

Developing Effective Fractions Instruction for Kindergarten Through 8th Grade

Instructional Tips for Educators Based on the Practice Guide

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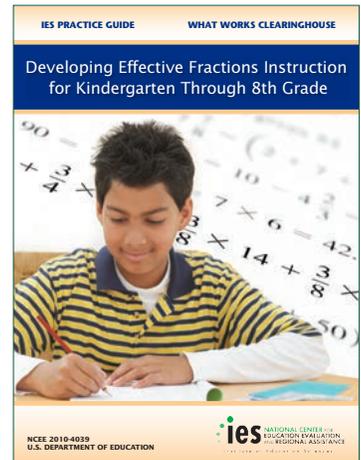


Instructional tips for

- Building on students' informal understanding of sharing and proportionality to develop initial fraction concepts
- Helping students recognize that fractions are numbers that expand the number system beyond whole numbers, and using number lines to teach this and other fraction concepts
- Helping students understand why procedures for computations with fractions make sense
- Helping students develop proportional reasoning skills before exposing them to cross-multiplication

About the WWC Instructional Tips

The instructional tips provide educators with how-to steps for carrying out these evidence-based recommendations from Institute of Education Sciences Educator's Practice Guides. The tips translate these recommendations into actions that educators can use in their classrooms. These tips are supported by research evidence that meets What Works Clearinghouse design standards and are based on a practice guide authored by a panel of experts: Robert Siegler, Thomas Carpenter, Frances (Skip) Fennell, David Geary, James Lewis, Yukari Okamoto, Laurie Thompson, and Jonathan Wray. To learn more about this evidence base, read the practice guide, *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade* (<https://ies.ed.gov/ncee/wwc/practiceguide/15>).



Tip for: Building on students' informal understanding of sharing and proportionality to develop initial fraction concepts

Students come to kindergarten with a rudimentary understanding of basic fraction concepts. They can share a set of objects equally among a group of people (that is, equal sharing) and identify equivalent proportions of common shapes (that is, proportional reasoning). By using this early knowledge to introduce fractions, teachers can build on what students already know. This helps students connect their intuitive knowledge to formal fraction concepts.

Developing Effective Fractions Instruction for Kindergarten Through 8th Grade recommends **using and extending equal-sharing activities** to develop students' understanding of ordering and equivalence of fractions.

Tip: Use equal-sharing activities to introduce the concept of fractions. Use sharing activities that involve dividing sets of objects as well as single whole objects.

- **Offer** a progression of sharing activities that build on students' existing strategies for dividing objects.
- **Ask students** to practice sharing a set of objects equally among a group of recipients.

Example: Have students draw a picture to share a set of objects evenly among recipients

State that 3 children want to share 12 cookies. Emphasize that each child will get the same number of cookies.

Have students draw the children and cookies to show the number of cookies each child will receive if each child received the same number of cookies.

Ask students how many cookies each child received.

Problem: Three children want to share 12 cookies so that each child receives the same number of cookies. How many cookies should each child get?

Examples of solution strategies: Students can begin to solve this problem by drawing three figures to represent the children. Next, students can draw a cookie by the first child, a cookie by the second child, and a cookie by the third child to represent that each child received one cookie. Students can continue to draw cookies next to the three children until they have drawn 4 cookies by each student, or a total of 12 cookies. Students can count the number of cookies next to each child to determine how many cookies each child receives. Other students may solve the problem by simply dealing the cookies into three piles, as if they were dealing cards.



Potential roadblock: Students do not share the cookies equally or they do not share all of the cookies.

Suggested approach: If students do not give each child the same number of cookies, tell students that they want to be fair and share the cookies equally, so that each child gets the same number of cookies. Encourage students to verify that they gave each child the same number of cookies.

If students do not share all of the cookies, help them understand that sharing scenarios require sharing all of the cookies—possibly by noting that each child wants to receive as much as possible—so no cookies are unaccounted for.

