
- Does the solution make sense? How can I verify the solution?
- Why did these steps work or not work?
- What would I do differently next time?

The practice guide also suggests that these types of prompts can help children monitor and reflect on their own thought process. You can use the prompts, as shown above, to support your child to become familiar with the problem and begin reasoning about the math problem.

For example, you can have your child read through the problem three times using prompts, which is often referred to as the "Three Reads" process. You can ask your child to read the problem once and answer what the problem is about. You can ask your child to read the problem again and this time ask what the problem is asking them to find out. Finally, you can ask your child to read the problem again and tell you what information in the problem is relevant to solve the problem. Sometimes word problems include numbers and other information that is irrelevant or not needed to solve the problem so taking time to reflect on what is relevant or needed to solve the problem can head off potential mistakes.

1st Read - Context (What is the story in this problem about?) The problem is about…
2nd Read - Purpose (What is the problem asking you to find out?) I need to find out…
3rd Read - Information (Which information in the problem is relevant?) The important information is…

The mathematics problem titled "Basketball Shots" includes these three prompts and specific guidance about how to support your child with the task. (See Basketball Shots Task with Three Reads below).

[3] Sample question list from p.19 of the *Improving Mathematical Problem Solving in Grades 4 Through 8* practice guide.

2. Help Children Use Visual Representations

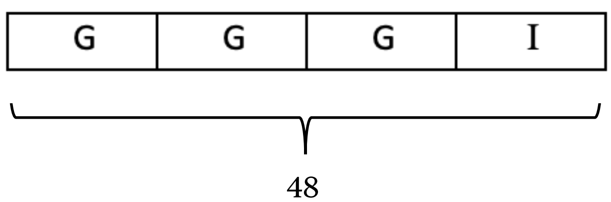
Visual representations help children solve problems by linking the relationships between quantities in the problem with the mathematical operations needed to solve the problem. According to the practice guide, “students who learn to visually represent the mathematical information in problems prior to writing an equation are more effective at problem solving.”[4]

Visual representations include tables, graphs, number lines, and diagrams such as strip diagrams, percent bars, and schematic diagrams. Parents or caregivers, along with the child, can review given materials and then ask the child to explain any representations that are provided and how they demonstrate relationships. Also consider how you or your child could represent your child's thinking about this problem in a drawing or diagram and include the important quantities and how they are related.

For example, consider the following mathematics problem:

Gus and Ike’s combined running distance this week was 48 miles.
Gus ran 3 times as far as Ike.
How many miles did Ike run?

For this problem, a tape diagram can show how far Gus ran and how far Ike ran.



Consider what this tape diagram shows. How does it represent the important quantities in the problem? How does it present information to help find the solution?

In this tape diagram, each segment with a “G” represents the distance Gus ran and segments with an “I” represents how far Ike ran. The total length of the tape diagram can show the 48 miles they ran in total. Knowing that each segment needs to represent the same distance, the problem solver can divide the 48 miles by four and find that Ike ran 12 miles.

Take time now to let the child consider an alternative way to represent this problem yourself. You, as parent or caregiver can ask, “Is there another visual representation that you would draw to solve this task?” Then you can create another visual representation to share with your child.

The mathematics problem titled “Sam’s Motorcycle” with specific guidance about how to support your child with the task is provided. (See Sam’s Motorcycle Task below).

[4] Jitendra et al. (1998); Xin, Jitendra, and Deatline-Buchman (2005); Jitendra et al. (2009); Selke, Behr, and Voelker (1991)

3. Encourage Using Multiple Problem-solving Approaches

Research has found that problem solvers who know how to use multiple strategies to solve problems may be more successful. Solve math problems with your child and then share how you solved the problem with one another–children can benefit from your ideas and sharing their own!

In addition, when children are regularly exposed to problems that require different strategies, they learn different ways to solve problems. As a result, children become more efficient in selecting appropriate ways to solve problems and can approach and solve math problems with greater ease and flexibility.

Teach children that problems can be solved in more than one way and that they should learn to choose between strategies based upon their ease and efficiency. To do this, make sure that children see, compare, and generate multiple strategies.

Solving a problem in multiple ways is important, no matter if the task is a word problem or not. Try the example below.

