Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students
Practice Guide Summary

Educators' Practice Guide Summary • WHAT WORKS CLEARINGHOUSE™

The three evidence-based recommendations in this WWC practice guide support teachers in helping students develop a deeper understanding of algebra.

Introduction

Algebra moves students beyond an emphasis on arithmetic operations to focus on the use of symbols to represent numbers and express mathematical relationships. Understanding algebra is key to success in future math courses, making it critical to identify strategies that improve algebra knowledge. The *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students* practice guide from the What Works Clearinghouse (WWC) presents three recommendations educators can use to help students develop a deeper understanding of algebra, promote process-oriented thinking, and encourage precise communication. The recommendations in the guide focus on:

- Incorporating solved problems into classroom instruction and activities,
- Utilizing the structure of algebraic representations, and
- Using alternative algebraic strategies when solving problems.

This summary introduces the recommendations and supporting evidence described in the full practice guide. For more practical tips and classroom examples, download your free copy of the guide at http://ies.ed.gov/ncee/wwc/practiceguide.aspx?sid=20.

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Recommendation 1.
Use solved problems to engage students in analyzing algebraic reasoning and strategies.

Compared to elementary mathematics work like arithmetic, solving algebra problems often requires students to think more abstractly and process multiple pieces of complex information simultaneously. Solved problems can minimize the burden of abstract reasoning by allowing students to see the problem and many solution steps at once—without executing each step—helping students learn more efficiently. Analyzing and discussing solved problems can help students develop a deeper understanding of the logical processes used to solve algebra problems. Discussion and the use of incomplete or incorrect solved problems can encourage students to think critically.

Definition and example of a solved problem

Solved problem: An example that shows both the problem and the steps used to reach a solution to the problem. A solved problem can be pulled from student work or curricular materials, or it can be generated by the teacher. A solved problem is also referred to as a "worked example."

Sample solved problem:

Solve for x in this equation: $3^{4x+3} = 81$	$3^{4x+3} = 81$ $3^{4x+3} = 3^4$ $4x + 3 = 4$ $4x = 1$ $x = \frac{1}{4}$

Abstract reasoning is processing and analyzing complex, non-concrete concepts.

How to carry out the recommendation

1. **Have students discuss solved problem structures and solutions to make connections among strategies and reasoning.** Create opportunities for students to discuss and analyze solved problems by asking students to describe the steps taken in the solved problem, explain the reasoning used, and decide whether that strategy is logical and mathematically correct. Foster extended analysis of solved problems by asking students to notice and explain different aspects of a problem's structure. This can help students recognize the sequential nature of solutions and anticipate the next step in solving a problem, improving their ability to understand the reasoning behind different problem-solving strategies.

2. **Select solved problems that reflect the lesson's instructional aim, including problems that illustrate common errors.** Presenting several solved problems that use similar solution steps can help students see how to approach different problems that have similar structures. Use incorrect solved problems to help students deepen their understanding of concepts and correct solution strategies by analyzing strategic, reasoning, and procedural errors. When analyzing an incorrect solved problem, students should explain why identified errors led to an incorrect answer so they can better understand the correct processes and strategies.

3. **Use whole-class discussions, small-group work, and independent practice activities to introduce, elaborate on, and practice working with solved problems.** The practice guide demonstrates many ways to incorporate solved problems into different classroom activities, from introducing a new solution strategy during whole-class discussion to integrating incomplete solved problems into independent practice assignments. See pages 12–14 of the practice guide for more examples.

Incomplete solved problems for independent practice activities

Include incomplete solved problems in students' independent practice, asking students to fill in the blank steps of the solved problems.		
$-x + 7 \geq 9$ $-x \geq 2$ ________	$3(x + 2) + 12 \leq 4(1 - x)$ ________ $3x + 18 \leq 4 - 4x$ $7x \leq -14$ $x \leq -2$	$2(x + 7) - 5(3 - 2x) \geq 7x - 4$ $2x + 14 - 15 + 10x \geq 7x - 4$ ________ $5x \geq -3$ $x \geq -\frac{3}{5}$