- **Ask students** to partition an object or set of objects into fractional parts. The focus of the problems shifts from asking students how many objects each person should get to asking students how much of an object each person should get. For example, when two children share one cookie, students have to think about how much of the cookie each child should receive.

Example: Show students how to divide one object evenly among recipients

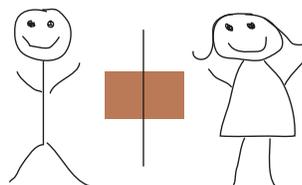
Describe that the two children will each receive the same amount of a chocolate bar to eat.

Ask students to draw how the two children will share the chocolate bar by dividing it equally.

Students might draw one rectangle to represent the chocolate bar and two figures to represent the two children. Students then might draw a line down the middle of the chocolate bar to partition it into two equal parts.

Ask students to use the images they drew to determine how much of the chocolate bar each child received.

Problem: Two children want to share one chocolate bar so that both have the same amount to eat. Draw a picture to show how much each child should receive.



Potential roadblock: Students are unable to draw equal-sized parts.

Suggested approach: Let students know that it is acceptable to draw parts that are not exactly equal, as long as they remember that the parts should be considered equal.

- **Challenge students** to share an object among a larger number of recipients or divide an object into larger or smaller pieces. Encourage students to notice that as the number of people sharing an object increases, the size of each person's share decreases. Ask students to say the amount of each share using formal fraction names and to compare the fractional pieces (for example, $\frac{1}{3}$ of a chocolate bar is greater than $\frac{1}{4}$ of it).

Summary of evidence for this tip

The WWC identified studies on relevant sharing and proportionality activities, but these studies did not meet WWC standards. Despite the limited evidence, the practice guide's panel of experts believes that students' informal knowledge of sharing and proportionality provides a foundation for introducing and teaching fractions concepts.

This tip is based on Steps 1 and 2 of Recommendation 1 in the [Developing Effective Fractions Instruction for Kindergarten Through 8th Grade](#) practice guide, which is to **build on students' informal understanding of sharing and proportionality to develop initial fraction concepts**. To learn more about the recommendation and the evidence, read the practice guide.

Tips for: Helping students recognize that fractions are numbers that expand the number system beyond whole numbers, and using number lines to teach this and other fraction concepts

Instruction to show that fractions are numbers

Understanding that fractions are numbers that provide a unit of measure helps students understand how fractions (1) expand the number system beyond whole numbers and (2) allow for more precise measurement than whole numbers. Teachers can use number lines to visually represent these concepts to students from the early grades onward.

The practice guide [Developing Effective Fractions Instruction for Kindergarten Through 8th Grade](#) recommends **using number lines** to help students understand that fractions are numbers and to improve students' **understanding of fraction concepts**, including **that equivalent fractions describe the same magnitude**, **that there is an infinite number of fractions between any two fractions (fraction density)**, and **that fractions can be negative**.

Tip: Use measurement activities and number lines to help students understand that fractions are numbers with all the properties that numbers share.

- **Ask students** to engage in activities that require they use fractions, rather than whole numbers, to solve problems.
- **Show students** how measuring tools (for example, a measuring cup or a ruler) have a measurement line with tick marks that indicate different values and that some of these values are fractions.
- **Ask students** to use measuring tools to measure objects that do not have a length that is equal to a whole number but to a fraction. This helps students understand that fractions describe quantities.
- **Explain** that fractions are numbers that provide a unit of measure, like whole numbers, but that they can be a more precise unit of measure than whole numbers. For example, if an object is slightly less than 4 inches long, $3\frac{3}{4}$ inches more precisely captures the length of the object.