Mandy and Erica[5] solved the problem differently, but they got the same answer. Why? Would you choose to use Mandy's way or Erica's way? Why?

Mandy's solution		Erica's solution	
$5(y+1) = 3(y+1) + 8$		$5(y+1) = 3(y+1) + 8$	
$5y + 5 = 3y + 3 + 8$	Distribute	$2(y+1) = 8$	Subtract on both
$5y + 5 = 3y + 11$	Combine	$y + 1 = 4$	Divide on both
$2y + 5 = 11$	Subtract on both	$y = 3$	Subtract on both
$2y = 6$	Subtract on both		
$y = 3$	Divide on both		

Both students balanced the equation but took different steps to find the value of y. While Mandy's solution showed how she distributed 5 before combining like terms, Erica subtracted the value of $(y + 1)$ when combining terms. Children should make sure to understand both solutions as well as notice similarities and differences.

Support your child in sharing multiple strategies and share your own strategies as well. When working with your child, take time to discuss similarities and differences among the various visual representations that you create together as well as different problem-solving approaches.

[5] Mandy's and Erica's solutions are presented in Example 15 on page 35 of the *Improving Mathematical Problem Solving in Grades 4 Through 8* practice guide. Consult this practice guide for additional examples of problems and multiple solutions.

Sam's Motorcycle Task

In this task, children reason about rates, specifically in the miles per hour on a motorcycle. The task uses double number lines to explore proportional relationships and present miles per hour visually. Double number lines can be a particularly powerful way to support children in reasoning proportionally and sharing their problem solving.

Learning Goal: Children practice reasoning proportionally and using diagrams to show relationships.

Age Range: 10-13 years old

Materials: Sam's Motorcycle task handout

Activity

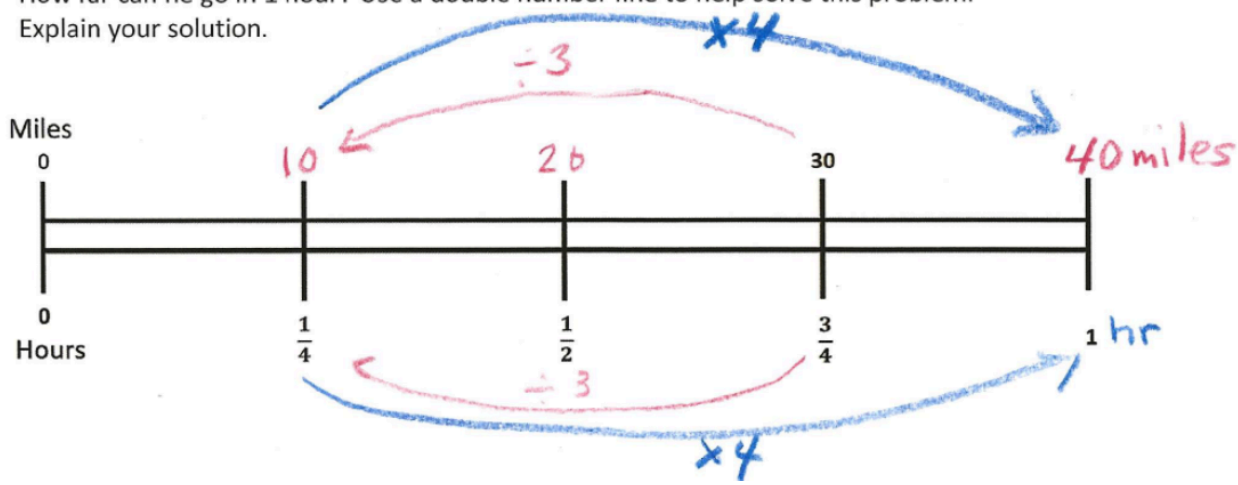
1. Read the Sam's Motorcycle math problem out loud, and then read the question in Part A below. Ask your child to read the numbers on the double number line. Make sure that your child notices that 30 miles is aligned with $\frac{3}{4}$ of an hour.
2. Ask your child to begin solving Part A. To facilitate problem solving, prompt your child to count the number of segments on the top number line and ask how to divide 30 into 3 segments. After dividing, prompt your child to label the number of miles traveled in ¼ of an hour on the top hash mark (10 miles). Next ask your child to label number of miles in ½ hour (20 miles). Based on this information, ask how far Sam can go in 1 hour (40 miles).
3. Have your child read the prompt for Part B or read it aloud. Ask your child what information from the task or from Part A could be used for this part of the task. Prompt your child to segment and label the number line, starting with labeling hours and making sure $3\frac{1}{2}$ hours is on the number line. Your child could use the number of miles in 1 hour, to help solve Part B.