Summary of evidence for Recommendation 1

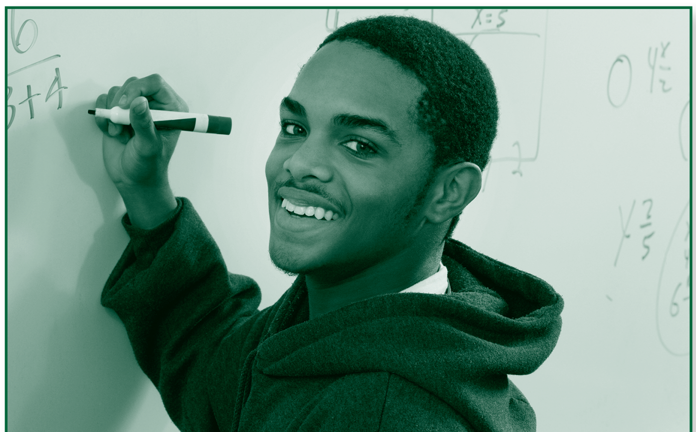
The WWC identified four studies that examined the effects of using solved problems in algebra instruction. Three studies showed positive effects on the conceptual knowledge of students in remedial, regular, and honors algebra classes. The remaining study found that solved problems had negative effects on conceptual and procedural knowledge when comparing students who studied solved problems to students who used reflective questioning (a practice suggested in Recommendation 2).

The studies demonstrating positive effects were promising, suggesting that when compared to asking students to solve practice problems alone, studying solved problems can improve achievement. However, the contexts of these studies were very limited: the use of solved problems was compared to the same instructional approach (additional practice problems)—rather than the diverse approaches used in algebra classrooms—and solved problems were only used for a short period of time in the classroom. This led the WWC to assign a **minimal** level of evidence rating to this recommendation. For more details, see Recommendation 1, page 5 of the practice guide.

Recommendation 2.
Teach students to utilize the structure of algebraic representations.

Paying attention to structure (an algebraic representation's underlying mathematical features and relationships) helps students make connections among problems, solution strategies, and representations that may initially appear different but are actually mathematically similar. An understanding of structure can simplify solving algebra problems, helping students understand the characteristics of algebra expressions and problems regardless of whether the problems are presented in symbolic, numeric, verbal, or graphic forms.

> An **algebraic expression** is a symbol or combination of symbols for variables, numbers, and arithmetic operations used to represent a quantity. Examples of algebraic expressions are $9 - a^2$ and $3x - 4y + 7$.



Consider these three equations:

$$2x + 8 = 14$$
$$2(x + 1) + 8 = 14$$
$$2(3x + 4) + 8 = 14$$

Though the equations appear to differ, they have similar structures:

In all three equations,
2
multiplied by a quantity,
plus 8,
equals 14.

How to carry out the recommendation

1. **Promote the use of language that reflects mathematical structure.** Precise mathematical language communicates the logical meaning of a problem's structure, operations, solution steps, and strategies. Using precise mathematical language is a key component to understanding structure and sets the foundation for the use of reflective questioning, multiple representations, and diagrams. See Examples 2.2 and 2.3 on page 18 of the practice guide for more information on how to model precise mathematical language.

Imprecise vs. precise mathematical language

Imprecise language	Precise mathematical language
Take out the *x*.	Factor *x* from the expression. Divide both sides of the equation by *x*, with a caution about the possibility of dividing by 0.
Move the 5 over.	Subtract 5 from both sides of the equation.
Use the rainbow method. Use FOIL.	Use the distributive property.
Plug in the 2.	Substitute 2 for x.
The numbers cancel out.	The numbers add to zero. The numbers divide to one.

2. **Encourage students to use reflective questioning to notice structure as they solve problems.** By asking themselves questions about a problem they are solving, students can think about the structure of the problem and the potential strategies they could use to solve the problem. For example, "What am I trying to solve for?" Educators will find reflective questions integrated into examples throughout the practice guide.

3. **Teach students that different algebraic representations can convey different information about an algebra problem.** Recognizing and explaining corresponding features of the structure of two representations can help students understand the relationships among several algebraic representations, such as equations, graphs, and word problems. Teachers can present students with equations in different forms and ask students to identify the similarities and differences. As needed, incorporate diagrams into instruction to demonstrate the similarities and differences between representations of algebra problems to students. See Examples 2.7, 2.8, and 2.9 on pages 21–23 of the practice guide for more ideas.

Equations of the same line in different forms

Compare different forms of equations for the same line.

	Similarities	Differences
Slope-intercept form $y = mx + b$ $y = 2x - 3$	Both are equations of straight lines It is easy to see that the slope is 2. It is hard to see what the *x*-intercept is.	**Slope-intercept form** makes it easy to see what the *y*-intercept is.
Point-slope form $y - y_1 = m(x - x_1)$ $y - 5 = 2(x - 4)$		**Point-slope form** makes it easy to see that the point (4, 5) is on the line.