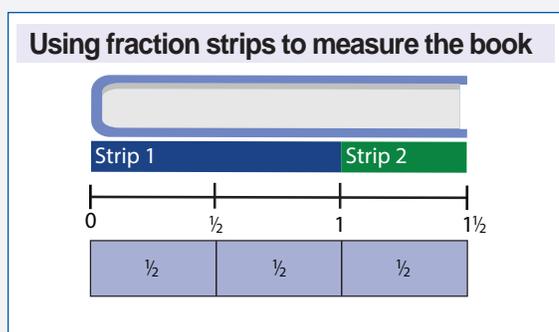
Example: Have students use fraction strips to measure classroom items

Provide students with strips of construction paper that each represent an initial unit of measure. Explain that each strip represents a whole. Ask students to use a whole strip that is the same length as the object to measure it. The object can be a small book, a marker, or a glue stick. Next, ask students to use a different whole strip that is smaller than the object to measure the same object.

When the length of an object is not equal to a whole number of strips, you can provide students with strips that represent fractional amounts of the original strip, such as halves and quarters. For example, a student might use one whole strip and a half strip to measure a small book.

By measuring the same object first using only whole strips and then using fractional strips, students can recognize how the length of the object remains the same but the fractional strips allow for more accuracy in the measurement. Discuss this with students.

By the end of the activity, students should realize that a half strip is equal to one-half the length of the original strip. Label a number line or version of the original full strip with tick marks and fractions to show how the smaller strips relate to the whole.



Potential roadblock: When attempting to partition a number line into three equal-sized parts, students may draw three internal hash marks rather than two or treat the whole number line as the unit and extend the length of the number line.

Suggested approach: Demonstrate that inserting two equally spaced hash marks between 0 and $1\frac{1}{2}$ divides the space into three equal segments, or three equal-sized parts. This rule can be generalized to divide number lines between any two numbers. Students can divide the number line into $\frac{1}{n}$ segments between two numbers by drawing $n - 1$ hash marks between them and making sure the hash marks are equally spaced.

Tip: Use number lines to help students understand fraction equivalence, fraction density, and negative fractions.

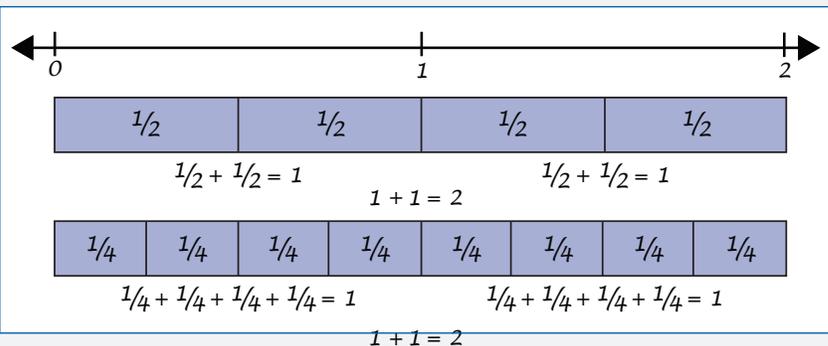
Fraction equivalence

- **Ask students** to look at two fraction strips divided into different fractional parts to show how they can use smaller or larger fractions to describe the same length.

Example: Have students identify the number of parts in each strip

Have students identify the number of parts in each strip. For example, the first strip has four parts, or is divided into fourths. The second strip has eight parts, or is divided into eighths.

Explain how they can use different fractions to describe the same length.

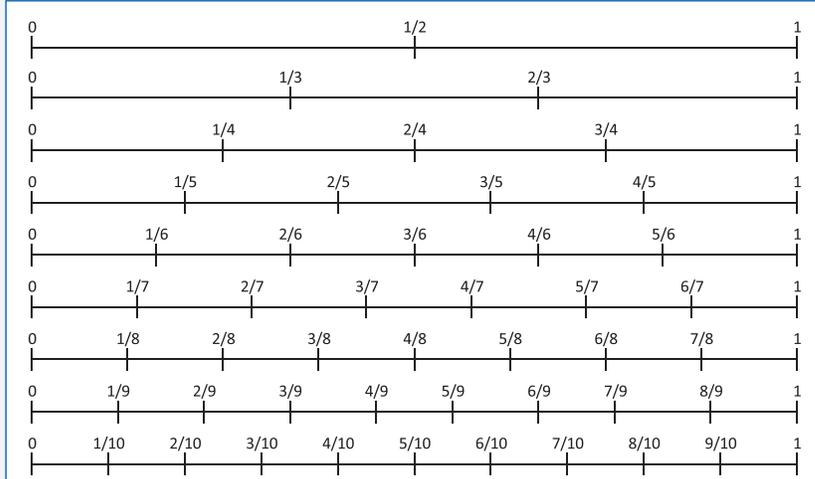


- **Ask students** to look at multiple stacked number lines marked into different fractional parts to show that students can use different fractions to describe the same magnitude.