Supporting your child

- Here are some suggestions for you as you facilitate your child's learning.
    - Talk about what "miles per hour" means in a familiar context. What is the speed limit on a nearby highway? What is the speed limit in a school zone?
    - Prompt your child to describe the double number line. How is the number line labeled? How can the number line be divided into equal sized segments?
- Use the double number line to show your own thinking. Ask your child, "What do you see in my diagram? How could you use this double number line to answer the question?
- Review sample answers below:

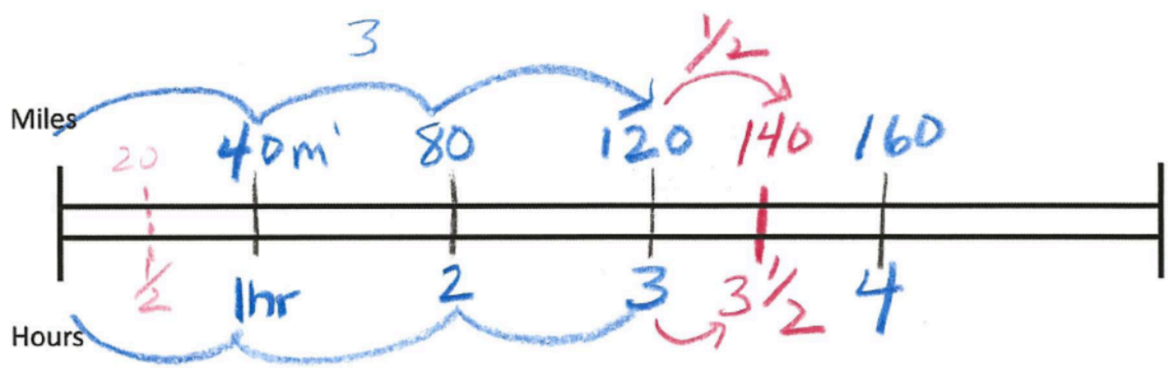
A)

How far can he go in 1 hour? Use a double number line to help solve this problem.
Explain your solution.



B)

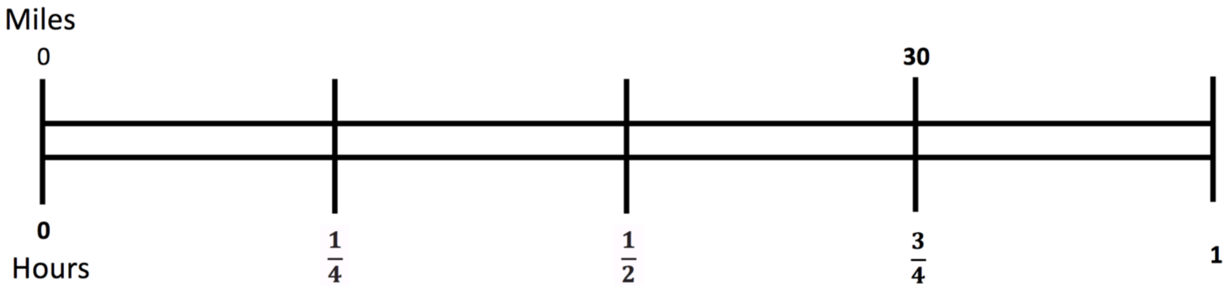
How far can he go in $3\frac{1}{2}$ hours? Use a double number line to help solve this problem.
Explain your solution.



