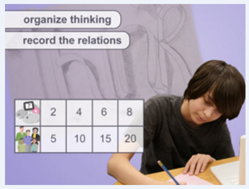
PRESENTATION
7:16 min

Full Details and Transcript



Developing Proportional Reasoning

May 2011

Topic DEVELOPING EFFECTIVE FRACTIONS INSTRUCTION FOR K-8

Practice RATIO, RATE, PROPORTION

Highlights

» Proportional reasoning is critical for success in algebra. Students need lots of practice with the multiplicative relationships of ratios, rates, and proportions *before* they learn cross-multiplication as an algorithm.

» Students need to understand that even though ratios are often expressed as fractions, ratios and fractions are not always the same thing.

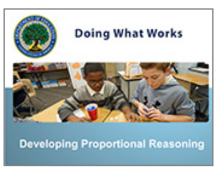
» Proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities stays constant while the values of the quantities change.

» Teachers can demonstrate a buildup strategy to show how to create equivalent ratios by the repeated addition or partitioning of the numbers in a ratio.

» Students can use a ratio table to organize their thinking and record the relations in a proportion problem.

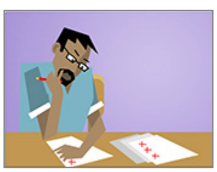
- » In a unit ratio approach, the quantities in a ratio are multiplied or divided by the same factor to maintain the proportional relationship.
- » Once students demonstrate that they can reason their way through problems involving ratios and proportions, they can then be taught cross-multiplication as a procedure that works regardless of the complexity of the numbers within ratios.
- » Cross-multiplication is based on the principle that when two equal fractions are converted into fractions with the same denominator, their numerators are also equal.

Full Transcript



Slide 1: Welcome

Welcome to the overview on Developing Proportional Reasoning.



Slide 2: Mr. Palm’s struggle

Mr. Palm‘s unit on ratios and proportional reasoning hasn’t been going well. He introduced his students to the cross-multiplication procedure, which they seemed to pick up quickly. Over time, however, Mr. Palm notices that his students are not developing a solid understanding of how and when to use cross-multiplication to solve problems. Even his most diligent students are often puzzled when confronted with real-world problems that involve proportions.

Slide 3: Good advice

Proportional reasoning is critical for success in algebra, so Mr. Palm asks the district math coach to help him restructure the unit.

The coach urges him to spend time developing students’ understanding of proportional relations before teaching cross-multiplication. Students need to understand that even though ratios are often expressed as fractions, ratios and fractions are not always the same thing. Students often struggle with this common misconception.

Slide 4: Mr. Palm's plan

Mr. Palm needs to build on and strengthen students' informal proportional reasoning about relationships between quantities and help them use multiple strategies, using different types of visual representations, to solve real-life problems.

Slide 5: Practice and understanding

Students need lots of practice with the multiplicative relationships of ratios, rates, and proportions *before* they learn cross-multiplication as an algorithm.

They need to understand that proportion is a relationship of equality between two ratios. In a proportion, the ratio of two quantities stays constant while the values of the quantities change.

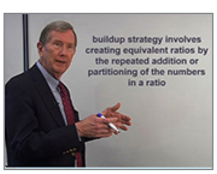
Slide 6: Real-world situations

Fortunately, multiplicative relationships occur all the time in daily life, and most students have an intuitive understanding of how to answer questions about them.

How can you adjust this recipe to serve more people?

Which car gets better mileage?

Which of these is the best buy?



Slide 7: Buildup strategy

Teachers can demonstrate a buildup strategy to show reasoning through questions such as these.

The buildup strategy involves creating equivalent ratios by the repeated addition or partitioning of the numbers in a ratio.

For example, two pounds of fish will serve five people; if we expect 15 people to come for dinner, how many pounds of fish should we buy?

Students might initially tackle that problem with counters or by drawing a diagram.

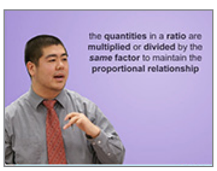
By using a buildup approach, students work up to the solution by repeatedly adding numbers.

Slide 8: Ratio tables

Students can use a ratio table to organize their thinking and record the relations in a proportion problem, helping them see that multiplication leads to the same solution as the buildup strategy.

The ratio table provides a visual representation of how multiplying both numbers in the 2-to-5 ratio by 3 reveals the number of pounds of fish needed for 15 people.

Once students have a good understanding of buildup strategies and ratio tables, teachers can present more challenging problems in which there is an advantage to using a unit ratio strategy.



Slide 9: Unit ratio approach

In a unit ratio approach, the quantities in a ratio are multiplied or divided by the same factor to maintain the proportional relationship.

This strategy is best shown by solving for x in a problem involving ratios without an integral relation. For example, take the problem 3 is to 9 as x is to 21.

The unit ratio approach involves reducing the known ratio to a form with a numerator of 1, that is, changing 3 is to 9 to 1/3. The next step is to determine the relationship between 1/3 and the ratio that includes the unknown factor.

In this case, multiplying both numerator and denominator of 1/3 by a factor of 7 gives us the value of x, which is 7.

The unit ratio strategy also may be recorded in a ratio table.

Slide 10: Real-world problems

Experts recommend that students get practice with applying buildup and unit ratio strategies to real-world problems such as

» Comparing the unit prices of various objects,

» Enlarging or reducing drawings or photos,

» Adjusting quantities in recipes, or

» Calculating the time, speed, and distance traveled by different vehicles.

Slide 11: Student goals

The goals for students are to

» Identify the features of problems that involve ratios and proportions,

» Notice the key information in a problem, and

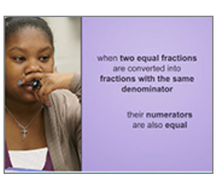
» Create diagrams or tables to track their thinking.

Students need many opportunities to explain their reasoning and compare their solutions to those of other students.

Teachers should encourage use of a variety of strategies and visual representations, including drawings, ratio tables, and double number lines.

Slide 12: Cross-multiplication

Once students demonstrate that they can reason their way through problems involving ratios and proportions, they can then be taught cross-multiplication as a procedure that works regardless of the complexity of the numbers within ratios.



Slide 13: Step by step

Cross-multiplication is based on this principle:

When two equal fractions are converted into fractions with the same denominator, their numerators are also equal.

Consider the ratios of 4 to 6 and 6 to 9.

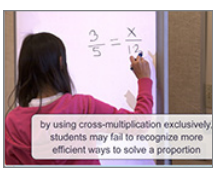
Multiply 4 over 6 by 9 over 9 (which is the same as multiplying by 1); then multiply 6 over 9 by 6 over 6.

Calculate and check that the denominators are equal. If two equal fractions have the same denominator, then the numerators are equal as well.

Slide 14: Understanding strategies

It's very important that teachers show how the cross-multiplication algorithm leads to the same answer as do reasoning strategies such as buildup and unit ratios.

Give students a chance to solve problems using all types of strategies, and discuss which methods are most efficient for different types of problems.



Slide 15: Follow-up

Once they have learned about cross-multiplication, students may attempt to apply the procedure without regard for whether a problem is set up correctly. Also, by using cross-multiplication exclusively, students may fail to recognize more efficient ways to solve a proportion.

It is important to continue to focus on problem structure, labeling of key information, and visual representations so that students develop good problem-solving habits. Students benefit when teachers point out the connections across problems with similar structures but from different contexts.

Slide 16: Conclusion

As always, if students are to develop the critical skill of proportional reasoning, remember what they've learned, and apply their knowledge to solve problems, they need a strong conceptual understanding of what they are doing and why.

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