Example: Have students look at multiple stacked number lines to identify equivalent fractions

Have students use a ruler to identify equivalent fractions, or fractions that are in the same location on the stacked number lines.

Explain to students how different fractions (such as $\frac{1}{2}$ and $\frac{2}{4}$) can describe the same value.



Source: Adapted from Shoseki (2010).

- **Challenge students** to represent the same number as a fraction, decimal, and percentage. Help students locate and compare fractions, decimals, and percentages on the same number line. This shows students that fractions, decimals, and percentages can all be used as different ways of representing the same number.

Potential roadblock: Students have difficulty understanding that two equivalent fractions are the same point on a number line.

Suggested approach: Show students one set of numerical labels above the number line and another set of labels below it. Thus, halves could be marked just above the line and eighths just below it. Point out to students the equivalent positions of $\frac{1}{2}$ and $\frac{4}{8}$, of 1 and $\frac{8}{8}$, of $1\frac{1}{2}$ and $1\frac{4}{8}$, and so on.

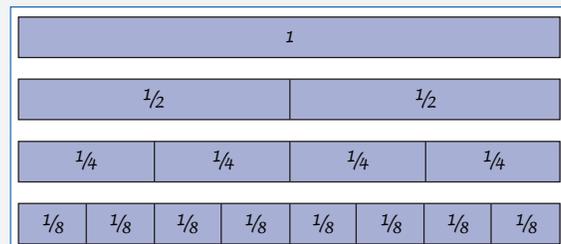
Another approach is for students to create a number line showing $\frac{1}{2}$ and another number line showing $\frac{1}{8}$ and then compare the two. Line up the two number lines and lead students in a discussion about equivalent fractions.

Fraction density

- **Strategically use** visuals to illustrate fraction density, the concept that there is an infinite number of fractions between any two numbers.

Example: Have students mark a number line to illustrate the concept of fraction density

Have students mark a number line to create smaller and smaller fractions to show there is an infinite number of fractions between any two fractions or numbers. For example, students can divide whole number segments in half to create halves, then divide each half into halves to create fourths, then divide each fourth into eighths, and so on.

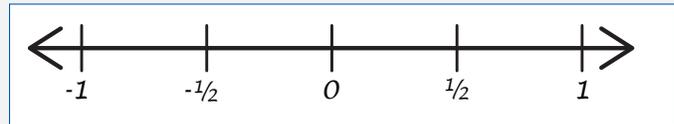


Negative fractions

- **Ask students** to use number lines, which can help students understand negative fractions.

Example: Use number lines to introduce students to negative fractions

Have students examine a number line that shows fractions less than zero and fractions greater than zero. Point out the negative fractions on the number line and explain to



students that a negative fraction can be the same distance away from 0 as a positive fraction. For example, show students how $-\frac{1}{2}$ is the same distance away from 0 as $\frac{1}{2}$ because $-\frac{1}{2}$ and $\frac{1}{2}$ have the same absolute value in the same way that -2 and 2 are the same distance away from 0 because -2 and 2 have the same absolute value.

- **Provide students** with stories that use positive and negative fractions, such as locations above or below sea level or about money gained or lost. Stories with both positive and negative fractions can help teachers illustrate the addition and subtraction of both types of fractions.