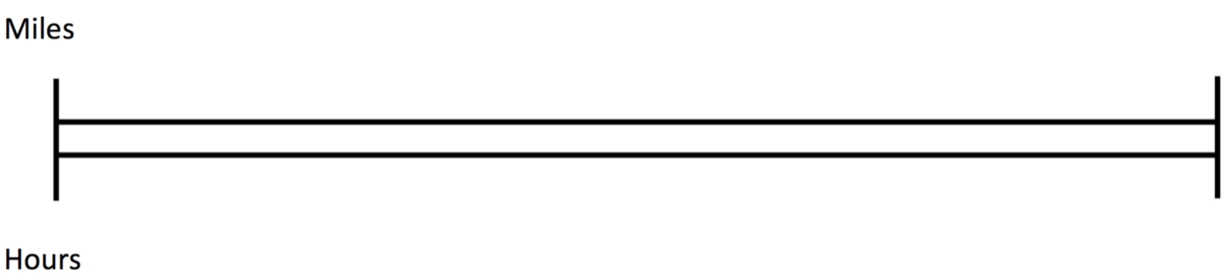
Sam's Motorcycle

Sam bought a used motorcycle. It was on sale because it could not go very fast. Sam was able to go 30 miles in $\frac{3}{4}$ of an hour.

A) How far can he go in 1 hour? Use a double number line to help solve this problem. Explain your solution.



B) How far can he go in $3\frac{1}{2}$ hours? Use a double number line to help solve this problem. Explain your solution.



Basketball Shots Task with Three Reads

In this math problem, children reason about proportional relationships, specifically in the context of playing basketball, looking at the number of shots the home team attempted and the number of shots they scored. The format of this math problem includes three structured readings of the task, where each reading has a different purpose and focus. Reading the task three times and with different purposes facilitates the process of understanding the meaning in word problems and supports learners in linking information in the problem to their work on drawing a diagram to solve the problem.

Learning Goal: Children practice reasoning proportionally and using diagrams to show relationships.

Age Range: 10-13 years old

Materials: Basketball Shots handout

Children are asked to draw a diagram to show their thinking and to demonstrate their reasoning. Diagrams are graphic creations that illustrate relationships among quantities. Examples of diagram types include number lines and tape diagrams. Diagrams are particularly helpful for exploring proportional relationships as diagrams can link quantities and relationships described verbally in the problem to mathematical operations needed to find a solution.

Activity

1. Read the math problem out loud, and then read the first prompt, “What is the story in this problem about?” The **first read** helps your child to *understand the "story"* or context of the text. The child should complete the sentence. They should not focus on the quantities or relationships between them during this reading. Rather, responses should be more like a title for the problem, such as “basketball” or “taking basketball shots.”
2. The purpose of the **second read** is to *identify the question or the goal of the task*. The math problem is read again in its entirety, looking specifically for information about what needs to be answered. Read the second prompt, “What is the problem asking you to find out?” Children should complete the sentence starter, “I need to find out….” Focus your child on what they need to find for an answer, and not just an operation (for example, encourage, “I need to find the number shots they tried to make,” rather than, “I need to subtract/divide”).
3. Read the math problem a final time. The **third read** of the text is *to gather important information that is needed to solve the problem*, such as specific quantities and their relationships. Read the third prompt, “Which information in the problem is relevant?” The child completes the sentence starter, “The important information is….” Make sure

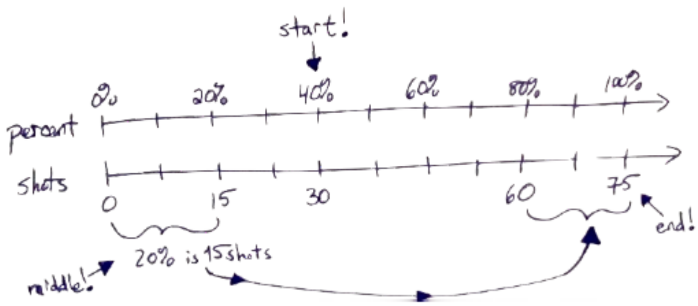
that your child knows the word “attempted,” which is a word they might not know or understand.

4. After completing all three reads, children should draw a diagram to show the important information for the task. Ask your child to discuss what they drew and share their solution.