Summary of evidence for Recommendation 2

The WWC identified six studies with diverse samples and settings that examined interventions related to teaching students to utilize the structure of algebraic representations. Two of the studies were conducted outside of the United States, and two study samples included students with specific learning challenges. Four of the six studies found positive effects on procedural knowledge, and three studies found positive effects on conceptual knowledge. However, none of the studies examined an important component of the recommendation: the use of language that reflects mathematical structure. As a result, there is a **minimal** level of evidence to support this recommendation. For more information, see Recommendation 2, page 17 of the practice guide.

Recommendation 3.
Teach students to intentionally choose from alternative algebraic strategies when solving problems.

A strategy involves a general approach for accomplishing a task or solving a problem. By learning from and having access to multiple algebraic strategies, students learn to approach algebra problems with flexibility, recognizing when to apply specific strategies, how to execute different solution strategies correctly, and which strategies are more appropriate for particular tasks. This can help students develop beyond the memorization of one approach, allowing them to extend their knowledge, think more abstractly, and select from different options when they encounter a familiar or unfamiliar problem. Comparing correct solution strategies can help deepen students' conceptual understanding and allow students to notice similarities and differences between problem structures and solution strategies.

How to carry out the recommendation

1. **Teach students to recognize and generate strategies for solving problems.** Provide students with examples that illustrate the use of multiple algebraic strategies, including standard strategies that students commonly use, as well as alternative strategies that may be less obvious. Solved problems can demonstrate how the same

Same problem solved using two different solution strategies*

Strategy 1: Devon's solution—apply distributive property first	
Solution steps	**Labeled steps**
$10(y + 2) = 6(y + 2) + 16$	Distribute
$10y + 20 = 6y + 12 + 16$	Combine like terms
$10y + 20 = 6y + 28$	Subtract $6y$ from both sides
$4y + 20 = 28$	Subtract 20 from both sides
$4y = 8$	Divide by 4 on both sides
$y = 2$	

Strategy 2: Elena's solution—collect like terms first	
Solution steps	**Labeled steps**
$10(y + 2) = 6(y + 2) + 16$	Subtract $6(y + 2)$ on both sides
$4(y + 2) = 16$	Divide by 4 on both sides
$y + 2 = 4$	Subtract 2 from both sides
$y = 2$	

Prompts to accompany the comparison of problems, strategies, and solutions

- What similarities do you notice? What differences do you notice?
- To solve this problem, what did each person do first? Is that valid mathematically? Was that useful in this problem?
- What connections do you see between the two examples?
- How was Devon reasoning through the problem? How was Elena reasoning through the problem?
- What were they doing differently? How was their reasoning similar?
- Did they both get the correct solution?
- Will Devon's strategy always work? What about Elena's? Is there another reasonable strategy?
- Which strategy do you prefer? Why?

*Adapted from Rittle-Johnson and Star (2007).

problem could be solved with different strategies and how different problems could be solved with the same strategy, emphasizing that strategies can be used flexibly. The practice guide offers several examples of how to help students develop skills for selecting which strategy to use and generating alternative strategies (see pages 28–32 in the practice guide).

2. **Encourage students to articulate the reasoning behind their choice of strategy and the mathematical validity of their strategy when solving problems.** Have students describe their reasoning while analyzing the problem structure, determining their solution strategy, solving a problem, and analyzing another student's solution. Describing their reasoning helps students understand the choices they make and goals they set when selecting a strategy. Students should communicate their reasoning verbally and through written work. Educators will find lists of prompts to accompany practice problems or initiate small-group discussion to encourage students to articulate their reasoning throughout Recommendation 3. For example, "What choices did you have to make in solving this problem?"

3. **Have students evaluate and compare different strategies for solving problems.** Encourage students to compare problem structures and solution strategies to discover the relationships among similar and different problems, strategies, and solutions. Use solved problems showing two strategies side by side to enable students to see the number, type, and sequence of solution steps, allowing them to compare solution strategies and consider the accuracy, efficiency, and applicability of various combinations of solution steps. Several examples in Recommendation 3 offer prompts for teachers to incorporate during whole-class discussion or independent work. For example, students can consider the prompt "What connections do you see between the two [problems/strategies/solutions]?"

Summary of evidence for Recommendation 3

The WWC identified six studies that examined the effects of teaching alternative algebraic strategies. Four studies showed positive effects of teaching alternative algebraic strategies, and two studies found negative or mixed effects. The two studies with mixed or negative results involved students with no prior knowledge of algebra, and they compared the use of multiple strategies to the use of just one strategy to solve a problem. These findings indicate that teaching alternative algebraic strategies can improve achievement, especially procedural flexibility, once students have developed some procedural knowledge of algebra. The WWC assigned a **moderate** level of evidence to support this recommendation. For more details, see Recommendation 3, page 27 in the practice guide.

For more practical tips and useful classroom examples, download a copy of the *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students* practice guide at
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