Summary of evidence for this tip

Five studies of students in preschool through grade 2 met WWC standards for being well-designed and well-implemented studies. These studies demonstrated that number line activities improved students' understanding of whole numbers; however, none of the studies examined using number lines in fractions instruction. Despite the lack of fractions applications, the panel believes that given the clear applicability of number lines to fractions and whole numbers, these findings indicate that number lines can improve fractions learning for elementary and middle school students.

This tip is based on Steps 1 and 3 of Recommendation 2 in the [Developing Effective Fractions Instruction for Kindergarten Through 8th Grade](#) practice guide. This recommendation focuses on **helping students recognize that fractions are numbers and that they expand the number system beyond whole numbers** and on **using number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward**. To learn more about the recommendation and the evidence, read the practice guide.

Tip for: Helping students understand why procedures for computations with fractions make sense

Students do best using computational procedures when they understand why a particular procedure helps them find the answer. Therefore, it is important to both explain how or why the procedures work to transform the fractions and emphasize the connection between conceptual understanding and the procedures.

Developing Effective Fractions Instruction for Kindergarten Through 8th Grade recommends using **visual representations** to improve students' understanding of formal computational procedures.

Tip: Use area models, number lines, and other visual representations to improve students' understanding of computational procedures.

- **Ask students** to use visual representations to find a common denominator when adding or subtracting fractions.

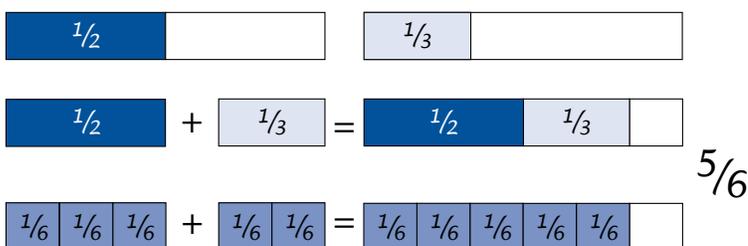
Example: Use fraction bars to show you must find a common denominator when adding or subtracting fractions

Show students visuals of the fractions that represent $\frac{1}{2}$ of the bar and $\frac{1}{3}$ of the bar. Students should see that the two pieces of the bar represent different proportions of the bar, making it difficult to add the two pieces together.

Next, show students visuals of the fractions that converted the $\frac{1}{2}$ piece and the $\frac{1}{3}$ piece of the bar to sixths. Have students count the five pieces of the sixths to get $\frac{5}{6}$.

Explain to students that to add fractions, the fractions need a common denominator.

Adding $\frac{1}{2} + \frac{1}{3}$ using fraction bars



Source: Adapted from Cramer and Wyberg (2009).

Potential roadblock: Students make computational errors. For example, students add fractions without first finding a common denominator.

Suggested approach: Carefully choose representations that demonstrate the need to add similar units and thus lead students to find a common denominator. Representations that hold units constant, such as a measuring tape with marked units, can help students see the need for common unit fractions.

Using some representations can actually reinforce misconceptions. In one study, having students use dot paper or graph paper led to them more often using the incorrect strategy of adding numerators without finding a common denominator. For example, when asked to solve $\frac{1}{4} + \frac{2}{3}$, a student drew four boxes on the dot paper and shaded in one box and then drew three boxes on the dot paper and shaded in two boxes. Rather than find the common denominator, the student added the shaded boxes and came to the inaccurate answer of $\frac{3}{7}$.

- **Ask students** to use visual representations to demonstrate how to multiply fractions. Multiplying two fractions requires finding a fraction of a fraction. For example, when multiplying $\frac{1}{4}$ by $\frac{2}{3}$, students could start with $\frac{2}{3}$ of the original unit and find $\frac{1}{4}$ of this fractional amount. Pictures can help students visualize this process to improve their conceptual understanding of multiplying fractions.

Example: Use pictures to show how to redefine the unit when multiplying fractions

Explain to students that they must first draw how much of the cake 1 cup of icing will cover. The problem says that 1 cup of icing will cover $\frac{2}{3}$ of the cake. Have students divide the cake into 3 equal parts horizontally and then shade 2 of the 3 parts.