Supporting your child

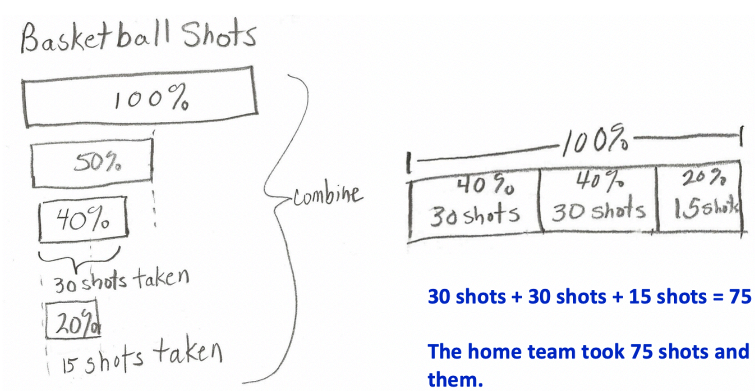
- Here are some suggestions for you as you facilitate your child’s learning.
    - Help your children as they complete the sentence prompts by asking questions about the problem.
    - Ask your children to draw what they can to show what they know about the quantities and relationships in the problem. Encourage them to draw a tape diagram or a number line to show how 40% relates to the 30 shots attempted.
- Draw your own diagram to show your thinking. Ask your child, “What do you see in my diagram?”
- Review sample answers below:

A)



The home team had a total of 75 shots, since this is the number when the percent is 100%.

B)



30 shots + 30 shots + 15 shots = 75 shots.

The home team took 75 shots and made 30 of them.

Basketball Shots

In a basketball game, the home team scored on 30 of its shots.

This was 40% of the shots attempted. How many shots were attempted?

1. Use the Three Reads Strategy for the Basketball task:

1st Read – Context (What is the story in this problem about?) The problem is about…
2nd Read – Purpose (What is the problem asking you to find out?) I need to find out…
3rd Read – Information (Which information in the problem is relevant?) The important information is…

2. Start drawing a diagram that shows the important information from the task and can be used to solve the task.

Appendix 1

This appendix provides the five recommendations identified in the *Improving Mathematical Problem Solving in Grades 4 Through 8* practice guide. These recommendations are evidence-based guidance to support mathematical problem-solving that can be used individually or in combination. It also includes links to the practice guide and related videos.

Recommendation 1. Prepare problems and use them in whole-class instruction.

1. Include both routine and non-routine problems in problem-solving activities.
2. Ensure that students will understand the problem by addressing issues students might encounter with the problem's context or language.
3. Consider students' knowledge of mathematical content when planning lessons.

Recommendation 2. Assist students in monitoring and reflecting on the problem-solving process.

1. Provide students with a list of prompts to help them monitor and reflect during the problem-solving process.
2. Model how to monitor and reflect on the problem-solving process.
3. Use student thinking about a problem to develop students' ability to monitor and reflect.

Recommendation 3. Teach students how to use visual representations.

1. Select visual representations that are appropriate for students and the problems they are solving.
2. Use think-alouds and discussions to teach students how to represent problems visually.
3. Show students how to convert the visually represented information into mathematical notation.

Recommendation 4. Expose students to multiple problem-solving strategies.

1. Provide instruction in multiple strategies.
2. Provide opportunities for students to compare multiple strategies in worked examples.
3. Ask students to generate and share multiple strategies for solving a problem.

Recommendation 5. Help students recognize and articulate mathematical concepts and notation.

1. Describe relevant mathematical concepts and notation and relate them to the problem-solving activity.
2. Ask students to explain each step used to solve a problem in a worked example.
3. Help students make sense of algebraic notation.

Website and Videos

The following links take you to the *Improving Mathematical Problem Solving in Grades 4 Through 8* practice guide in PDF and associated resources, including an eight-page set of instructional tips, videos: https://ies.ed.gov/ncee/wwc/PracticeGuide/16

Instructional tips document:
https://ies.ed.gov/ncee/wwc/Docs/practiceguide/wwc_mps_tips_072517.pdf

Improving Mathematical Problem Solving in Grades 4 Through 8 Background Video, The Components of Problem Solving narrated by John Woodward, University of Puget Sound.
https://www.youtube.com/watch?v=Bx1ajfXsAEg&feature=youtu.be

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