The Importance of Fractions Instruction
Robert S. Siegler, Ph.D., December 2010

DEVELOPING EFFECTIVE FRACTIONS INSTRUCTION FOR K-8

» The panel chair describes the composition of the Practice Guide panel.

» He delineates the sequence of learning about rational numbers across the grades from early elementary to middle school, from concepts of fair sharing through ratio, rate, and proportion.

» He discusses why there is currently so much emphasis on the importance of fractions instruction based on the difficulty that U.S. children experience with fractions operations compared to children in other countries.

» Major misconceptions that are barriers to understanding rational numbers include the tendency to apply knowledge of whole numbers to operations with fractions.

» The U.S. math curriculum is shallow in terms of rational numbers.

» It is important to strengthen teachers’ understanding of rational numbers so that they can build students’ understanding rather than have them rely on memorization.

» Teachers need to work simultaneously on developing students’ conceptual understanding along with procedural knowledge.
My name is Robert Siegler. I’m the Teresa Heinz Professor of Cognitive Psychology at Carnegie Mellon University.

I was the chairman of the panel that wrote the Practice Guide on teaching fractions, and on this panel we had people with a variety of kinds of expertise. We had a mathematician. We had several teachers of mathematics who had won awards for their excellence in teaching. We had several math educators who taught at universities. And we had several psychologists who studied how children learn fractions.

When children come to school they already have some basic understanding of fractions. For example, if you ask them to give you half and me half, they can do that by counting one for you, one for
me, one for you, one for me, and so on. The next advances come when schools start to teach children more about fractions in a somewhat more formal way, and depending on the school district, this might occur in second grade or third grade or fourth grade. And the children start learning some general ideas about fractions, such as what the numerator means and what the denominator means and that increasing numerators means that the fraction gets bigger and increasing denominators means that the fraction gets smaller if you keep the numerator the same size. So they learn these when they’re taught the concepts, and that ranges from second through fourth grade.

By about fourth grade and continuing into fifth and sixth grade, children are learning about fraction arithmetic. They tend to learn addition and subtraction of fractions earlier and multiplication and especially division of fractions later. So by about sixth grade, children have been exposed to not only what fractions are but how to combine them in arithmetic ways. Children also, when they get into sixth grade and increasingly in middle school, are going to be exposed to concepts like proportions and ratios and rates, and they learn to solve those kind of problems primarily in middle school.

There are a variety of reasons why fractions are receiving a great deal of emphasis right now. Probably the most important is that U.S. children do poorly in this aspect of mathematics learning. U.S. children have a great deal of difficulty understanding fractions. This is true in fraction arithmetic, where all four operations present difficulty, where the children often confuse the operations and make mistakes. But it also includes simpler aspects of fractions, for example, locating a fraction on a number line or deciding which of two fractions is bigger than the other. The major misconception that children have with fractions is that they treat them like whole numbers. For example, a problem like $\frac{3}{5} + \frac{5}{6}$, they might give an answer like $\frac{8}{11}$ because they add the numerators and add the denominators as if they were whole numbers.

We don’t stay with any one concept long enough for the children to really master it and understand it deeply. Compared
with other countries, the U.S. teaches about four times as many mathematical concepts in any one year, but obviously when you look at the bottom-line result, they’re not learning four times as much math; they’re actually learning less math.

03:39 In the panel’s research we found that there were a variety of reasons why teachers need professional development in fractions in particular. One reason is that, for many teachers, their understanding of fractions is relatively shallow. They understand that a given procedure such as invert-and-multiply works, but they have little understanding of why it works, why that’s a legitimate thing to do in solving fraction division problems. Without this kind of understanding, they are unable to convince many students that this is a legitimate thing to do, and the students are reduced to a kind of rote memorization. The problem with rote memorization is that people tend to forget it.

04:28 Children cannot learn the procedures very well because they lack the basic conceptual understanding of what fractions are. So if you have no idea how big 12/13 is and how big 7/8 are, you’re as likely to say that 12/13 + 7/8 = 19 or 21 as you are to say that it’s about 2. And, in fact, that’s exactly what happened in a National Assessment of Educational Practice question. Children very often made mistakes on the procedures because they didn’t understand the underlying concepts. In addition to getting a much better conceptual understanding of fractions than children in the U.S. typically get, we also need to help children translate between the conceptual understanding and how that plays out in the arithmetic procedure. And we probably need to give the children more practice with the procedures that are now being built on this solid conceptual base than we typically do at present.