Explain to students that this image shows that 1 cup of icing would cover $\frac{2}{3}$ of the cake—but the problem asks how much of the cake $\frac{1}{4}$ cup of icing will cover. To

determine this, tell students they need to divide the cake into 4 parts. They can do this by drawing 3 vertical lines to divide the cake into 4 columns, shown in the second image of the cake.

Then have students use a second color to shade the amount of the iced cake that shows how much of the cake $\frac{1}{4}$ cup of icing will cover. As shown in the third image, students should shade in 2 of the 8 iced pieces of cake, which represents $\frac{1}{4}$ of $\frac{2}{3}$ of the cake.

To determine how much of the cake $\frac{1}{4}$ cup of icing can cover, have students count the number of parts that are shaded in the second color to get the numerator and the total number of parts to get the denominator. Students should answer that $\frac{2}{12}$ of the cake is covered with $\frac{1}{4}$ of the icing.

This approach shows students how to redefine the unit. Students initially treat the full cake as the whole unit when determining how much of the cake can be covered with 1 cup of icing. Then they redefine the unit and treat the $\frac{2}{3}$ of the cake that was covered by the cup of icing as the whole unit when determining how much of the cake can be covered with $\frac{1}{4}$ cup of icing.

Lori is icing a cake. She knows that 1 cup of icing will cover $\frac{2}{3}$ of a cake. How much cake can she cover with $\frac{1}{4}$ cup of icing?

$\frac{2}{3}$ of a cake

$\frac{1}{4} \times \frac{2}{3} = \frac{2}{12}$ of a cake

$\frac{1}{4}$ of $\frac{2}{3}$ of a cake

- **Ask students** to use visual representations to divide a number into fractional parts. Dividing fractions is conceptually similar to dividing whole numbers, in that students can think about how many times the divisor goes into the dividend. For example, $\frac{1}{2} \div \frac{1}{4}$ can be represented in terms of “How many $\frac{1}{4}$ s are in $\frac{1}{2}$?” Representations, like ribbons, can help students model the practice of dividing fractions.

Example: Show students how to divide fractions using representations

Ask students to think about how many times the divisor goes into the dividend. For example, “How many $\frac{1}{4}$ s are in $\frac{1}{2}$?”

Have students cut two ribbons of equal size.

Have students cut one ribbon in half and mark each part of the ribbon with $\frac{1}{2}$.

Have students cut the second ribbon into fourths and mark each part with $\frac{1}{4}$.

Have students find out how many fourths of a ribbon fit onto one-half of a ribbon.

Show students how to solve the problem computationally.

$$\frac{1}{2} \div \frac{1}{4} =$$

$$\frac{1}{2} \times \frac{4}{1} =$$

$$4 \div 2 = 2$$

Using a ribbon to solve $\frac{1}{2} \div \frac{1}{4}$

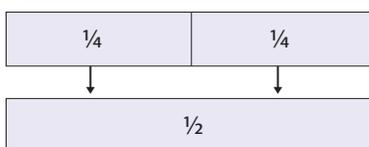
Step 1. Divide a ribbon into halves.



Step 2. Divide another ribbon of the same length into fourths.



Step 3. Find out how many fourths of a ribbon can fit into one-half of the ribbon.



Two fourths fit into one-half of the ribbon, so $\frac{1}{2} \div \frac{1}{4} = 2$.

Summary of evidence for this tip

Studies that meet WWC standards for being well-designed and well-implemented show that using area models, number lines, and other visual representations improve students’ understanding of formal computational procedures. Two studies that meet WWC standards demonstrate that using manipulatives and visual representations, such as fraction squares and fraction strips, improved students’ computational skills with fractions. Three other studies that meet WWC standards reported a favorable effect on computational skills with decimals when students were taught conceptual understanding along with computational skill.

This tip is based on Step 1 of Recommendation 3 in the [Developing Effective Fractions Instruction for Kindergarten Through 8th Grade](#) practice guide, which is to **help students understand why procedures for computations with fractions make sense**. To learn more about the recommendation and the evidence, read the practice guide.

