The Components of Problem Solving

John P. Woodward, Ph.D., August 2011

Topic

IMPROVING MATHEMATICAL PROBLEM SOLVING IN GRADES 4 THROUGH 8

Highlights

» Dr. Woodward explains that grades 4-8 are critical periods in learning to solve problems and that, in countries that do well on math assessments, a significant portion of instruction involves math problem solving.

» Problem solving goes beyond word problems, and includes symbol manipulation and visual analysis.

» Dr. Woodward describes the IES Panel’s recommendations, beginning with planning for problem solving as a central part of whole-class instruction so that it isn’t left to homework or independent seat work. It is important to make sure that problems vary in difficulty level and are appropriate for students’ language and background knowledge.

» The second recommendation is to help students monitor their strategies as they solve problems.

» The third recommendation is related to visualizations (e.g., diagram, number strips) to help break down a problem.

» The fourth recommendation is to ensure that students see and listen to variety of strategies for solving a problem in order to build greater cognitive flexibility.
» The final recommendation is about connecting problem solving to mathematical notation.

» Key issues for teachers to think about include the amount of time allocated to problem solving, attending to the linguistic and cultural background of students, and thinking about the different purposes for using particular problems.

About the Interviewee
John P. Woodward is currently distinguished professor and dean of the School of Education at the University of Puget Sound, in Tacoma, Washington. Since 1984, he has conducted over $12 million of research into the instructional needs of students with disabilities as well as academically at-risk students in elementary, middle, and high school. The majority of his research since 1989 has focused on mathematics education and technology-based instruction. One of his recent projects was a collaborative, five-year program of research that examined methods for helping students with disabilities succeed in standards-based mathematics instruction in grades 4-8. More recent projects have included software development as well as curriculum research in mathematics. His work is cited in considerable detail in the recent Instructional Practices Report from the National Mathematics Panel (2008) and The What Works Clearinghouse publication Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools (2009). He also chaired the recent panel for Improving Mathematical Problem Solving in Grades 4 through 8: A Practice Guide (2011).

Full Transcript
00:05 I’m John Woodward. I’m the dean of the School of Education at the University of Puget Sound, and I was chair of the Math Problem Solving Guide for grades 4 through 8.

00:13 Grades 4 through 8 are critical periods for kids learning mathematics. We even see that by looking at the international data. And one of the things that’s indicative of mathematics instruction in other countries in the world, particularly countries like Japan, that have always done well on math assessments, is that a significant
portion of their classroom instruction across a week’s period of time, for example, involves problem solving.

00:36 Math problem solving has traditionally been thought of as just word problem solving, and that’s true, we can look at an example where students are asked to configure a pen for horses using the same perimeter and different area. But we can also think about math problem solving as symbol manipulation. For example, students can look at different solutions to algebraic equations and talk about the different strategies used to solve those equations.

01:03 Another example is where students are simply given a simple visual setup—how many squares on a checkerboard—and asked to answer that problem using some kind of visual analysis.

01:15 The checkerboard problem is a really good problem, in part because people think the answer is obvious. Anybody who has played checkers or chess, look at the board and say, “Well, okay, let me remember. Yeah, that’s right, they are 8x8, or 64 squares.” But it’s obviously more complicated that when you start to analyze it visually. You’ve got the larger square that frames the checkerboard and then as you think about it further, you see patterns, you see 2x2s, 3x3s, 4x4s. There are all kinds of square patterns inside that checkerboard.

01:50 The panel derived over time five major recommendations that are supported by empirical evidence. But for the first one, where there isn’t a lot of research, all of them have moderate or strong levels of evidence that support their practices.

02:04 One of the first recommendations that’s quite important is that teachers plan for problem solving as part of whole-class instruction. The idea is problem solving is not to be left to homework or independent seat work. That creates its own problem because we can’t specify how many minutes per day or how many hours per week; we simply feel that mathematical problem solving should be a more central portion of the instruction throughout, say, a unit of instruction. So the recommendations begin with planning very clearly for problem solving, making sure that the problems are appropriate to the content
that kids are learning, that they vary in their difficulty level, that they’re sensitive to kids’ background knowledge, their language use, those aspects—before they even begin the problem solving.

02:52 A second recommendation has to do with helping kids monitor their strategies as they solve the problem. This may begin with a task list of some kind, where kids can think about strategies that they might use in solving the problems. We also recommend that teachers in the initial phases of problem solving talk out loud or model how they go about working through a complex problem.

03:16 A third recommendation has to do with the visualizations of problems using appropriate representations. These can be number strips, these can be diagrams; a variety of things come into play that help kids break down a problem and visualize it as they work their way toward a solution.

03:31 A fourth recommendation has to do—and this is really quite critical—that kids spend time seeing and listening to variety of strategies for solving a problem. That’s where that “Aha, I’ve never thought about this before” comes from. It makes sense that students are immersed in their own way of thinking about a problem and solving it and come to think that that’s the only way to solve it. What becomes critical is for kids afterwards to hear different solution strategies. The importance of this is this builds greater cognitive flexibility and different ways of looking at how they can solve problems.

04:06 And a final recommendation has to do with connecting the problem solving with the mathematical notation. A lot of times people think, well, they've solved the problem, they've gone through the process, kids have talked about strategies, but then they forget to connect it to the formal mathematics that kids may be studying at that point in time.

04:23 I can think of a number of key issues that teachers need to consider when they think about problem solving. The first and foremost one is the amount of time they’re going to allocate to the problem solving. That needs to shift. That’s not to say again that there is going
to be 15 minutes or 20 minutes per day, but it needs to have a greater presence across the week, across the unit of instruction. The second one, which also involves planning, has to do with ensuring that you’re meeting and attending to the linguistic background, the cultural background of the students you’re working with. The third thing that I would think that would be very important for teachers to consider is just the types of problems; ask themselves what’s the purpose of the problems that they want to use. For example, should they provide several problems on a new topical area, or is the purpose of problem solving to develop that persistence to get them to be able to work with a problem that’s novel and has an unusual set of characteristics, irrelevant information that they have to struggle with.

05:22  Students need to persist in their problem solving. You see this very clearly by the time they get to high school. So the ability to stick with a problem, deal with aspects of a problem that are unknown or unclear at first is a central part of mathematical problem solving and success in mathematics.