Effective Problem-Solving Instruction, Part 1: Visual Representations

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**Topic**  
Improving Mathematical Problem Solving in Grades 4 Through 8

**Practice**  
Problem-Solving Instruction

**Highlights**

» Teachers can use specific strategies during problem-solving instruction to build students’ understanding of core mathematics concepts and skills. The three important strategies that apply at all grade levels and in all areas of mathematics are: use of visual representations, encouragement of multiple approaches to solving problems, and linking mathematical and algebraic notation to intuitive approaches.

» Using visual representations prior to introducing equations brings several benefits to students, helping them organize the information in a problem, distinguish relevant from irrelevant information, clarify the goal of the problem, see relationships, and then focus on mathematical reasoning.

» If teachers encourage use of select visual representations consistently, students will more likely apply them on their own and will rely less on drawing narrative pictures. Commonly used representations are tables, strip diagrams, percent bars, and number lines. Strip diagrams are especially useful for comparisons; tables are helpful for ratio and proportion problems.
» Overall, the goal should be to help students articulate mathematically valid explanations of their reasoning.

Full Transcript

Slide 1: Welcome

Slide 2: Opportunities in process

When teaching students how to solve mathematics problems, teachers need to be continually alert for instructional opportunities. As students share their reasoning while working through a challenging problem, observant teachers will have instructional “openings” that can be used to introduce or reinforce familiar mathematics concepts and methods.

Taking advantage of these opportunities to further students’ skills and understanding is at the core of effective teaching.

Slide 3: Three instructional strategies

There are three major strategies that teachers can use to guide students as they learn how to tackle mathematics problems.

» Teach students how to create visual representations of relevant information in a problem,

» Encourage students to explore multiple ways to solve a problem, and

» Demonstrate how to break a problem into steps that can then be expressed through formal mathematical notation.

Slide 4: Summary of strategies

These three core strategies—visual representations, multiple approaches to problems, and mathematical notation—are the teacher's
primary tools for taking advantage of the instructional opportunities available when problem solving.

All three can be applied at all grade levels and with all mathematical topics.

**Slide 5: Visual representations**

Research suggests that students who develop visual representations prior to working with equations are more effective problem solvers. This may be because visual representations help students develop a deeper understanding of the problems they are working with.

The right type of representation can help a student get a coherent view of the problem by identifying and organizing pieces of relevant mathematical information. Specifically, the visualization helps students summarize what key information is known and see what the problem is asking them to solve for.

Use of an appropriate visual can also reveal the relationships between the quantities identified in the problem. Once students grasp these relationships, they can focus their attention on mathematics reasoning and the problem-solving process. Students are also in a better position to express a problem using equations.

**Slide 6: Familiarity with powerful visuals**

Teachers are advised to consistently use a few powerful visual representations. A powerful model or representation is one that has a variety of applications, such as a number line or strip diagram.

If students work with a particular visual representation when they encounter a certain type of problem, they are more likely to grow comfortable with that tool and use it on their own. Students are also then less likely to use narrative pictures, which can distract them from the essential mathematical information in the problem.
Slide 7: Types of visuals

Tables, number lines, strip diagrams, percent bars, and schematic diagrams are among the most frequently used visuals. Schematic diagrams use abstract graphic symbols rather than realistic pictures and include only relevant problem elements. Of course, some visuals are better suited for particular types of problems.

For example, strip diagrams work well for problems that involve comparisons. In this example, Cheri is using a strip diagram to find out how much chili was served for dinner when the family has consumed 3/4 of the chili and there is 1 1/2 cups left.

Tables work well for ratio and proportion problems. Here, Pedro and Sally have drawn a ratio table to figure out how much paint of each color is needed to paint five classrooms if one gallon of yellow and five gallons of beige are required to paint two classrooms.

Slide 8: Talking through the visual

To help students learn how to employ visual representations, teachers can talk aloud about what they are thinking and the decisions they are making as they reason through a problem.

During the thinkaloud, teachers should demonstrate how they identify what information will be placed in the diagram and what aspects of the problem are irrelevant.

It is essential that teachers explain why the representation they are using is appropriate for the problem at hand.

It is just as important for students to explain to the teacher how they are setting up a diagram and representing the quantities in the problem.

By listening to their students’ reasoning, teachers are better able to identify and address possible misconceptions. As teachers grow to
understand how their students are thinking about problems, they will be able to introduce them to the idea that there is more than one way to think about a problem.

**Slide 9: Learn more**

To learn more about Effective Problem-Solving Instruction, please see the other videos on Multiple Strategies and Mathematical Notation in this series.