Tip for: Developing students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to solve such problems

Proportional reasoning is a critical skill that prepares students to develop more advanced topics in mathematics. When students think proportionally, they understand the multiplicative relationship between two quantities. For example, understanding the multiplicative relationship in the equation $Y = 2X$ means Y is two times as large as X .

[*Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*](#) recommends developing students' understanding of proportional relations before teaching computational procedures.

Tip: Develop students' understanding of proportional relations by building on students' developing strategies for solving ratio, rate, and proportion problems.

- **Strategically use** a progression of problems that build on students' ability to develop strategies for proportional reasoning. Pose problems that allow students to use the additive strategy (also known as the buildup strategy) and the unit ratio strategy and progress to problems that are easier to solve through cross-multiplication.
- **The additive strategy.** Pose story problems that have students repeatedly add the numbers within one ratio to solve the problem. Ensure that the first problems involve ratios for which students can easily apply an additive strategy, such as the example below.

Example: Using the additive strategy to solve a problem

Sample problem. If Steve can purchase 3 baseball cards for \$2, how many baseball cards can he purchase with \$10?

Solution approach. Students can build up to the unknown quantity by starting with 3 cards for \$2 and repeatedly adding 3 more cards and \$2, thus obtaining 6 cards for \$4, 9 cards for \$6, 12 cards for \$8, and finally 15 cards for \$10.

- **Present** similar problems as shown in this example but use a larger number for the unknown quantity: for example, "If Steve can purchase 3 baseball cards for \$2, how many baseball cards can he purchase with \$24?" Using similar problems with larger quantities will show students the advantage of multiplying and dividing rather than depending upon repeated addition.

- **The unit ratio strategy.** Present problems that students cannot solve through repeated addition or through multiplying or dividing a given number by a single integer, such as the one in the following example. These problems encourage students to use the unit ratio strategy, in which they multiply or divide quantities in a ratio by the same factor to maintain the proportional relationship.

Example: Using the unit ratio strategy to solve a problem

Sample problem. Yukari bought 6 balloons for \$24. How much will it cost to buy 5 balloons?

Solution approach. Students might figure out that if 6 balloons cost \$24, then 1 balloon costs \$4.

To solve this problem, students must find the cost of one balloon, which they can do by dividing 24 by 6 = 4. Then they multiply \$4 by 5 balloons to find that 5 balloons cost \$20.

- **Present** problems as shown in this example that require students to use similar reasoning, but for which the answer is not a whole number. For example, “Susan is making dinner for 6 people and wants to use a recipe that serves 8 people. The recipe for 8 people calls for 2 cups of cream. How much cream will she need to serve 6?” Students could solve this problem by reasoning that because 2 cups of cream serve 8 people, 1 cup of cream would serve 4 people, and $1\frac{1}{2}$ cups of cream would serve 6.
- **The cross-multiplication strategy.** Present problems that students cannot solve through repeated addition or when it would be difficult to use the unit ratio strategy, like with the following example. These problems encourage students to use cross-multiplication, in which they multiply the numerator of each (or one) side by the denominator of the other side to create equivalent ratios. Specifically, when faced with a problem that can be written as an equality of two ratios $a/b = c/d$, the problem can be rewritten using cross-multiplication as $(a \times d = b \times c)$.

Example: Using the cross-multiplication strategy to solve a problem

Sample problem. Luis usually bikes the 2 miles to his school in 25 minutes. However, today one of the streets on his usual path is being repaired, so he needs to take a 3-mile route. If he bikes at his usual speed, how much time will it take him to get to his school?

Solution approach. This problem can be solved in two stages. First, because Luis is biking at his “usual speed,” students know that $\frac{2}{25} = \frac{3}{a}$. Second, use cross-multiplication to solve for a, or the amount of time it would take Luis to reach school.

$$(25 \times 3) = (2 \times a)$$

$$\frac{(25 \times 3)}{2} = \frac{(2 \times a)}{2}$$

$$2 \qquad 2$$

$$37.5 = a$$

Therefore, it would take Luis 37 minutes and 30 seconds to reach school using the route he took today.

- **Explaining why cross-multiplication works.** Show students that when they convert two equal fractions into fractions with the same denominator, the numerators are also equivalent. Explain that the cross-multiplication procedure is a way to create equivalent denominators.

Example: Showing why cross-multiplication works

Step 1. Start with two equal fractions, for example $\frac{2}{3} = \frac{4}{6}$.

Step 2. Find a common denominator using each of the two denominators.

- First, multiply $\frac{2}{3}$ by $\frac{2}{2}$, which is the same as multiplying $\frac{2}{3}$ by 1 because $\frac{2}{2} = 1$.
- Next, multiply $\frac{4}{6}$ by $\frac{1}{1}$, which is the same as multiplying $\frac{4}{6}$ by 1 because $\frac{1}{1} = 1$.

Step 3. Calculate the result: $\frac{(2 \times 2)}{(3 \times 2)} = \frac{(4 \times 1)}{(6 \times 1)}$

Step 4. Check that the denominators are equal. If two equal fractions have the same denominator, then the numerators of the two equal fractions must be equal as well; thus, $2 \times 2 = 4 \times 1$.

Note that $2 \times 2 = 4 \times 1$ is the result of applying the $(a \times d = b \times c)$ cross-multiplication strategy.

As a result, students can see that the original proportion, $\frac{2}{3} = \frac{4}{6}$, can be solved using cross-multiplication, $2 \times 2 = 4 \times 1$, as a procedure to create equivalent denominators efficiently.

Potential roadblock: Many students misapply the cross-multiplication strategy.

Suggested approach: Carefully present several examples of the type shown in the example above. These examples can help students understand the logic behind the cross-multiplication procedure and why the ratios within the problem must be in the correct form for the procedure to work. Making sure that students understand the logic of each step in the demonstration takes time, but it can prevent many future errors and misunderstandings.

Tip: Encourage students to use visual representations to solve ratio, rate, and proportion problems.

- **Encourage** students to use visual representations that are likely to elicit insight into a particular aspect of ratio, rate, and proportion concepts, for example, a ratio table.

Example: Using a ratio table to solve a proportion problem

Sample problem. A recipe calls for 1 cup of flour to serve 8 people. How many cups of flour are needed to serve 32 people?

Cups of flour	1	2	3	4
Number of people served	8	16	24	32

Solution approach. Students can use the ratio table to see that multiplying the ratio of 1 cup of flour to serve 8 people, or $\frac{1}{8}$ by $\frac{4}{1}$ (in other words, 4 times the recipe) provides the amount of flour needed for 32 people.

Using the ratio table, explain to students that multiplication leads to the same solution as the additive strategy.

Potential roadblock: Students do not generalize strategies across different ratio, rate, and proportion contexts.

Suggested approach: In addition to providing students with problems across a variety of contexts and teaching a variety of rate, ratio, and proportion problem-solving strategies, link new problems with previously solved ones and have students judge when the same solution strategy could be used for different types of problems. For example, demonstrate how information in two types of problems, such as recipes and mixture problems, can be organized in the same way and then compare solution procedures for the two types of problems side by side.

- **Encourage students to create** their own visual representations to solve a broad range of ratio, rate, and proportion problems.

Summary of evidence for this tip

Three studies that meet WWC standards for being well-designed and well-implemented studies demonstrated that manipulatives and visual representations can be beneficial teaching tools for teaching proportions to students in grades 4, 5, and 7. Three additional studies showed that students use a variety of strategies when solving proportional reasoning problems, but these studies did not meet WWC standards because they did not examine whether instruction on these strategies improved students' learning.

This tip is based on Steps 1 and 2 of Recommendation 4 in the [Developing Effective Fractions Instruction for Kindergarten Through 8th Grade](#) practice guide, which is to **develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems.** To learn more about the recommendation and the evidence, read the practice guide.